



Universal Multiple Octet Coded Character Set
International Organization for Standardization
Organisation internationale de normalisation
Международная организация по стандартизации

Doc Type: Working Group Document
Title: Additional Mathematical Symbols
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Status: Expert contribution
Action: For consideration by JTC1/SC2/WG2 and UTC

The set of mathematical symbols proposed for addition to ISO/IEC 10646-1 and Unicode is based on an extensive search of existing mathematical literature. By its very nature, such a search can never be exhaustive, least of all for a notation that is a living and productive as mathematical notation. In order to have a workable proposal, an initial set was frozen at around the time it was first submitted to WG2.

In the process of refining the initial set, some much needed character candidates were found to have been inadvertently dropped. In parallel to work on refining this initial set, the literature search progressed and generated a list of additional candidate characters.

An ad-hoc group consisting of the authors of this document, with input from Michael Everson, Ken Whistler and Murray Sargent, has narrowed these candidate characters down to the list presented here and provided suggested code locations, names and annotations for use in the Unicode Standard.

Format of this document

While the format of this document follows that of a Unicode character names list, many of the annotations are provided mainly for the readers and reviewers of this document; they are not intended to become part of the standard.

In some cases compatibility ‘mappings’ with novel tags can be found in the names list. These are indications of glyphic or semantic relations between characters, and not suggested formal decompositions. In the final standard they would be replaced by simple cross-references.

Use of Mathematical Symbols with MathML

Since the original proposal that is now part of PDAM1, the W3C has essentially completed its work on Version 2.0 of MathML and many vendors are busy implementing support for it. In moving away from a model that is entirely based on SGML-style entities towards stronger reliance on character codes MathML makes reference to the characters in PDAM1, but also needs to be able to map some of the existing legacy entity sets to character codes. Some of the characters in this proposal are urgently needed for this purpose. Adding these characters to the standard quickly, enables W3C to proceed with MathML without having to add cumbersome temporary work-arounds for these entities.

Proposed code Allocation

Where characters can fill existing holes in the standard (or in PDAM1) the proposed code allocation is indicated directly in the list of characters. Characters that would fit into new blocks are shown with temporary code positions that use private use space.

Three new blocks are to be created to contain the following sequence of characters:

- *F540 – F55C Additional Math Operators
- *F580 – F597 Supplemental Geometric Shapes
- *F570 – F57F Supplementary Arrows-A

The actual location of these blocks is expected to be in the range 2B00 – 2E7F. The code points for the characters proposed here will then have to be adjusted to match the agreed upon location of the blocks.

Notes on proposed characters

2063 INVISIBLE SEPARATOR

The invisible separator (comma) is intended for use in index expressions and other mathematical notation where two adjacent variables form a list and are not implicitly multiplied. In mathematical notation, commas are not always explicitly present, but need to be indicated for symbolic calculation software to help it disambiguate a sequence from a multiplication. For example in the following sequence:

$$\sigma^2 \boxed{,}^n$$

is distinct from

$$\sigma^{2n}$$

which could be mis-interpreted as the singly superscripted

$$\sigma^{(2n)}$$

instead of the functionally equivalent

σ^{2,n}

which is doubly superscripted as intended. Use of the invisible comma would hint to a math layout program to typeset a small space between the variables.

A fuller discussion of the uses of adjacency in mathematical notation can be found in the forthcoming Unicode Technical Report #28, *Unicode Support for Mathematics*.

This character is currently supported in widely selling mathematical software.

2052 COMMERCIAL MINUS SIGN

The use of the commercial minus sign is attested in commercial or tax related forms or publications in German, where it is pronounced “abzüglich” and is also cited in other sources for the same usage in Scandinavia. The *./.* form of this character appears to be something of a fallback representation; while there are citations of both forms in the same publication, they occur with different fonts and without apparent change in meaning. Therefore it is proposed to unify these two forms. If a requirement can be demonstrated for accessing the second form explicitly it can be added as a glyph variation accessible via VS1.

Additional usage: the sign appears in the Finno-Ugric Phonetic Alphabet (FUPA) to mark a structurally related borrowed element of different pronunciation. In Finland, the dingbats *ℳ* and *ℹ* are always used for “correct” and “incorrect” respectively in correcting a student’s paper. This contrasts with e.g. American practice, where *✓* and *✗* might be used for “correct” and “incorrect” respectively in the same context. The symbol may also appear as marginal note in letters, denoting enclosures. One variation replaces the top dot with a digit indicating the number of enclosures.

Since use of this character is not part of standard mathematical notation, it is proposed to encode the character in the General Punctuation block.

F549 MULTIPLICATION ON-LINE

Unlike the period (full stop) this dot is used as an operator, indicating multiplication. The glyph needs different spacing from the glyph for period and the character is explicitly an operator and not sentence terminal punctuation.

Note that use of the full stop in mathematical expressions is not uncommon in its two functions of decimal point and sentence terminal punctuation. The latter use occurs when a displayed expression ends a sentence.

F54A ON-LINE DOT

Unlike F549, this is used in pairs as a fence. Fences have different spacing than other characters of otherwise similar appearance and need to be handled specially in math layout. Glyphs are usually designed with different alignment and spacing.

F576 – F57F Long Arrows

The long forms of arrows are commonly used in mathematical papers. Encoding of these via the VS1 character had been considered, but they are proposed as separate characters here because they represent distinct semantics, rather than mere stylistic glyph differences. One example of a semantic distinction is that the shorter forms are used in connection with limits and the longer with mappings. Their use is so common that they were assigned entities in the ISOAMSA entity set, one of the several mathematical symbol entity sets ISO defined.

MathML is an XML application, and MathML is intended to support the full legacy collection of the ISO math entity sets. This is in part because publishers who use forms of the ISO 12087 standards for mathematics would reasonably expect it to. The STIX project of the STIPUB group of publishers included those ISO sets in its collection of mathematical characters that formed the source list submitted to the UTC for standardization.

These characters are needed to complete the mapping to the entity set in question.

29D8 LEFT WIGGLY FENCE

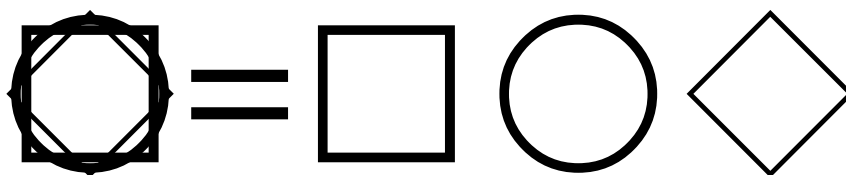
The proposed character looks superficially similar to FE34 PRESENTATION FORM FOR VERTICAL LOW LINE, which is intended for legacy support as an ‘underlining’ character in vertical context. It has a compatibility mapping to 005F in Unicode. This represents a very different use and properties from the standard use of fence characters in mathematical notation — therefore the characters cannot be unified.

29E6 LARGE SQUARE WITH BLACK MEDIUM SMALL CIRCLE

This character looks superficially like a larger version of 22A1 SQUARED DOT OPERATOR – but the latter is unambiguously an operator, whereas the proposed LARGE SQUARE WITH BLACK MEDIUM SMALL CIRCLE is an object and larger.

Geometrical Shapes

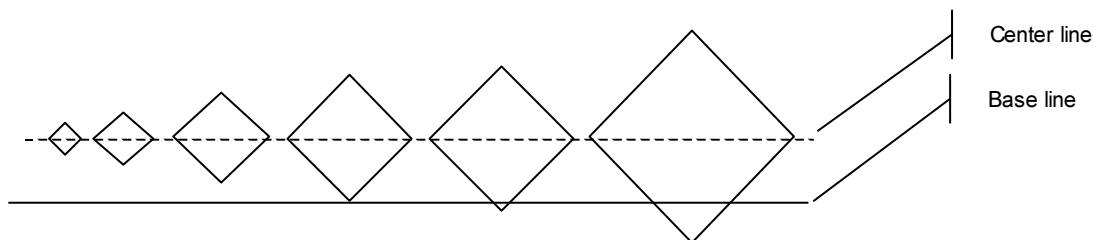
Ideal sizes. Mathematical usage requires at least four distinct sizes of simple shapes, and sometimes more. The size gradation must allow each size to be recognized, even when it occurs in isolation. In other words shapes of the *same* size should ideally have roughly the same visual “impact” as opposed to same nominal height or width. For mathematical usage simple shapes ideally share a common center. The following diagram shows which size relationship *across* shapes of the same nominal size is considered ideal.



Please note that neither the current set of glyphs in the standard nor the glyphs from many commonly available non-mathematical fonts show this kind of size relation.

Actual sizes. The sizes of existing characters and their names are not always consistent. For mathematical usage, therefore, the MEDIUM SMALL SQUARE should be used together with the MEDIUM size of the other basic shapes, and correspondingly for the other sizes. The basic shapes from the ZapfDingbats font match the unmarked size for triangle, diamond and circle and the MEDIUM size for the square. To achieve the correct size relation, mathematical fonts may need to deviate in minor amounts from the sizes shown in the character charts.

Positioning: For a mathematical font, the centerline should go through the middle of a parenthesis, which should go from bottom of descender to top of ascender. This is the same level as the minus or the middle of the plus and equal signs. For correct positioning, the glyph will descend below the baseline for the larger sizes of the basic shapes as in the following schematic diagram:



The standard triangles used for mathematics are also center aligned. This is different from the positioning for the reference glyphs of existing characters shown in the charts. Mathematical fonts may need to deviate in positioning of these triangles.

Relative sizes of simple geometrical shapes

Size	Circles	Squares	Diamonds	Lozenges	Triangles
Large					
Unmarked					
Medium					
Med.Small					
Small					
Very Small	/ /				
Tiny					

Size	Circles	Squares*	Diamonds	Lozenges	Triangles
Large	25EF	2588	F588	F591	
Unmarked	25CF	25A0	25C6	29EB / 25CA	25B2
Medium	F59A	25FC	F58B	F594	F580
Med. Small	2981	25FE	F58D	F596	
Small	2022 / 25E6	25AA	F58F	22C4* / F598	25B4
Very Small	2219 / 2218 / 00B7*	F59C			
Tiny	22C5*				

Characters already coded or in the PDAM are shown in black in the upper table with bold code points in the lower table. Where a white symbol is shown the black symbol is not currently encoded, or the relation between white and black symbol is not obvious from the name. Symbols in blue correspond to the current proposal.

The light green comparison glyphs are from the ZapfDingbats font.

The left column in both tables gives a size indication. Where possible it matches the size indication in the character name. Sometimes the names do not indicate a size, in which case, the characters have been interfiled according to relative sizes of their glyph (e.g. as done for the characters that consist of smaller circles).

Notes:



- DIAMOND OPERATOR is a misnomer for *SMALL LOZENGE
- The squares are consistently one size larger than their nominal size
- Circles and dots occupy the same series. 00B7 may also be rendered in tiny size

	200	201	202	203	204	205	206
0							
1							
2						2052 %	
3							2063 ,
4							
5							
6							
7							
8							
9							
A							
B							
C							
D							
E							
F							

- 2052 ∕. COMMERCIAL MINUS SIGN
= abzüglich (German), med avdrag av (Swedish), piska (Swedish, "whip")
- a common glyph variant and fallback representation looks like ./.
 - may also be used as a dingbat to indicate correctness
 - used in Finno-Ugric Phonetic Alphabet to indicate a related borrowed form with different sound
- 0025 % percent sign
→ 066A ∕. arabic percent sign











Invisible Operator

- 2063 ∕. INVISIBLE SEPARATOR
= invisible comma
- contiguity operator indicating that adjacent mathematical symbols form a list, e.g. when no comma is used between multiple indices

	20D	20E	20F
0			
1			
2			
3			
4			
5			
6			
7			
8			
9		 20E9	
A		 20EA	
B			
C			
D			
E			
F			

Combining characters

- 20E9 ⌘ COMBINING WIDE BRIDGE ABOVE
= contraction operator
• this character extends the full width of the base character
→ 0346̄ combining bridge above
- 20EA ⇐ COMBINING LEFTWARD ARROW OVERLAY












	268	269	26A	26B	26C	26D	26E	26F
0	 2680							
1	 2681							
2	 2682							
3	 2683							
4	 2684							
5	 2685							
6	 2686							
7	 2687							
8	 2688							
9	 2689							
A								
B								
C								
D								
E								
F								

Dice

- 2680 ☐ DIE FACE-1
- 2681 ☐ DIE FACE-2
- 2682 ☐ DIE FACE-3
- 2683 ☐ DIE FACE-4
- 2684 ☐ DIE FACE-5
- 2685 ☐ DIE FACE-6

Go markers

- 2686 ○ WHITE CIRCLE WITH DOT RIGHT
- 2687 ○ WHITE CIRCLE WITH TWO DOTS
- 2688 ● BLACK CIRCLE WITH WHITE DOT RIGHT
- 2689 ● BLACK CIRCLE WITH TWO WHITE DOTS

	298	299	29A	29B	29C	29D	29E	29F
0								
1								
2								
3								
4								
5								
6								
							29E6	
7								
		2997						
8								
		2998				29D8		
9								
						29D9		
A								
						29DA		
B								
						29DB		
C								
							29FC	
D								
							29FD	
E								
							29FE	
F								
							29FF	

Brackets

- 2997 (LEFT BLACK TORTOISE SHELL
BRACKET
≈ <black> 3014 [left tortoise shell bracket
- 2998) RIGHT BLACK TORTOISE SHELL
BRACKET
≈ <black> 3015] right tortoise shell
bracket

Fences

- 29D8 † LEFT WIGGLY FENCE
→ FE34 ⎵ presentation form for vertical
low line
- 29D9 † RIGHT WIGGLY FENCE
→ FE34 ⎶ presentation form for vertical
low line
- 29DA †† LEFT DOUBLE WIGGLY FENCE
- 29DB †† RIGHT DOUBLE WIGGLY FENCE

Other Symbol

- 29E6 ◻ LARGE SQUARE WITH BLACK
MEDIUM SMALL CIRCLE
→ 29C7 ◻ squared small circle
→ 22A1 ◻ squared dot operator










Brackets

- 29FC < LEFT POINTING CURVED ANGLE
BRACKET
→ 2329 < left pointing angle bracket
- 29FD > RIGHT POINTING CURVED ANGLE
BRACKET
→ 232A > right pointing angle bracket

Other Symbols

- 29FE + TINY
- 29FF - MINY

	2A0	2A1	2A2	2A3	2A4	2A5	2A6	2A7
0								
1								
2								
3								
4								
5								
6								
7								
8								
9								
A								
B								
C								
D								
E							$\frac{*}{=}$	
F							2AGE	

	2A8	2A9	2AA	2AB	2AC	2AD	2AE	2AF
0								
1								
2								
3								
4								
5								
6								
7								 2AF7
8								 2AF8
9								 2AF9
A								 2AFA
B								 2AFB
C								 2AFC
D								 2AFD
E								 2AFE
F								 2AFF

2A6E \approx^* EQUALS WITH ASTERISK

Relations

2AF7 \lll STACKED VERY MUCH LESS-THAN
 \approx <stacked> 22D8 \lll very much less-than

2AF8 \ggg STACKED VERY MUCH GREATER-THAN
 \approx <stacked> 22D9 \ggg very much greater-than

2AF9 \leqslant VARIANT LESS-THAN OVER EQUAL TO
 \approx <variant> 2266 \leqslant less-than over equal to

2AFA \geqslant VARIANT GREATER-THAN OVER EQUAL TO
 \approx <variant> 2267 \geqslant greater-than over equal to

2AFB \equiv TRIPLE SOLIDUS BINARY RELATION
 = triple slash binary relation
 → 2AF4 \equiv triple vertical bar binary relation


























Operators

2AFC \equiv LARGE TRIPLE VERTICAL BAR OPERATOR
 often n-ary
 → 2AF4 \equiv triple vertical bar binary relation
 → 2980 \equiv triple vertical bar delimiter

2AFD \parallel DOUBLE SLASH OPERATOR
 = tangential to

2AFE \perp WHITE VERTICAL BAR
 = Dijkstra choice

2AFF \perp N-ARY WHITE VERTICAL BAR
 = N-ary Dijkstra choice

	F54	F55
0	 F540	 F550
1	 F541	 F551
2	 F542	 F552
3	 F543	 F553
4	 F544	 F554
5	 F545	 F555
6	 F546	 F556
7	 F547	 F557
8	 F548	
9	 F549	
A	 F54A	
B	 F54B	
C	 F54C	
D	 F54D	
E	 F54E	
F	 F54F	

Relation

F540 \equiv GLEICH STARK
= tautological equivalent

F556 \leftarrow LONG LEFT TACK
F557 \uparrow UP TACK WITH CIRCLE ABOVE
= radial component
 \rightarrow 2AF1 \downarrow down tack with circle below

Operators

F541 \wedge AND WITH DOT
F542 \diamond LOZENGE DIVIDED BY HORIZONTAL RULE
F543 \cup ELEMENT OF OPENING UPWARDS
 \rightarrow 2AD9 \cap element of opening downwards
F544 \lrcorner LOWER RIGHT CORNER WITH DOT
= pullback
F545 \ulcorner UPPER LEFT CORNER WITH DOT
= pushout
F546 \bowtie LEFT OUTER JOIN
F547 \bowtie RIGHT OUTER JOIN
F548 \bowtie FULL OUTER JOIN

Operator

















F549 \cdot MULTIPLICATION ON-LINE
= often omitted
 \rightarrow 2062 \cdot invisible times
 \rightarrow 22C5 \cdot dot operator
 \rightarrow 2024 \cdot one dot leader
 \rightarrow 002E \cdot full stop
 \rightarrow F54A \cdot on-line dot

Fence

F54A \cdot ON-LINE DOT
= paired, used as fence
 \rightarrow F549 \cdot multiplication on-line

Operators

F54B \diamond WHITE CONCAVE-SIDED DIAMOND
= never (modal operator)
F54C \diamond WHITE CONCAVE-SIDED DIAMOND WITH LEFTWARDS TICK
= was never (modal operator)
F54D \diamond WHITE CONCAVE-SIDED DIAMOND WITH RIGHTWARDS TICK
= will never be (modal operator)
F54E \square WHITE SQUARE WITH LEFTWARDS TICK
= was always (modal operator)
F54F \square WHITE SQUARE WITH RIGHTWARDS TICK
= will always be (modal operator)
F550 \perp LARGE UP TACK
 \approx \langle large \rangle 22A5 \perp up tack
F551 \top LARGE DOWN TACK
 \approx \langle large \rangle 22A4 \top down tack
F552 \equiv LEFT AND RIGHT DOUBLE TURNSTILE
F553 \mp LEFT AND RIGHT TACK
F554 \leftarrow LEFT MULTIMAP
 \rightarrow 22B8 \rightarrow multimap
F555 \leftarrow LONG RIGHT TACK
= discrete Fourier transform

	F57
0	 F570
1	 F571
2	 F572
3	 F573
4	 F574
5	 F575
6	 F576
7	 F577
8	 F578
9	 F579
A	 F57A
B	 F57B
C	 F57C
D	 F57D
E	 F57E
F	 F57F



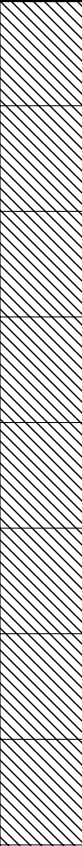































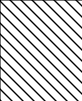
Arrows

- F570 \Updownarrow UPWARDS QUADRUPLE ARROW
 F571 \Downarrow DOWNWARDS QUADRUPLE ARROW
 F572 \circlearrowleft ANTICLOCKWISE GAPPED CIRCLE ARROW
 → 21BA \curvearrowleft anticlockwise open circle arrow
 F573 \circlearrowright CLOCKWISE GAPPED CIRCLE ARROW
 → 21BB \curvearrowright clockwise open circle arrow
 F574 \rightarrow RIGHT ARROW WITH SMALL CIRCLE
 F575 \oplus RIGHT ARROW WITH CIRCLED PLUS

Long Arrows

These are used for mapping whereas the short forms would be used in limits. They are also needed for MathML to complete mapping to the ASMA sets.

- F576 \longleftarrow LONG LEFTWARDS ARROW
 F577 \longrightarrow LONG RIGHTWARDS ARROW
 F578 \longleftrightarrow LONG LEFT RIGHT ARROW
 F579 \Leftrightarrow LONG LEFTWARDS DOUBLE ARROW
 F57A \Rrightarrow LONG RIGHTWARDS DOUBLE ARROW
 F57B \Leftrightarrow LONG LEFT RIGHT DOUBLE ARROW
 F57C \longleftarrow LONG LEFTWARDS ARROW FROM BAR
 = maps from
 F57D \longrightarrow LONG RIGHTWARDS ARROW FROM BAR
 = maps to
 F57E \Leftrightarrow LONG LEFTWARDS DOUBLE ARROW FROM BAR
 F57F \Rrightarrow LONG RIGHTWARDS DOUBLE ARROW FROM BAR

	F58	F59	F5A	F5B
0	 F580	 F590		
1	 F581	 F591		
2	 F582	 F592		
3	 F583	 F593		
4	 F584	 F594		
5	 F585	 F595		
6	 F586	 F596		
7	 F587	 F597		
8	 F588	 F598		
9	 F589	 F599		
A	 F58A	 F59A		
B	 F58B	 F59B		
C	 F58C	 F59C		
D	 F58D	 F59D		
E	 F58E	 F59E		
F	 F58F			

Triangles

Triangles exist in regular and small sizes. An additional intermediate size is needed.

- F580 ▲ BLACK UP-POINTING MEDIUM TRIANGLE
 F581 △ WHITE UP-POINTING MEDIUM TRIANGLE
 F582 ► BLACK RIGHT-POINTING MEDIUM TRIANGLE
 F583 ▷ WHITE RIGHT-POINTING MEDIUM TRIANGLE
 F584 ▼ BLACK DOWN-POINTING MEDIUM TRIANGLE
 F585 ∇ WHITE DOWN-POINTING MEDIUM TRIANGLE
 F586 ◀ BLACK LEFT-POINTING MEDIUM TRIANGLE
 F587 ◁ WHITE LEFT-POINTING MEDIUM TRIANGLE

Diamonds

Diamonds are needed in several sizes, including one larger size. The sizes here match the sizes of the lozenges. Widely available fonts contain black diamonds in six sizes down to tiny.

- F588 ◆ BLACK LARGE DIAMOND
 F589 ◇ WHITE LARGE DIAMOND
 F58A ⬠ WHITE DIAMOND CONTAINING BLACK SMALL DIAMOND
 F58B ◆ BLACK MEDIUM DIAMOND
 F58C ◇ WHITE MEDIUM DIAMOND
 F58D ◆ BLACK MEDIUM SMALL DIAMOND
 F58E ◇ WHITE MEDIUM SMALL DIAMOND
 F58F • BLACK SMALL DIAMOND
 F590 ◊ WHITE SMALL DIAMOND

Lozenges

Lozenges are needed in several sizes, including one larger size. The sizes here are chosen to interpolate between the two already coded sizes. Widely available fonts contain black diamonds in six sizes down to tiny.

- F591 ◆ BLACK LARGE LOZENGE
 F592 ◇ WHITE LARGE LOZENGE
 → 25CA ◊ lozenge
 → 29EB ◆ black lozenge
 F593 ⬠ WHITE LOZENGE CONTAINING BLACK SMALL LOZENGE
 F594 ◆ BLACK MEDIUM LOZENGE
 F595 ◇ WHITE MEDIUM LOZENGE
 F596 ◆ BLACK MEDIUM SMALL LOZENGE
 F597 ◊ WHITE MEDIUM SMALL LOZENGE
 F598 • BLACK SMALL LOZENGE
 → 25C4 ◀ diamond operator

Circles

Circles exist in many sizes, but there is one gap corresponding to medium size. The medium size here corresponds to the size of the inner circles in 25C9 and 25CE. Medium small and large circles only exist in one color.

- F599 ● BLACK MEDIUM CIRCLE
 F59A ○ WHITE MEDIUM CIRCLE
 F59B ◦ WHITE MEDIUM SMALL CIRCLE
 → 2981 • z notation spot
 F59C ● LARGE BLACK CIRCLE
 → 25EF ○ large circle

Squares

For the same size designation, the size of encoded squares are one step larger than those of other shapes. A very small size is needed to match up with the small sizes for lozenge, diamond, triangle and circle.

- F59D ▪ BLACK VERY SMALL SQUARE
 F59E ◻ WHITE VERY SMALL SQUARE