

Exploratory proposal to encode the Kaktovik numerals

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Introduction. This proposal explores the addition of the numeral system adopted by indigenous communities in Alaska and (potentially) elsewhere in the arctic region.

Origins and design principles. According to [Wikipedia](#), the numbering system emerged in the town of Kaktovik, Alaska, on December 1994. The teacher was William Clark Bartley, his students expressed their dissatisfaction with base 10 numerals, when their language expressed numeric quantities in base 20.

At first they tried making distinct glyphs for the digits ten through nineteen, but it resulted cumbersome and slow to remember all the shapes and values.

The teacher had a background on linguistics and so the small class settled on a few design principles:

- 1) The symbols should be *easy to remember*.
- 2) There should be a *clear relationship between the symbols and their meanings*.
- 3) It should be *easy to write* the symbols. For example, being able to be written without lifting the pencil, and should be able to be *written quickly*.
- 4) They should *look very different from Arabic numerals* so there would not be any confusion between the two systems.
- 5) They should be pleasing to look at.

Requirement 2 enforces the featural nature of the system, whilst requirement 3 enforces features common in cursive systems (this only applies to the glyphs of each digit, i.e. they don't join by default with adjacent characters).

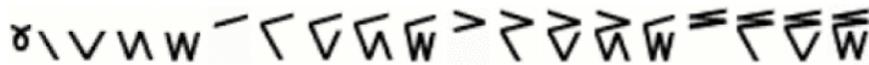
Description. The number system is base 20, which reflect the nature of the Iñupiaq language. The design of the glyphs follows a sub-base 5, with the number and position of the strokes following a logical progression to construct them (this is all very similar to Mayan numerals).

All digits greater than zero, are composed by two kinds of stroke: semi-vertical and semi-horizontal. They aren't truly vertical or horizontal, so that they can be drawn in zigzag patterns (ensuring that they can be written without raising the pen). Only the digit zero is different, having the shape of a small loop.

Each semi-vertical stroke represents an increment of one, and they can be grouped up to four. After four, a semi-horizontal stroke is drawn in the upper part of the character cell. Each semi-horizontal stroke represents an increment of five and up to three of them can form part of a digit. The value of the digit is deduced by multiplying the amount of strokes by their respective multipliers and summing both amounts.

When it comes to stroke order, the priority is always granted to the top-most semi-horizontal strokes; which may start at either the upper right or upper left of the cell. For digits that are less than five (or for when there are no more semi-horizontal strokes to draw), priority is given to the leftmost stroke; with these always starting from the top right, but always leaving extra space above, for the possible ad-hoc addition of semi-horizontal strokes.

Glyph design considerations. Here is a font of the 20 digits, as it appears in a [Tex discussion thread](#). We shall use it for the remainder of the document.



Notice how the slant of the strokes with respect to their analogues, may increase as the value increases. This is desirable if one wants to keep the advance width between digits constant, but don't want to leave "ugly" whitespace padding. Fonts may have consistent slanting of their strokes, if the widths of the characters are inconsistent (as it happens in many fonts that show the digit 1 as narrower than other digits).

It is important, that the digits 1-4 have the extra space above, where semi-horizontal strokes would have been drawn. This is in order to avoid the non-intuitive situation, where the digits 1-4 appear taller than the larger digits.

Digit	Glyph	No. of Strokes
ZERO	ø	1
ONE	\	1
TWO	√	2
THREE	∞	3
FOUR	W	4
FIVE	∟	1
SIX	∟	2
SEVEN	∟	3
EIGHT	∟	4
NINE	∟	5
TEN	∟	2
ELEVEN	∟	3
TWELVE	∟	4
THIRTEEN	∟	5
FOURTEEN	∟	6
FIFTEEN	∟	3
SIXTEEN	∟	4
SEVENTEEN	∟	5
EIGHTEEN	∟	6
NINETEEN	∟	7

As one can see, the stroke count is always between 1 and 7, with an average of 3½.

Name. Here I discuss various possible names. This is perhaps the most contentious part of the proposal, since any name proposed here was suggested based on the limited information online. Since I have no way of contacting the intended user community, I hope this exploratory proposal will be enough to reach out to the artic communities for their feedback.

- **Kaktovik Inupiaq numerals:** This name is the one that is used on the internet, and presumably the current favorite. It makes reference to both the town of origin as well as the language it was made to represent.

However, it doesn't seem like a good idea to use this on the character names, since there is simply no precedent for such a naming scheme. The name implies there is either more than one type of Kaktovik numerals or more than one type of Iñupiaq numerals. There is no need to have the two specifiers at the same time, all it does is make unnecessarily long names that would inconvenience software developers.

This does not at all imply that the long name would suddenly be "forbidden" or recommended against, but simply that displaying the longer name would be left to the front-end (as it always should). It is quite common to have characters names, that don't reflect the common name for the character in the user community (even if we only count English language names).

Having a more neutral name, would also invite other communities to adapt the numerals, even if they aren't from Kaktovik or Iñupiaq speakers.

- **Inupiaq numerals:** This makes reference to the language it was meant to, but not to the town (note that while the correct spelling is Iñupiaq, Unicode character names can only include ASCII letters, which excludes "ñ"). While this runs the risk of excluding other artic languages communities, it may well be the case that they will adopt the numerals anyway, and they won't feel hampered on using alternatives names on their languages.

Nevertheless I still believe it would be counter-productive to go for such a name. The numerals best qualities are broader than the Iñupiaq language per se.

- **Kaktovik numerals:** This makes references to the town they are from, but not the language. Such a name would be unprecedented, since geographically based names, typically make reference to bigger geographical area than a single town.

The name however, has the benefit of preserving the due credit to the original creators, which would be guarded by the stability policies of Unicode. For this reason, it is unlikely that any other person that isn't from Kaktovik, would interpret this name as being exclusionary.

- **Alaskan numerals:** This is also based on geography, but by covering a much bigger area, it does have precedents on the naming schemes. While being more inclusive than the previous name, it still excludes Canadians and Greenlanders.
- **Arctic numerals:** This is not a good suggestion. While including the entire probable user base, it may have the unintended consequence of associating the numerals, with other things that shouldn't be necessarily related to the users: Like extreme cold, seal hunting, and many more.
- **Inuit numerals:** This option and the "Kaktovik numerals" option are my personal favorites.

Inuit is a blanket term, intended for indigenous people of the artic region. It is both a reference to ethnicity, as well as a language family.

A slight disadvantage is that in the US, this term is [interpreted](#) as excluding the Yupiks and Iñupiat. However their preference for the word "Eskimo" is extremely problematic for the user community at large, and therefore not considered.

Attestations. Here I show the attestations that I could find on the web of these numerals:

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(continued from previous page)

ers. Consequently, there are Inupiaq speakers who have had only a rudimentary understanding of their own traditional number system.

The Kaktovik Inupiaq numerals began as an ordinary math enrichment activity at Kaveelook Middle School on Barter Island, but because of the remarkable simplicity of the system, it has caught on as a way of expressing, in symbols, the numbers of the Inupiaq language. It has gained recognition not only on the North Slope and in Alaska generally, but it has also gained attention nationally as well as internationally. In early September of 1994 at Harold Kaveelook School, students were exploring base-2 numbers in their middle-school math class. Some students mentioned that Inupiaq, their Eskimo dialect, has a base-20 system. They then decided to try to write the Inupiaq numbers with regular Arabic numerals, but found there were not enough symbols to write the Inupiaq numbers.

Upon creating ten extra symbols, the students found that the new symbols were difficult to learn and remember. They discussed the problem and tried different approaches. Finally they hit upon a system that was conceptually simple and reflected the Inupiaq oral counting system. After fine-tuning their new numeral symbols, the students then began to do simple addition and subtraction problems with them. To their amazement, they discovered that their numerals had a number of distinct advantages. It was easier to add and subtract with them than with Arabic numerals. Often the numerals almost gave the students the answer.

The students enjoyed the challenge of converting decimal numbers into the base-20 Kaktovik Inupiaq numerals. As they tried to convert increasingly larger numbers, they found that conversion was easier using counters with place value. This idea was then extended into a form of a base-20 abacus. The students discussed the ideal structure of their abacus, got beads from the art teacher, experimented and finally built abacuses in the school shop. Since that time, they have found that because the base-20 Inupiaq abacus represents numbers in a similar way to their new numerals, it is easy to work with the abacus not only to convert, but also to add, subtract, multiply and even to divide. Their Inupiaq abacus has become an important component of math education using the Kaktovik Inupiaq numerals. Inupiaq mathematics, to the extent that it now exists as a scholastic discipline, was born as a twin, on the heels of the Kaktovik Inupiaq numerals. As the students began to perform mathematical operations with their numerals more and more, they discovered that the symbols were powerful enough to be manipulated as symbols. It is as though the symbol itself is a kind of graphic math manipulative.

When the class began to experiment with division, they did it the same way they did when dividing decimal numbers. However, a few students noticed that part of the process can be simplified because of the visual nature of the numerals they invented. Soon they had figured out how to do long division almost as though it was short division. Quite frequently, as students work with the numerals they have discovered shortcuts in math that cannot be done so easily with the Arabic numerals.

In the spring of 1995, the North Slope Borough Board of Education invited the students from Kaveelook School to fly to Barrow to present and explain their invention. Those who attended that presentation were impressed with the exciting educational possibilities opened up by this system. It is a system which is a direct reflection of the way one counts in Inupiaq. The underlying genius of the Inupiaq language has been crystallized in these numerals, making them useful for practical purposes.

As the 1995-96 school year began in August, the ECE immersion class in Barrow and the Inupiaq language classes in Wainwright and Point Lay began introducing the numerals into the classrooms. Teachers in other grades at the elementary school, the middle school, and even the high school in Barrow began introducing the system to their students. Ilisagvik, the local college, began introducing the numerals and their use to students across the North Slope by adding Inupiaq mathematics into its curriculum and its catalog and compressed video classes. By this time, a great deal had been discovered about the practical potential of the Kaktovik Inupiaq numerals, and the students and their teacher had managed to collect a great deal of material about other Arctic and Native American counting systems. The numerals have also been used exclusively (to the exclusion of Arabic numerals) in an ECE immersion program in Barrow and a complete textbook is being developed in the Inupiaq language to teach math, using the numerals, in the first-grade immersion classes. ♦

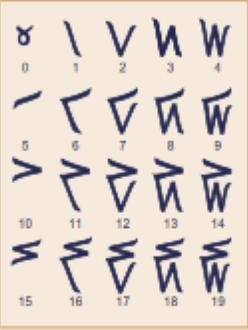


Figure 1. Part of an article titled "Making the Old Way Count" by Wm. Clark Bartley, of the newsletter "Sharing Our Pathways" a publication directed to and made by the Alaskan community.

VOL. 2, ISS. 1, January-February 1997

<http://www.ankn.uaf.edu/sop/SOPv2i1.pdf> (page 13)

Resolution 9: REGARDING KAKTOVIK NUMERALS

WHEREAS there is no widely-accepted means of representing (with simple numeric symbols) the traditional base-twenty counting systems used in Inuit languages; and

WHEREAS variations of a base-twenty counting system are part of our common Inuit cultural heritage, but these are being lost because fewer and fewer Inuit learn and use the traditional numbers; and

WHEREAS students in the Inuit community of Kaktovik have developed a base-twenty counting system which they desire to further research as a part of their on-going math education; and

WHEREAS the ICC recognizes the right of each community to its own numbering system;

THEREFORE BE IT RESOLVED THAT the Inuit Circumpolar Conference endorse further research into the use and development of the Kaktovik Numbering System, as well as any other local Inuit numbering system; and

BE IT FURTHER RESOLVED THAT the ICC encourage all member communities to try to make broader use especially in education of their own local base-twenty counting tradition in order to preserve and to revitalize the traditional Inuit counting systems.

Figure 2. Resolution 9 of the 1998 resolutions of the ICC-C (Inuit Circumpolar Council-Canada), encouraging the use of the numerals.

<https://web.archive.org/web/20170202234626/http://www.inuitcircumpolar.com/resolutions7.html>

<https://www.inuitcircumpolar.com/icc-canada/>

Inupiaq numerals

Designed by students of the Harold Kaveolook School on Barter Island, Alaska, in September 1994, the Kaktovik Inupiaq numerals made it easier to do calculations in the base-20 of the Inupiaq language numeral system. It is now widely used among Alaskan Iñupiat.



Figure 3. Entry describing the numeral system in a blog about the numeral system in various languages.
<https://www.languagesandnumbers.com/how-to-count-in-Inupiaq/en/esi/>

Characters used are U+E5A0 to U+E5B3, starting at 1 through 19 then 0.

Numeral	Denary
ᐅᐅᐅᐅ	0
ᐅᐅᐅᐅ	1
ᐅᐅᐅᐅ	2
ᐅᐅᐅᐅ	3
ᐅᐅᐅᐅ	4
ᐅᐅᐅᐅ	5
ᐅᐅᐅᐅ	6
ᐅᐅᐅᐅ	7
ᐅᐅᐅᐅ	8
ᐅᐅᐅᐅ	9
ᐅᐅᐅᐅ	10
ᐅᐅᐅᐅ	11
ᐅᐅᐅᐅ	12
ᐅᐅᐅᐅ	13
ᐅᐅᐅᐅ	14
ᐅᐅᐅᐅ	15
ᐅᐅᐅᐅ	16
ᐅᐅᐅᐅ	17
ᐅᐅᐅᐅ	18
ᐅᐅᐅᐅ	19

If it's more convenient to copy & paste, this is 0 through 19 in a sequence: □□□□□□□□□□□□□□□□□□

Figure 4. Screenshot from a GitHub repository to design a Font for the numerals that use *Private Use* codepoints (all my browsers failed to display the numerals, despite the font being installed and the browsers restarted).
<https://github.com/0xcf843ecf802c722f434d56/InupiaqNumbers>
<https://github.com/0xcf843ecf802c722f434d56/InupiaqNumeralConverter>
 (Script meant to force display of the numerals on websites, using the font)

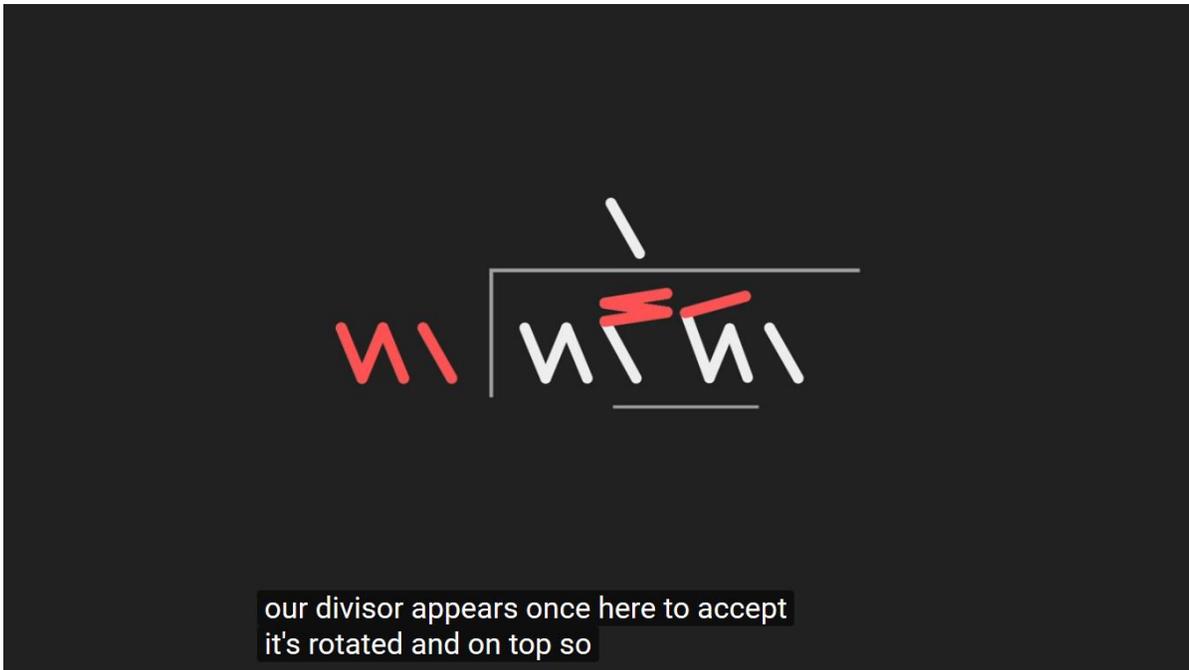


Figure 7. Screenshot of a video made by a language enthusiast, demonstrating how long division is made more efficient by the use of the numerals.
 “Why These Are The Best Numbers!” by *Artifexian*
<https://www.youtube.com/watch?v=EyS6FczH0Q>

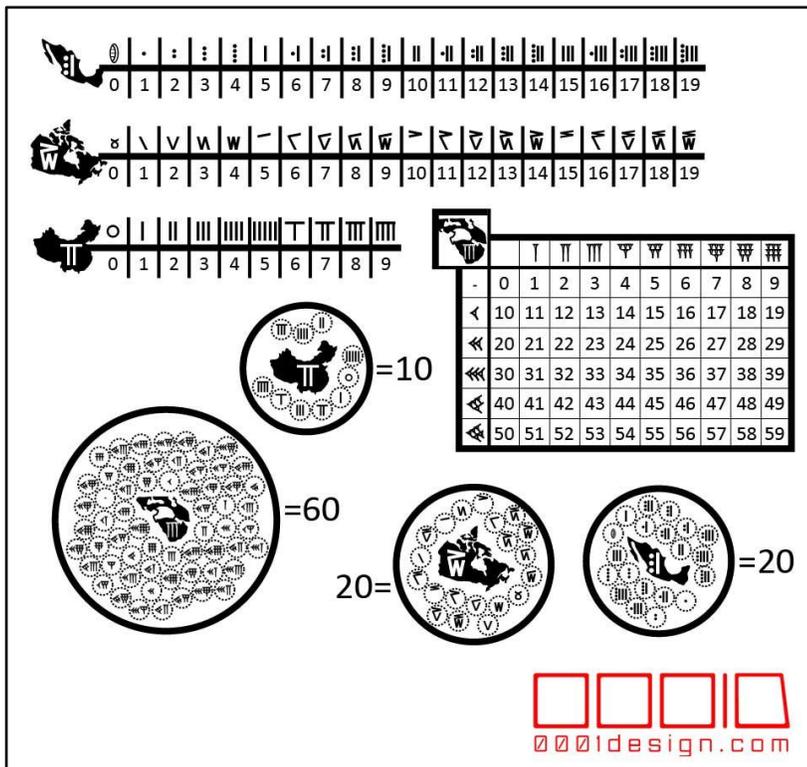


Figure 8. Illustration comparing different featural numeral systems (Mayan, Kaktovik, Cuneiform and Chinese rod numerals)
<https://www.nickpisca.com/sherpa/?p=539>



Figure 9. Photo of a classroom in Utqiagvik, Alaska (formerly known as Barrow); showing the numerals along with their respective readings in Inupiaq.

<https://www.adn.com/commentary/article/alaska-native-teachers-lead-classes-language-they-have-not-mastered/2016/01/17/#>

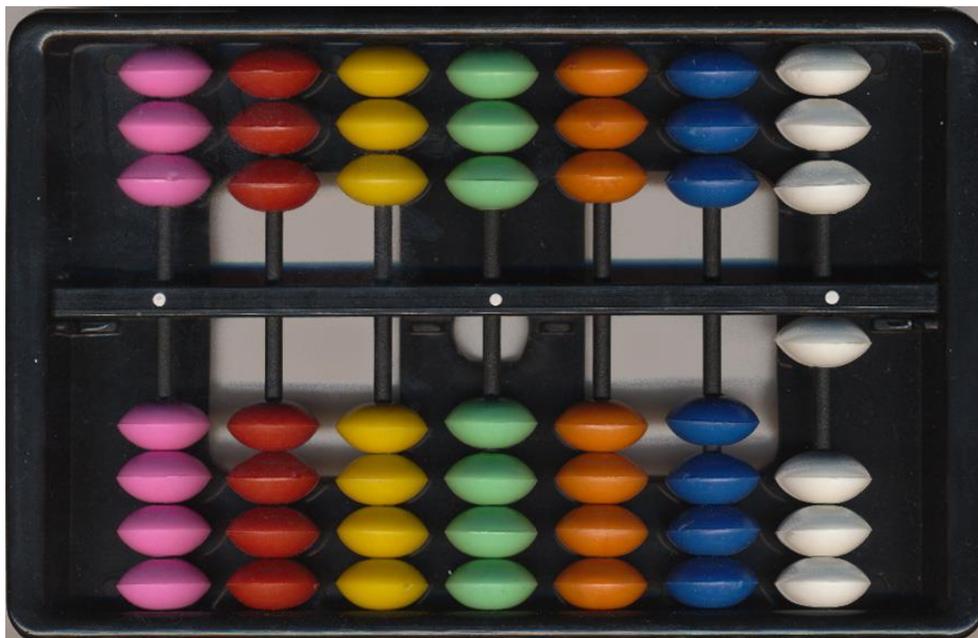


Figure 10. Photo of an abacus designed to work with base-20, as it is used by the aforementioned community.

https://commons.wikimedia.org/wiki/File:Abacus_Inuit.jpg#/media/File:Abacus_Inuit.jpg

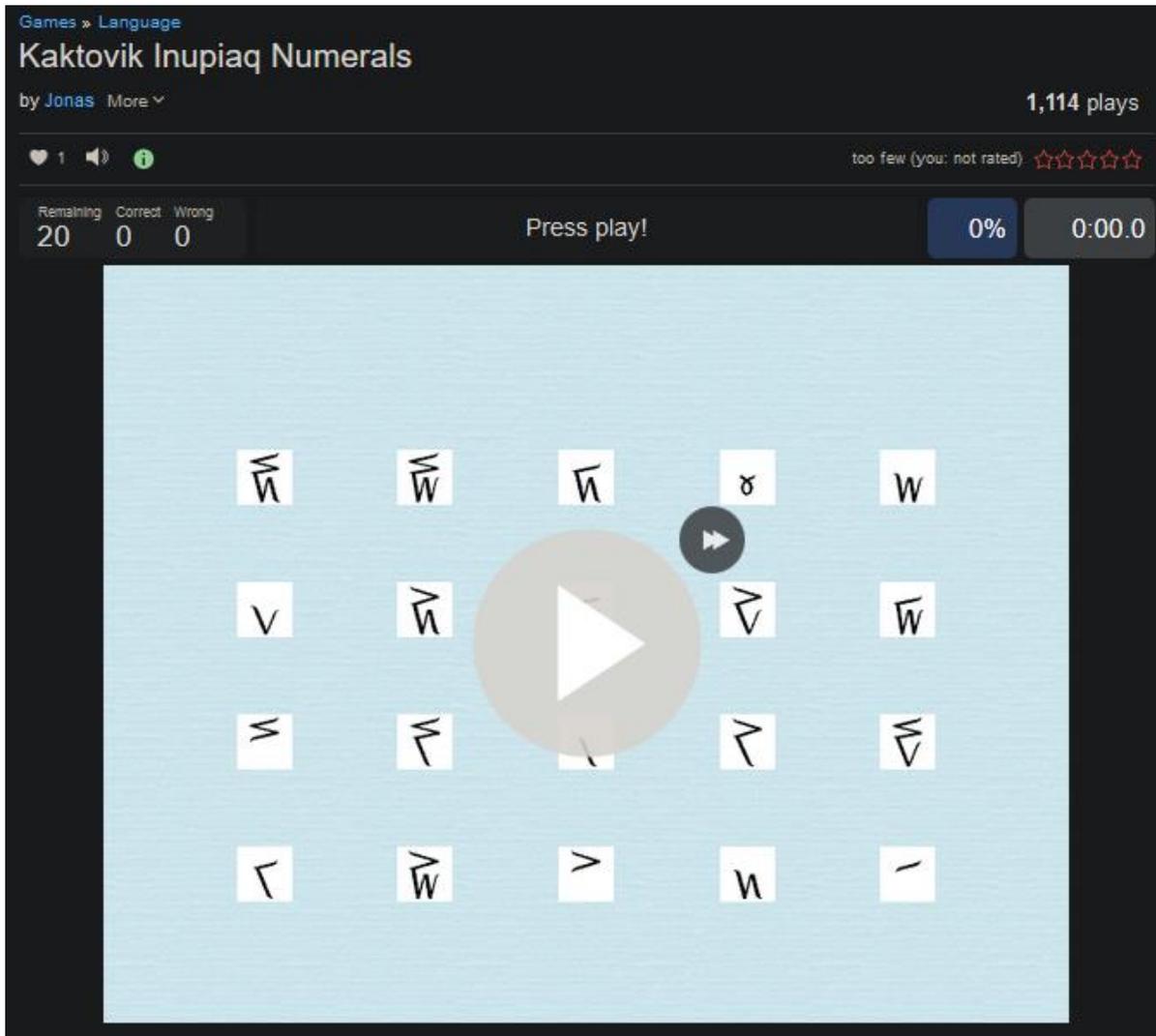


Figure 11. Simple game based on the quick recognition of the glyphs.

<https://www.purposegames.com/game/Kaktovik-Inupiaq-numerals-game>

Future Work. In order for the encoding of the numerals to go ahead, these steps must be taken.

- 1) Consult the user community to reach a consensus regarding the name and representative glyphs.
- 2) Obtain attestations of the numbers appearing in plain text, as well as in arithmetic operations.
- 3) Agree with the technical committee on the codepoint range, so that these numerals can be placed in the Roadmap.

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