

Universal Multiple-Octet Coded Character Set
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Title: Proposal to encode 11 cossic characters

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1. Background

With this document we present a new version of our **Cossic characters** proposal (L-2438) which was edited on September 17, 2024. About that document we received recommendations by D. Anderson and A. Freytag, on Febr. 19 and Febr. 25, 2025.

In this new version we will discuss these comments and implement the suggestions accordingly.

2. About Coss or cossic characters

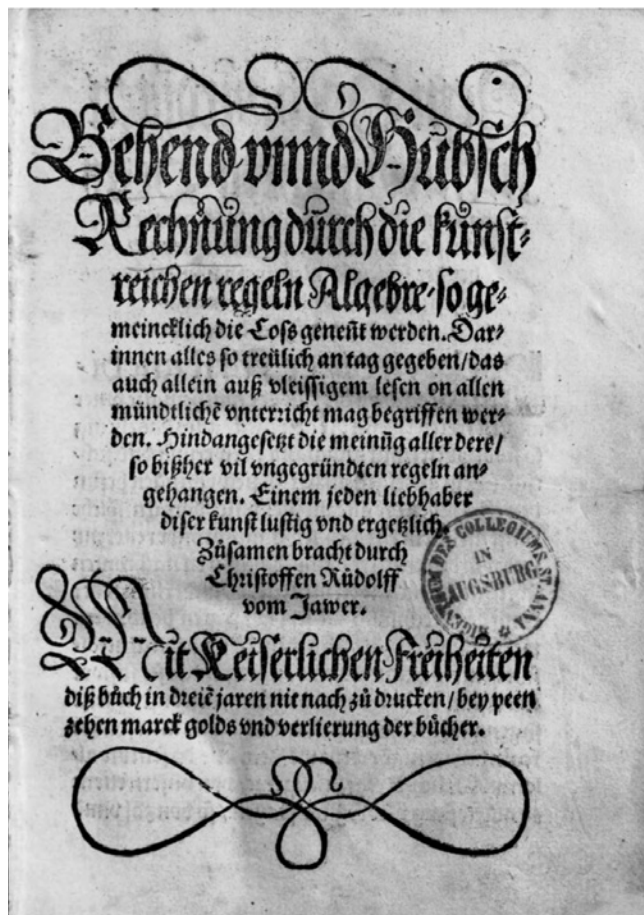
“Coss” (or “Cofs”, historic) is a German term for written or printed treatises about Algebra. It derives from Italian cosa (“thing”) which was used to denote variables in calculations. The first printed “Coss” was a book by German mathematician Christoff Rudolff (ca. 1500 – before 1543): *Behend und hübsch Rechnung durch die kunstreichen regeln Algebre, so gemeinlich die Cofs geneñt werden*. (“Handy and neat calculation by the artful Algebre rules, commonly so called the Coss.”) The work was based on older algebra manuscripts which the author studied in Vienna. The book was released in Straßburg in 1525 and was out of stock shortly thereafter. Because it was such a desired title, Michael Stifel edited a new and extended version of Rudolff’s Coss in 1553.

In the 1525 edition the character “ $\sqrt{}$ ” was used the first time for radix in print. For the expression of powers (up to ninth) Rudolff used a set of special abbreviation characters. Some of them were common in writing at the time (and used for different purposes), some were rather special additions. Since this set of cossic characters appears explicitly for a longer time in mathematical literature, we see a need to have them encoded as a whole, in order to enable precise content encoding in facsimile transcriptions of the historic sources.

Title page of Rudolff's *Coss*,
edited in Straßburg 1525.

Source:

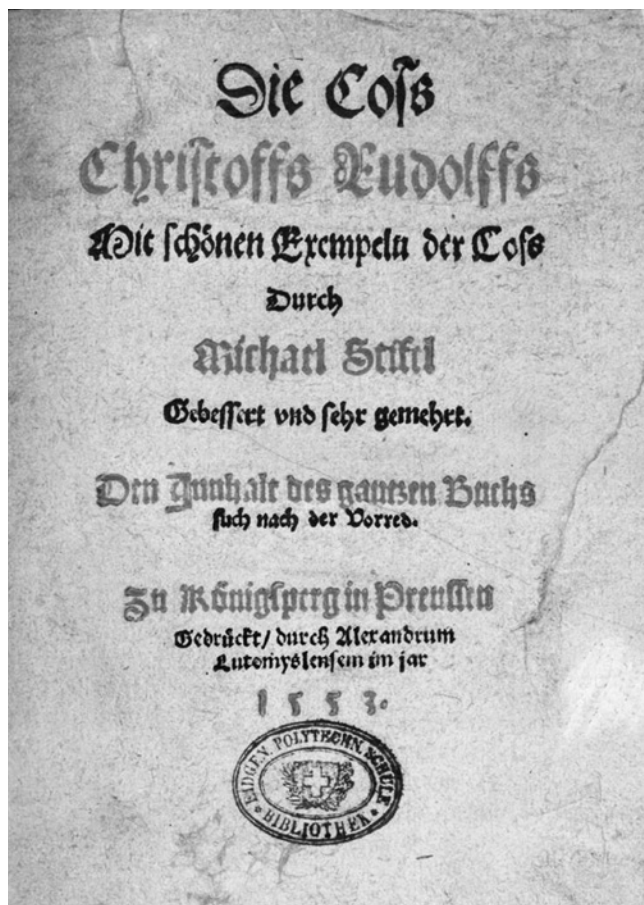
Münchner Digitalisierungszentrum



Title page of Stifel's new edition of
Rudolff's *Coss*,
printed in Königsberg 1553.

Source:

ETH Zurich



We see a case for encoding this whole set of characters in its particular form and determined by its special meaning and function. For editions of historic sources in facsimile mode it is necessary to encode these symbols accurately and unambiguously.

3. Characters

The cossic characters set consists of two sub-groups. If this proposal gets accepted, the following new characters will exist.

Group 1 is a range of 8 Latin abbreviation characters, derivatives from Latin letters c, d, r, f and z. They represent the initials of the names of roots or powers. These characters are comparable to other already encoded abbreviation characters, like ℥ (*libra*, 2114), ℥ (*per*, 214C), ℥ (*denarius/penny*, 20B0), ℥ (*prae-*, A755) or ℥ (*-rum*, A75D) which show a combination of a modified basic shape with some sort of graphic attachment, like scriptive loops directly connected to or crossing the base glyph.

We prefer to encode them as *mathematical symbols*, rather than as *Latin letters*. The aspect of case pairing is not relevant in all of the cases since no capital variants of these characters have ever been used anywhere. “Lowercase” in the proposed character names is merely to indicate the proper ‘parent’ characters. The cossic characters do not occur as abbreviations in general Latin writing but exclusively in calculation contexts. Hence their specific shapes in combination with very specific meaning should justify their encoding, even if an apparent close optical ‘neighbourhood’ to existing characters can be observed.



LOWERCASE C WITH SMALL SLASH

= cubus

- denotes cube of the unknown



LOWERCASE C WITH RIGHT LOOP

= cubus

- denotes cube of the unknown



LOWERCASE C WITH DESCENDER

= census

- denotes square of the unknown



LOWERCASE D ROTUNDA WITH CROSSING LOOP

= dragma

- denotes numerus / constant
- 1E9F δ latin small letter delta
- A77A d latin small letter insular d
- 20B0 ℥ german penny sign



LOWERCASE R ROTUNDA WITH LOOP

= res, radix

- denotes the unknown
- A75D r latin small letter r rotunda
- A75D r latin small letter rum rotunda
- A776 R latin letter small capital rum
- 221A $\sqrt{\quad}$ square root

- ß SHARP S WITH DESCENDER**
 = sursolidum
 • denotes fifth power of the unknown
 • glyph shows a ligature of long s and s
 • also used as abbreviation for solidus, semis
 → 017F ß latin small letter long s
 → 0073 s latin small letter s
 → 00DF ß latin small letter sharp s
 → A7D7 ß latin small letter middle scots s
- ſ LOWERCASE LONG S WITH TOP LOOP**
 = sursolidum
 • denotes fifth power of the unknown
 → 017F ſ latin small letter long s
 → 1E9C ſ latin small letter long s with diagonal stroke
 → 1E9D ſ latin small letter long s with high stroke
- ꝛ LOWERCASE KURRENT Z SIGN**
 = zensus
 • denotes square of the unknown
 → 0292 ꝛ latin small letter ezh

Group 2 is a set of *root* or *radix* symbols, their shapes beeing related to √ (221A).

The glyphs have been revised, according to recommendation by D. Anderson, so that they show their kinship with 221A better now.

℥ RADIX SIGN 1
 = fourth root
 → 221A √ square root

℥℥ RADIX SIGN 2
 = eighth root
 → 221A √ square root

℥℥℥ RADIX SIGN 3
 = sixteenth root
 → 221A √ square root

4. The *cubus* characters

In group 1 there are two different characters for “cubus”: ꝑ and ꝑ̄. Although the meaning is the same, the representative glyphs differ considerably. These typographic differences are strongly tied to certain writing or publishing traditions. Therefore we propose to encode two characters, thus being in line with a principle which has been followed in e.g. the alchemical characters block, where also (in some cases) two or three different characters bear (basically) the same meaning. This character pair situation is also evident with some other characters of the cossic set.

5. *Radix* characters

The LOWERCASE R ROTUNDA WITH LOOP ʀ occurs frequently with the meaning of “res” or “radix”. The left part of the glyph is derived from the shape of the capital R, in a similar way as

the left parts of the R ROTUNDA and RUM ROTUNDA characters (A75A to A75D) are derived from R. The distinctive feature of ʒ is its right half with a prominent crossing loop moving down as a descender. In this form, the character unambiguously denotes the mathematical meaning in contrast to the syllabic meaning “-rum” of A75C/A75D as well as of A776.

But, as the sources show, the small capital R with stroke ʀ (A776, LATIN SMALL CAPITAL RUM) has also been used in the set of cossic characters. Therefore we additionally propose the addition of new anotations to this character, as follows:

A776 ʀ LATIN LETTER SMALL CAPITAL RUM

- cossic sign for res, radix
- A75D ʒ latin small letter rum rotunda
- XXXX ʒ lowercase r rotunda with loop

6. *Sursolidum* characters

“Sursolidum” is also represented by two different characters: β and β° . This dual track situation has evolved historically by different local notation traditions. On the one hand, in an edition of historic sources it would not be tolerable to encode e.g. β (or even β , 00DF) instead of β° .

In the comments by D. Anderson and A. Freytag these alternative names are suggested:

MATHEMATICAL ITALIC SHARP S
SHARP S WITH HOOK

We do not favour to use the name part “Italic” because in the sources this glyph is upright in most cases, cursive only in very few cases. Hence cursiveness is not a significant feature. (Often type-setters lacked the sorts in their cases which they may have preferred but borrowed some similar-looking letter from somewhere else.)

The name part “sharp s” seems not a lucky choice because the β character evolved as a ligature of f and s in the function of an Latin abbreviation for *fursolidum* or (sometimes) *femis*. Thus it has a *different* root than the German sharp s (ß), the similar look is actually accidental. On the other hand, for the modern user it seems advisable to point at the relation with the existing β (00DF) and so we consider two options for a meaningful naming:

MATHEMATICAL SHARP S
or
SHARP S WITH DESCENDER

MATHEMATICAL SHARP S would be consistent with the naming convention in the Mathematical Alphanumerics block and appropriate when the character would be located there.

SHARP S WITH DESCENDER would be less suggestive and a bit more descriptive. We would prefer this second name and also prefer an allocation together with the other cossic symbols.

“... it is important to follow existing practice for encoding letter-like mathematical symbols.”

It seems questionable to divide the cossic set into “math alphanumerics” and “math symbols”. It doesn’t seem to make much sense. Even the *sursolidum* ‘sharp s’ is no mathematical alphanumeric variables character, but an abbreviation for Latin words.

In the 1525 edition, mathematical characters are set in italics, thus we find a c (U+1D450 MATHEMATICAL ITALIC SMALL C) in the abbreviation for cubus de cubo.

It is right that the survey of the historic sources reveal some inconsistencies. In the 1525 edition of Rudolff the *cossic* characters are not set in Italics but rather as an autonomous letter series modelled on an upright Roman style.

In the 1553 edition, no such contrast is made; the abbreviation for *Cubus de cubo* is an ordinary *c* set in the same blackletter font as the rest of the text.

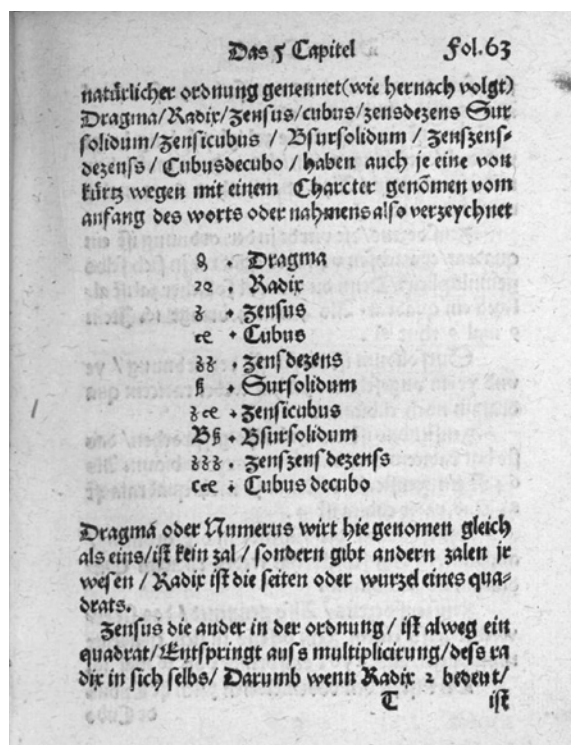
Similarly, the usage of the proposed character for *fursolidum* in the 1525 edition parallels the usage of an ordinary *ß* (which appears as an *f-z* ligature) as the abbreviation for *Surfolidum* in the 1553 edition.

This examples shows the inconsistency mentioned, they were obviously short of the right sorts in 1553 and so the borrowed the common Fraktur *ß* and the common *c* into the cossic set. But this makeshift practice is not considered authoritative.

This evidence suggests that the character should be called *MATHEMATICAL ITALIC SMALL SHARP S*, and have a decomposition to *U+00DF* of type **.

Such a decomposition is critical to for searchability, which is one of the stated purposes of this encoding initiative (see L2/24-141, p. 2), as it enables a text search using loose matching to match the 1553 edition against the 1525 one.

As stated above, we would follow that approach, with the exception of ‘Italic’, which is of no significance and possibly misleading.



Fol. 63 of the 1553 edition of Rudolff's Coss. Here we spot a slight inconsistency among the cossic characters. The use of the ‘ordinary’ *ß* instead of *ß* for *sursolidum* is neither typical nor decisive (it's actually a composing mistake).

7. Census characters

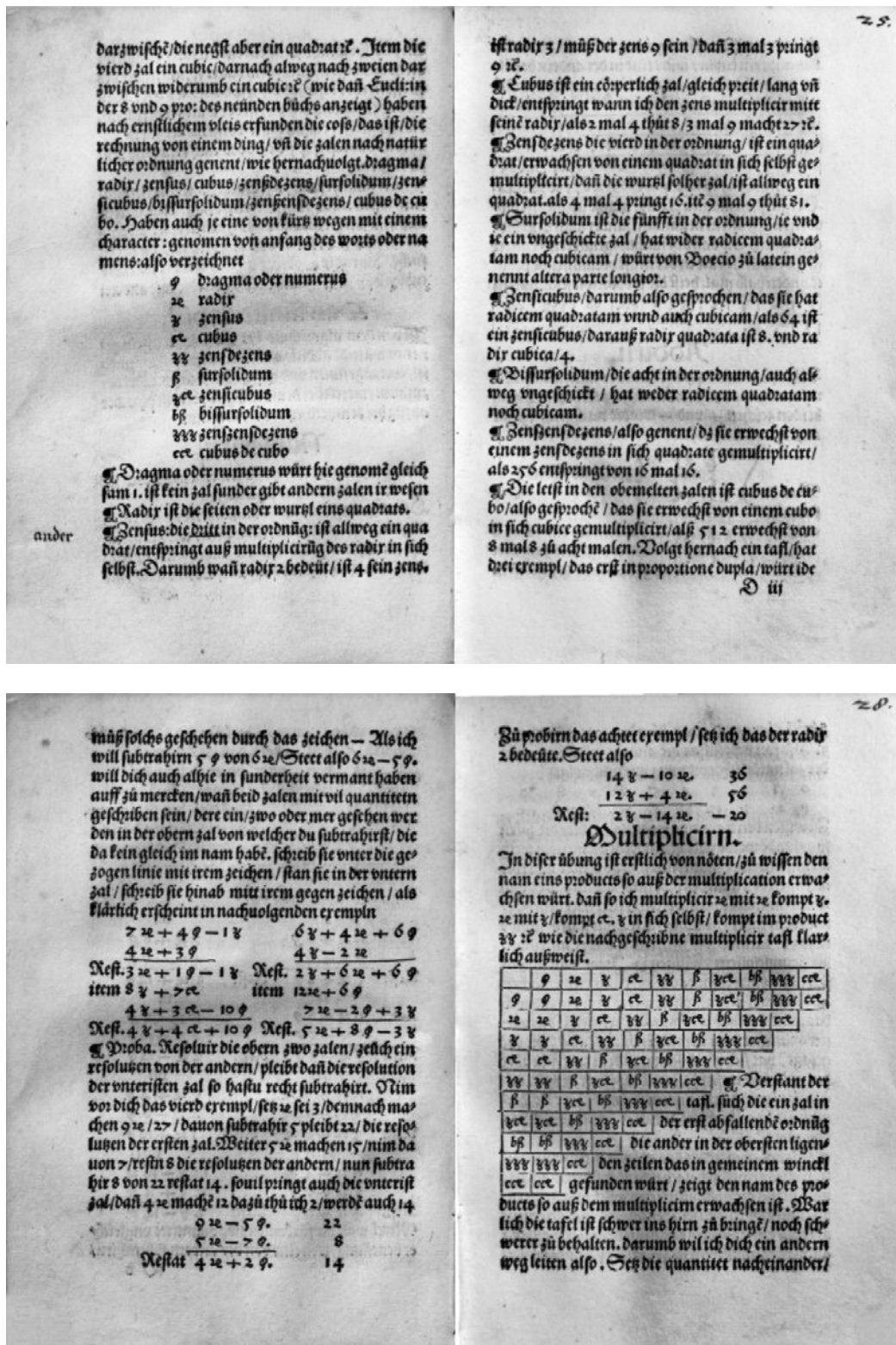
There are also two different characters for “census/zensus”: *ç* and *ꝥ*, related either to *c* or to *z*.

We do not recommend proposing *LOWERCASE KURRENT Z SIGN* as a new character, but instead as a variation sequence on *U+1D4CF*...

We are not in favour of this solution because the proposed character is completely different in shape and function from the alphanumeric variable character *1D4CF*. To draw on a somewhat shared root ‘*z*’ as the main criterion seems a bit arbitrary; moreover, it would break the new character out of the connection with the other characters of the cossic set. Last but not least we question if the novelty of only one or two newly incorporated lowercase Standardized Variation Sequences – for which the *1D400* block has no precedent – would be desirable for that code block. We propose giving the *ꝥ* and *ß* characters regular new codepoints and **keep them in a complete set of encoded cossic characters** for the sake of consistency would be the sound solution.

8. Figures and further explanations

The *Coss* characters became a widely adopted set of characters for denoting powers and roots, in the 16th and 17th century. We show a couple of instances from printed sources and also a piece of manuscript evidence by Leibniz. See page 17 for a synopsis of all characters belonging to the first group.



Christoff Rudolff: *Behend und hübsch Rechnung durch die kunstreichen regeln Algebre, so gemeinlich die Coss genennt werden*. Straßburg 1525. fol. 24v-25r, 27v-28r.

resolution der vnteristen zal / so hastu recht gemul-
tiplicirt. Als ich wil probirn dz drit exempl / setze den
werdt radialis 5. demnach würt die erst der obern:
nemlich 6 2e + 8 9: resoluirt in 38. die ander in 18 /
multiplicirt 38 mit 18 komen 684. so wil bedüt auch
das facit / daß 30 8 machen 750 / dauon subtrahir
10 / als den werdt 222 / mer subtrahir 56 / bleiben 684
hab ich wellen probiren.

Diuidirn.

Wan du hast diuidirt die grösser quantitet durch
die kleiner / wilt wissen den nam des quocients / Bee
in die nachuolgedt rasi / süch die grösser quantitet in
der obern / die kleiner in der vntern überck hengen
den zeilen so würt dir in gemeinem winkel der nam
des quocients angezeigt.

9	2e	8	ce	38	β	3ce	68	388	ce
9	9	2e	8	ce	38	β	3ce	68	388
2e	9	2e	8	ce	38	β	3ce	68	388
8	9	2e	8	ce	38	β	3ce	68	388
ce	9	2e	8	ce	38	β	3ce	68	388
38	9	2e	8	ce	38	β	3ce	68	388
β	9	2e	8	ce	38	β	3ce	68	388
3ce	9	2e	8	ce	38	β	3ce	68	388
68	9	2e	8	ce	38	β	3ce	68	388
388	9	2e	8	ce	38	β	3ce	68	388
ce	9	2e	8	ce	38	β	3ce	68	388

30.
Wan ein quantitet würt geteilt durch ein andere in
im namen gleich / kompt alweg 9 im quociens. würt
ein quantitet durch 9 geteilt / so kompt solch quanti-
tet im quociens. daß 9 verändert keine quantitet wi-
der im multiplicirn nach diuidirn.

Ein ander weiß / den nam des quocients zu süchē
wan die grösser geteilt würt durch ein kleinere oder
gleiche quantitet.

Schreib die quantitet nacheinander vñ verzeichē
sie mit der zal natürlicher ordnung / wie du im mul-
tiplicirn gelernt hast also.

0 1 2 3 4 5 6 7 8 9
9 2e 8 ce 38 β 3ce 68 388 ce

Subtrahir die zal der kleinern von der zal der gröf-
fern quantitet / durch dz übrig würt künde der nam
des quocients. Als ich diuidir 8 durch ce / subtrahir
3 von 5 resten 2 / zeigt das der quociens sei 2. Item
ich diuidir 38 durch β / subtrahir 5 von 6 restat 1 /
zeigt im quociens 2. Item ich diuidir ce durch ce /
subtrahir 3 von 3 restat 0 / zeigt 9. Des zu merern
verstandi nim dise exempl. Ich diuidir 6 8 durch 2
22 Stet also.

6 8 (fa: 32. Item 5 ce (fa: 2 1/2. Item 12 8 (fa: 4 9
22e 22e 38

Proba. Resoluir beide zalen. Diuidir ein resolu-
ren durch die ander / das auß solcher teilung komen
würdt / müß gleich sein des quocients resoluten.
Zu einer prob nim das erst exempl / set den werde

diuidirt werde durch 3. Darumb laß dich alhie nie-
tiren / was oben im fünfften capitel des ersten teils
bey der diuision gesagt ist.

Die erst equation oder regl der cos.

Wan zwo quantiteten natürlicher ordnung einan-
der gleich werden / diuidir die kleiner in die grösser
quantitet / der quociens zeigt an den werdt 12. Als
in disen exempln.

Die teiler

3 2e	6 9
4 8	8 2e
5 ce	10 8
6 38	12 ce
7 β	14 38 facit 12. 29
8 3ce	16 β
9 68	18 3ce
10 388	20 68
11 ce	22 388

Die ander equation.

Wan zwo quantiteten einander gleich werden / zwi-
schen welchen eine: natürlicher ordnung nach be-
griffen: geschwigen ist. Diuidir die kleiner in die
grösser quantitet / radir quadrata des quociens zeigt
an den werdt 12. Als

54.
Die teiler

2 8	8 9
3 ce	12 2e
4 38	16 8
5 β	20 ce fa: 12. 29
6 3ce	24 38
7 68	28 β
8 388	32 3ce
9 ce	36 68

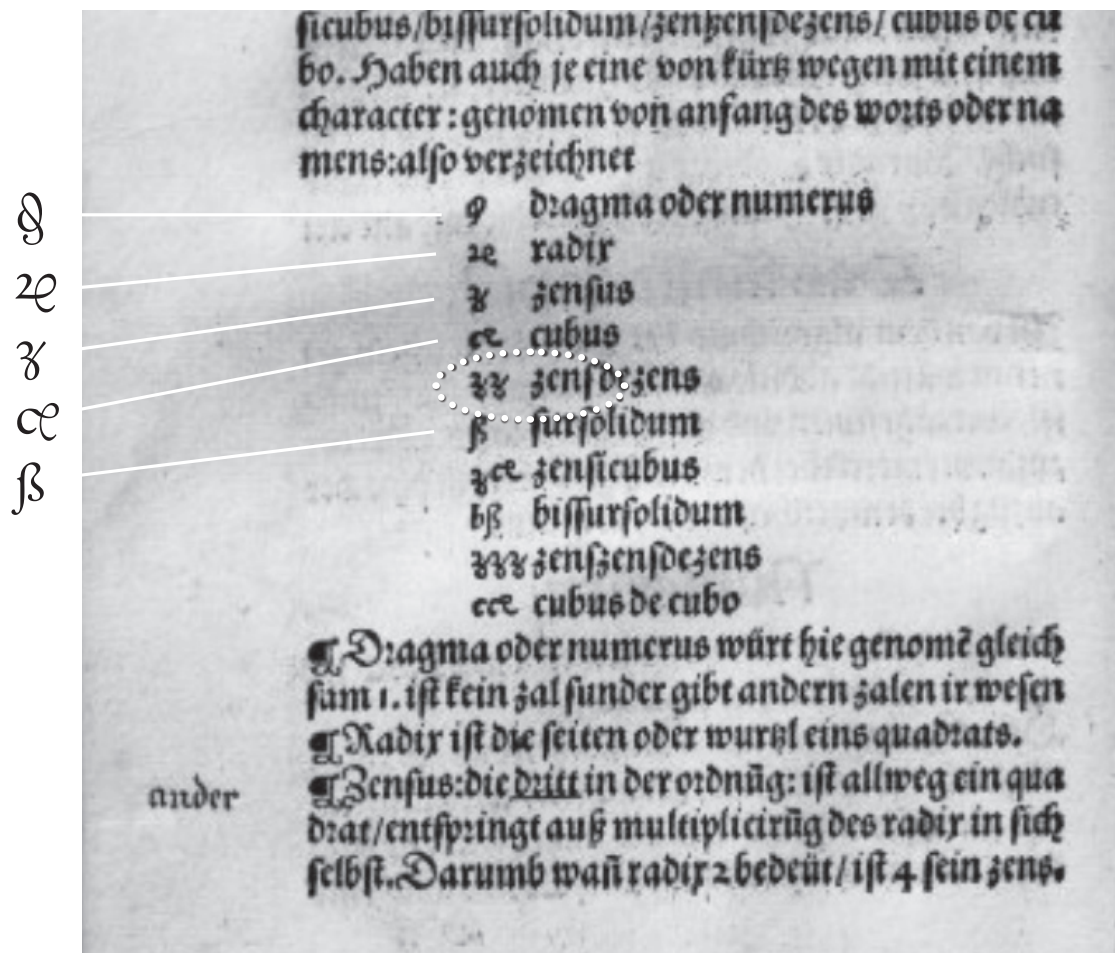
Die dritt equation.

Wan zwo quantiteten einander gleich werden / zwi-
schen welche zwo andere: natürlicher ordnung nach
begriffen: geschwigen sein. Diuidir die kleiner in die
grösser quantitet / radir cubica des quociens zeigt an
den werdt 12. Als

2 ce	16 9
3 38	24 2e
4 β	32 8
5 3ce	40 ce fa: 12. 22 3
6 68	48 38
7 388	56 β
8 ce	64 3ce

Die viert equation

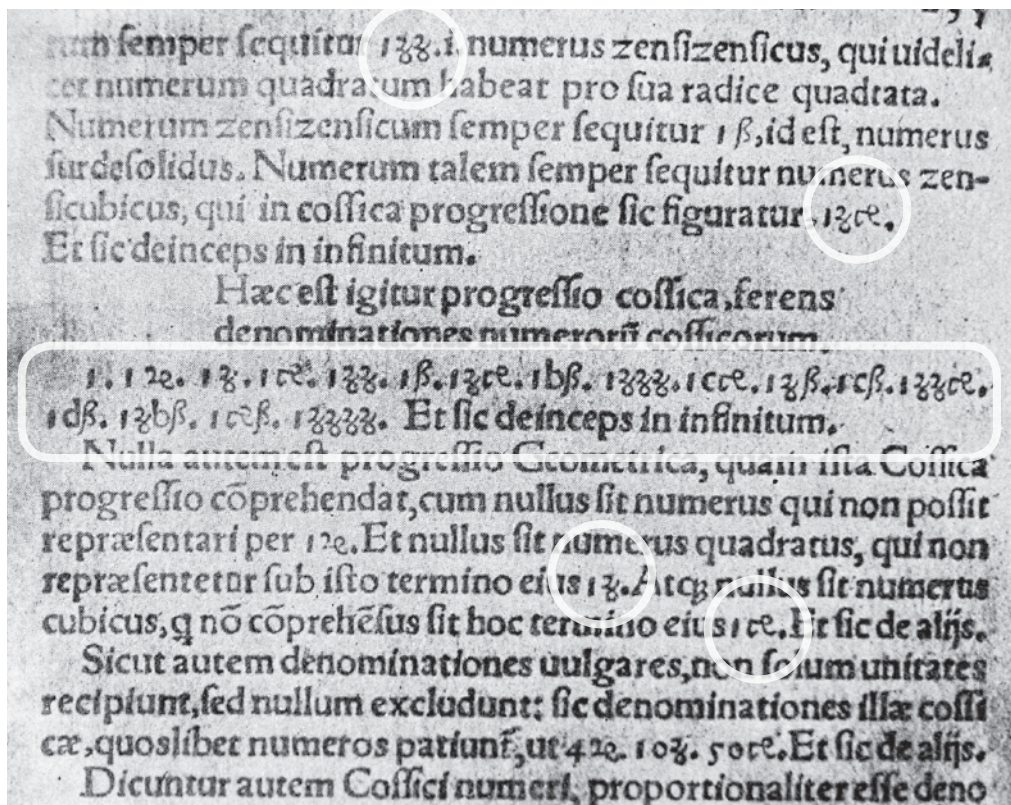
Wan zwo quantiteten einander gleich werden / zwi-
schen welche drey andere natürlicher ordnung nach
begriffen: geschwigen sein. Diuidir die kleiner in die
grösser quantitet / Radir quadrata von radice qua-
drata des quociens / zeigt an den werdt 12. Als



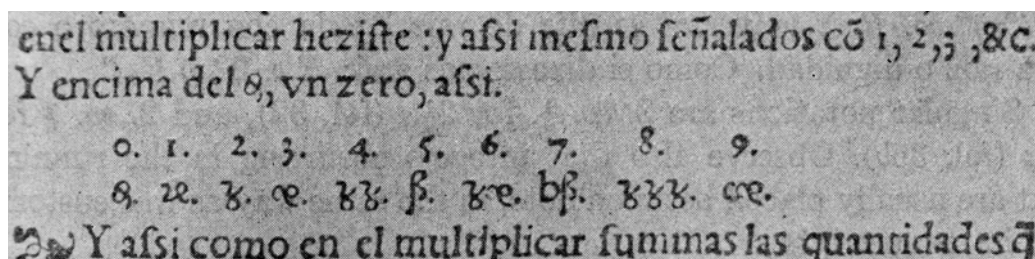
Christoff Rudolff: *Behend und hübsch Rechnung* ... Straßburg 1525, part of fol. 24v. In this chapter Rudolff introduces the set of root and power symbols by samples and explanations. We can see: 3 LOWERCASE KURRENT Z SIGN, § LOWERCASE D ROTUNDA WITH CROSSING LOOP, 2℞ LOWERCASE R ROTUNDA WITH LOOP, ℄ LOWERCASE C WITH RIGHT LOOP and ß SHARP S WITH DESCENDER.

This print demonstrates the deliberate distinction between the cossic character 3 and the normal fraktur 3 (see at ☉). Whereas in other scenarios this two shapes could be seen as ‘just’ glyph variants without semantic distinction, in this case the form difference is clearly an indicator for a specific meaning. The character 3 LOWERCASE KURRENT Z SIGN is denoting *zensus*. It is semantically determined by this one unambiguous meaning, and graphically characterized by a) a round-shaped upper part (mostly), and b) a prominent loop descender which crosses upwards. The origin of its shape is neither *Fraktur* type nor Latin script style but the German *Kurrent* writing style. Therefore the character is neither to be unified with *ezh* (0292) nor with any of the mathematical alphanumeric characters 1D4B5 etc^a, in order to secure its specific semantic content.

The character ß should not be unified with LATIN SMALL LETTER SHARP S (00DF, its somewhat obscure origin going back to medieval long-s abbreviation characters and later becoming typographically a ligature of long s and z in blackletter styles). Whereas the ß character is clearly and unambiguously based on the ligation of f and s in order to facilitate a crisp abbreviation sign for frequently occurring words like *femis*, *solidus* or *furfolidus*. Since the German B appears in various shapes nowadays in typefaces (e.g. in Times – like seen here – it is definitely not a f-s ligature), it would be misleading to assign 00DF to the special usage shown in these examples.



Stifel 1544 (after Cajori). This sample shows ꝛ LOWERCASE KURRENT Z SIGN, ʀ LOWERCASE R ROTUNDA WITH LOOP, c LOWERCASE C WITH RIGHT LOOP and ꝑ SHARP S WITH DESCENDER.



Aurel 1552, fol. 73B (after Cajori). This sample shows ꝛ LOWERCASE KURRENT Z SIGN (2., 4., 6., 8.), ꝑ LOWERCASE D ROTUNDA WITH CROSSING LOOP (0.), ʀ LOWERCASE R ROTUNDA WITH LOOP (1.), c LOWERCASE C WITH RIGHT LOOP (3., 6., 9.), and ꝑ SHARP S WITH DESCENDER (5., 7.).

These samples also show how those characters were used in combination to express the powers 4th and so on.

nous fournit de termes consecutiz, pour exposer les nombres Radicaus e leurs Singes: comme vous voyez par la Table ici mise.

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16,
 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16,
 11, 12, 13, 14, 15, 16.
 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16,
 2048, 4096, 8192, 16384, 32768, 65536.

L'ordre des Exposans composez.

4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24,
 25, 26, &c.

L'ordre des Singes composez.

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, &c. La ou vous noterez, que le Canlique est toujours participant, ou le Cube redouble.

L'ordre des Exposans incomposez.

Exemple de la Diuision.

Je veu diuiser 308 m. 58^{re}, p. 24, par 5^{re} m. 3.
 La posicion sera comme vous voyez,

40
 308 m. 58^{re} p. 24
 5^{re} m. 3.
 —————
 308 m. 18^{re}.

Je di donc ainsi : 5 an 30 font com-

Three extracts from Peletier 1554: ç LOWERCASE C WITH DESCENDER, cç LOWERCASE C WITH RIGHT LOOP and ß SHARP S WITH DESCENDER.

These samples also demonstrate the usage of ʀ (A776) as part of the cossic set, as well as the use of slashed figures (on which we elaborate in another proposal).

to uoide the tedious repetition of these woordes: is equalle to: I will sette as I doe often in woorkes bfe, a paire of paralleles, or Gemowe lines of one lengthe, thus:=====, bicause noe. 2. thynges, can be moare equalle. And now marke these numbers.

1. 14.ze. + 15.g=====71.g.
2. 20.ze. — 18.g=====102.g.
3. 26.z — 10ze=====9.z — 10ze — 213.g.
4. 19.ze — 192.g=====10z — 108g — 19ze
5. 18.ze — 24.g.=====8.z. — 2.ze.
6. 34z — 12ze — 40ze — 480g — 9.z.
1. In the firste there appeareth. 2. numbers, that is
14.ze.

Another Example of Addition.

$$\begin{array}{r}
 5.z.c. + 20.c. = 3.fz. \\
 \hline
 5.z.c. + 3.fz. \quad \text{to} \quad 20.c. = 6.fz. \\
 \hline
 6.c.c. \quad \quad \quad 6.c.c. \\
 \hline
 6.c.c.
 \end{array}$$

That is in les-
ser termes.

$$\begin{array}{r}
 5.c. + 20.g. = 3.z. \\
 \hline
 6.z.c.
 \end{array}$$

Here is noe multiplication, no2 reduction to one common denominato2: sith thei bee one all ready: no2

Two extracts from Recorde 1557 (after Cajori): c LOWERCASE C WITH RIGHT LOOP, § LOWERCASE D ROTUNDA WITH CROSSING LOOP, ze LOWERCASE R ROTUNDA WITH LOOP and z LOWERCASE KURRENT Z SIGN.

anis, a diuerſe *Arithmetike* from the other. Practiſe bryngeth in, here, diuerſe compounding of Numbers: as ſome tyme, two, three, foure (or more) *Radicall* nũbers, diuerſly knit, by ſignes, of More & Leſſe: as thus $\sqrt{8} 12 + \sqrt{c} 15$. Or thus $\sqrt{8} 8 19 + \sqrt{c} 12 - \sqrt{8} 2$. &c. And ſome tyme with whole numbers, or fractions of whole Number, amõg them: as $20 + \sqrt{8} 24. \sqrt{c} 16 + 33 - \sqrt{8} 10. \sqrt{8} 8 44 + 12 - + \sqrt{c} 9$. And ſo infinitely, may hap the varietie. After this: Both the one and the other.

Example from Dee 1570 (after Cajori): Ꝣ LOWERCASE C WITH RIGHT LOOP and ꝥ LOWER-CASE KURRENT Z SIGN.

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
 1, Ꝣ, Ꝣ, Ꝣ, Ꝣ, Ꝣ, Ꝣ, Ꝣ, Ꝣ, Ꝣ, Ꝣ,
 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024,
 11, 12, 13, 14, 15, 16,
 Ꝣ, Ꝣ, Ꝣ, Ꝣ, Ꝣ, Ꝣ, Ꝣ, Ꝣ, Ꝣ, Ꝣ,
 2048, 4096, 8192, 16384, 32768, 65536.

From Peletier 1620.

C A P. XXVIII. 159

Sit rursus Binomium primum $72 + \sqrt{8} 2880$. Maius nomen 72 .
 ſecabitur in duas partes producentes 720 . quartam partem quadrati
 2880 . maioris nominis, hac ratione.
 Semifis maioris nominis 72 . eſt 36 . a
 cuius quadrato 1296 . detracta quarta
 pars prædicta 720 . relinquit 576 . cu-
 ius radix 24 . addita ad ſemiſſem no-
 minatam 36 . & detracta ab eadem, fa-
 cit partes quaſiſas 60 . & 12 . Ergo ra-
 dix Binomij eſt $\sqrt{8} 60 + \sqrt{8} 12$. quod
 hic probatum eſt per multiplicationem radicis in ſe quadratè.

Sit quoque elicienda radix ex hoc reſiduo ſexto $\sqrt{8} 60 - \sqrt{8} 12$.
 Maius nomen $\sqrt{8} 60$. diſtribuetur in duas partes producetes 3 . quar-
 tam partem quadrati 12 . minoris nominis, hoc pacto. Semifis ma-
 ioris nominis $\sqrt{8} 60$. eſt $\sqrt{8} 15$. a cuius quadrato 15 . detracta nomi-
 nata pars quarta 3 . relinquit 12 . cuius radix $\sqrt{8} 12$. addita ad ſe-
 miſſem $\sqrt{8} 15$. prædictam, & ab eadem ſublata facit partes $\sqrt{8} 15 +$
 $\sqrt{8} 12$. & $\sqrt{8} 15 - \sqrt{8} 12$. Ergo radix dicti Reſidui ſexti eſt $\sqrt{8} (\sqrt{8} 15 + \sqrt{8} 12) - \sqrt{8} (\sqrt{8} 15 - \sqrt{8} 12)$ quod hic probatum eſt.

$\sqrt{8} (\sqrt{8} 15 + \sqrt{8} 12)$	$— \sqrt{8} (\sqrt{8} 15 - \sqrt{8} 12)$
$\sqrt{8} (\sqrt{8} 15 + \sqrt{8} 12)$	$— \sqrt{8} (\sqrt{8} 15 - \sqrt{8} 12)$
Quadrata partium. $\sqrt{8} 15 + \sqrt{8} 12$ & $\sqrt{8} 15 - \sqrt{8} 12$	
$— \sqrt{8} 3$	
$— \sqrt{8} 3$	
Summa. $\sqrt{8} 60 - \sqrt{8} 12$	

Clavius 1608 (after Cajori): ꝥ LOWERCASE KURRENT Z SIGN. In this ſetting of Roman type ſtyle the common z character will have the uſual Greek-Latin ‘Zeta’ ſhape, z, whereas the ſymbol for *zensus* retains not only the z initial (in this Latin treatiſe one may expect *census* inſtead), but alſo the ſpecific *kurrent* ſcript form of the letter.

2ℓ
 3ℓ
 4ℓ
 5ℓ

72		De Notatione Algebraica.		C A I	
Nomina.		Characteres.		Pot.	
Radix	2ℓ	R	A	a	a
Quadratum	28	Q	Aq	aa	a ²
Cubus	2ℓ	C	Ac	aaa	a ³
Quad. quadratum	2828	QQ	Aqq	aaaa	a ⁴
Surdefolidum	ℓ8	S	Aqc	&c.	a ⁵
Quad. Cubi.	282ℓ	QC	Acc		a ⁶
2 ^m Surdefolidum.	Bℓ8	hS	Aqqc		a ⁷
Quad. quad. quad.	282828	QQQ	Aqcc		a ⁸
Cubi cubus	2ℓ2ℓ	CC	Accc		a ⁹
Quad. Surdefol.	28ℓ8	QS	Aqqcc		a ¹⁰
3 ^m Surdefolidum	Cℓ8	cS	Aqccc		a ¹¹
Quad. quad. cubi	28282ℓ	QQC	Acccc		a ¹²
4 ^m Surdefolidum	Dℓ8	dS	Aqqccc		a ¹³
Quad. 2 ⁱ Surdefol.	28Bℓ8	QbS	Aqcccc		a ¹⁴
Cubus Surdefol.	2ℓℓ8	CS	Accccc		a ¹⁵
Quad. quad. quad. quad.	28282828	QQQQ	Aqqcccc		a ¹⁶
&c.					

From Wallis, Operum mathematicorum, 1657 (after Cajori); shows the use of ℓ LOWERCASE LONG S WITH TOP LOOP for “sursolidum”.

The 3 LOWERCASE KURRENT Z SIGN has been given a sort of ‘Latinization treatment’ here, based rather on the Greek/Roman zeta shape. We regard this as a glyph variant with no distinctive meaning.

see also
next page

1 ① sec ① Produict d'une prime quantité par une prime
quantité secondement posée.
5 ④ ter ② Produict de cinq quartes quantitez par une
seconde quantité tiercement posée.
Les caracteres signifians racines de quels l'expli-
cation se trouve à la 29 & 30 definition sont tels :

- ✓ Racine de quarré.
- ✓✓ Racine de racine de quarré.
- ✓✓✓ Racine de racine de racine de quarré.
- ✓✓✓✓ Racine de racine de racine de racine de quarré.
- ✓ ③ Racine de cube.
- ✓✓ ③ Racine de racine de cube.
- ✓ ④ Racine de quarte quantité.
- ✓✓ ④ Racine de racine de quarte quantité, &c.

Le caractere signifiant la separation entre le si-
gne de racine & la quantité, duquel l'explication se
trouve à la 34. definition, est tel.

Χ, Comme ✓ 3 Χ ② n'est pas le mesme que ✓ 3 ②,
comme dict est à ladicte 34. definition.

Les caracteres signifians plus & moins, comme à
la 36 definition, sont tels :

- + Plus.
- Moins.

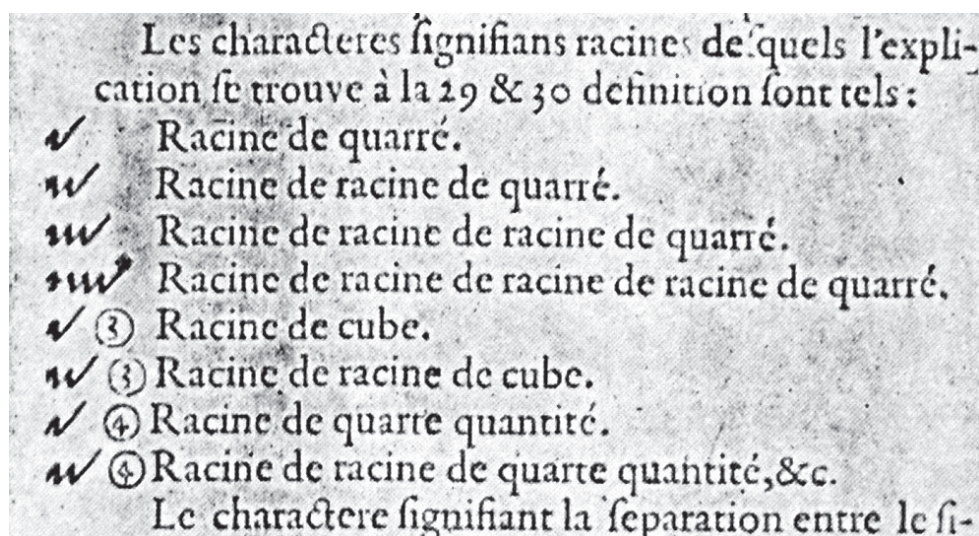
Et pour expliquer la racine d'un multinomie
(qu'aucuns appellent racine universelle) nous use-
rons le vocable du multinomie, comme:

- ✓ bino 2 + ✓ 3, c'est à dire racine quarrée de bino-
mie, ou de la somme de 2 & ✓ 3.
- ✓ trino ✓ 3 + ✓ 2 — ✓ 5, c'est à dire racine quarrée
de trinomie, ou de la somme de ✓ 3 & ✓ 2 & —
✓ 5.
- ✓ ③ bino ✓ 2 + ✓ 3, c'est à dire racine cubique de

✓ RADIX SIGN 1, ✓✓ RADIX SIGN 2, ✓✓✓ RADIX SIGN 3

These characters can be seen related to the established radix symbol ✓ (221A).

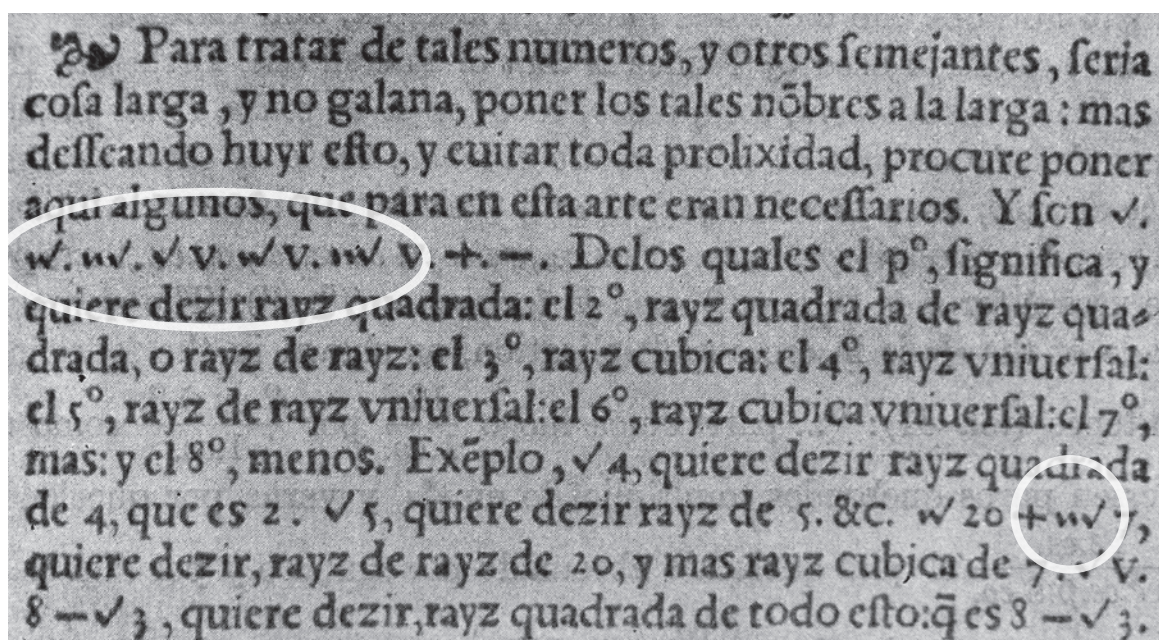
Simon Stevin, L'arithmétique in Œuvres mathématiques, 1634 (after Cajori)



Simon Stevin, L'arithmétique in Œvres mathématiques, 1634 (after Cajori)

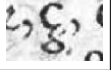
The number of ascending lines indicates how often an operation of root determination is performed on an expression. In the Stevin example the combination with an encircled number indicates, which type of root is meant. If there is no such number, the square root is to be considered. For example, the combinations denote the following:

- ✓✓ square root of square root, which corresponds to the forth root;
- ✓✓✓ square root of square root of square root, which corresponds to the eighth root;
- ✓✓✓✓ square root of square root of square root of square root, which corresponds to the sixteenth root;
- ✓ (3) cubic root of cubic root, which corresponds to the ninth root;
- ✓ (4) forth root of forth root, which corresponds to the sixteenth root.



✓✓ RADIX SIGN 1, ✓✓✓ RADIX SIGN 2, ✓✓✓✓ RADIX SIGN 3
Marco Aurel, Arithmetica algebrica, 1552 (after Cajori)

9. Synopsis (Group 1)

	Glyph								
	Character	LOWERCASE D ROTUNDA WITH CROSS- ING LOOP	LOWERCASE R ROTUNDA WITH LOOP	LOWERCASE KURRENT Z SIGN	LOWERCASE C WITH DESCENDER	LOWERCASE C WITH SMALL SLASH	LOWERCASE C WITH RIGHT LOOP	SHARP S WITH DE- SCENDER	LOWERCASE LONG S WITH TOP LOOP
	Meaning	dragma	radix	zensus	census	cubus	cubus	solidus sursolidum semis	sursolidum
1	Rudolf 1525								
2	Stifel 1544								
3	Aurel 1552								
4	Peletier 1554								
5	Recorde 1557								
6	Dee 1570								
7	Peletier 1620								
8	Clavius 1608/12								
9	Beeckmann 1628								
10	Wallis 1657								
11	Leibniz MS 1676								
12	MS Leiden 17. c.								
13	MS Ham- burg 17. c.								

Comparative survey of Coss characters in various sources, 1525 to 1676.

10. Unicode Character Properties

```
xi01;LOWERCASE C WITH SMALL SLASH;Sm;0;ON;;;;N;;;;;
xi02;LOWERCASE C WITH DESCENDER;Sm;0;ON;;;;N;;;;;
xi03;LOWERCASE C WITH RIGHT LOOP;Sm;0;ON;;;;N;;;;;
xi04;LOWERCASE D ROTUNDA WITH CROSSING LOOP;Sm;0;ON;;;;N;;;;;
xi05;LOWERCASE R ROTUNDA WITH LOOP;Sm;0;ON;;;;N;;;;;
xi06;SHARP S WITH DESCENDER;Sm;0;ON;;;;N;;;;;  ſ
xi07;LOWERCASE LONG S WITH TOP LOOP;Sm;0;ON;;;;N;;;;;
xi08;LOWERCASE KURRENT Z SIGN;Sm;0;ON;;;;N;;;;;
xi09;RADIX SIGN 1;Sm;0;ON;;;;N;;;;;
xi10;RADIX SIGN 2;Sm;0;ON;;;;N;;;;;
xi11;RADIX SIGN 3;Sm;0;ON;;;;N;;;;;
```

ſ In case this char. becomes a part of the 1D400 block, the UCP suggestion would be:
xi06;MATHEMATICAL SHARP S;Lu;0;L; 00DF;;;;N;;;;;

“x” stands for *unspecified codespace*. “i” refers to our internal characters classification, see N5277.

11. Bibliography

LAA – refers to: Leibniz, Gottfried Wilhelm: Sämtliche Schriften und Briefe. (‘Leibniz-Akademie-Ausgabe’, many volumes)
LBr – refers to: Leibniz’s original correspondence papers, GWLB Hanover
LH – refers to: Leibniz’s original manuscripts, GWLB Hanover

Aurel, Marco: Arithmetica algebratica, Valencia 1552
Bombelli, Rafael: L’Algebra. Bologna 1579
— : L’Algebra. Milan 1966
Cajori, Florian: A history of mathematical notations. Chicago 1928
Cardano, Gerolamo: Opera omnia. Lyon 1663
Clavius, Christophorus: Algebra. Rome 1608
Dee, John: [preface] in: Euclid: The elements. Henry Billingsley (ed.), London 1570
Descartes, René: La Géométrie. Leiden 1637
Dulaurens, François: Specimina Mathematica. Paris 1667
Ghaligai, Francesco: Pratica d’Arithmetica, Florence 1552
Peletier, Jaques: L’Algèbre. Lyon 1554
Probst, Siegmund: Édition des symboles de Leibniz. PDF, Hanover 2023 (presentation Paris 2023)
Rudolff, Christoff: Behend und hübsch Rechnung durch die kunstreichen regeln Algebre, so gemeincklich die Coß genennt werden. Straßburg 1525
Stevin, Simon: Œvres mathématiques. Leiden 1634
Stifel, Michael: Arithmetica integra. Nürnberg 1544
Wallis, John: De sectionibus conicis nova methodo expositis tractatus. Oxford 1655
— : Operum mathematicorum, Oxford 1657
— : Treatise of Algebra. London 1685

**ISO/IEC JTC 1/SC 2/WG 2
PROPOSAL SUMMARY FORM TO ACCOMPANY SUBMISSIONS
FOR ADDITIONS TO THE REPERTOIRE OF ISO/IEC 10646¹**

Please fill all the sections A, B and C below.

Please read Principles and Procedures Document (P & P) from <http://std.dkuug.dk/JTC1/SC2/WG2/docs/principles.html> for guidelines and details before filling this form.

Please ensure you are using the latest Form from <http://std.dkuug.dk/JTC1/SC2/WG2/docs/summaryform.html>.

See also <http://std.dkuug.dk/JTC1/SC2/WG2/docs/roadmaps.html> for latest Roadmaps.

A. Administrative

1. Title:	Proposal to add 11 cossic characters to the UCS		
2. Requester's name:	Uwe Mayer, Siegmund Probst, David Rabouin, Elisabeth Rinner, Andreas Stötzner, Achim Trunk, Charlotte Wahl		
3. Requester type (Member body/Liaison/Individual contribution):	Individual (work group)		
4. Submission date:	2025-03-24		
5. Requester's reference (if applicable):	LUCPL-2509		
6. Choose one of the following:			
This is a complete proposal:			Yes
(or) More information will be provided later:			

B. Technical – General

1. Choose one of the following:			
a. This proposal is for a new script (set of characters):			No
Proposed name of script:			
b. The proposal is for addition of character(s) to an existing block:			Yes
Name of the existing block:	since no space is available in the various Math symbols blocks, we propose a new block Miscellaneous Mathematical Symbols-C or similar. This new block can also accomodate other related new character sets we propose (see N5277)		
2. Number of characters in proposal:			11
3. Proposed category (select one from below - see section 2.2 of P&P document):			
A-Contemporary	B.1-Specialized (small collection)	Yes	B.2-Specialized (large collection)
C-Major extinct	D-Attested extinct		E-Minor extinct
F-Archaic Hieroglyphic or Ideographic			G-Obscure or questionable usage symbols
4. Is a repertoire including character names provided?			Yes
a. If YES, are the names in accordance with the "character naming guidelines" in Annex L of P&P document?			Yes
b. Are the character shapes attached in a legible form suitable for review?			Yes
5. Fonts related:			
a. Who will provide the appropriate computerized font to the Project Editor of 10646 for publishing the standard?	Andreas Stötzner		
b. Identify the party granting a license for use of the font by the editors (include address, e-mail, ftp-site, etc.):	Andreas Stötzner Gestaltung, Klaufügelweg 21, 88400 Biberach/R., Germany, as@signographie.de		
6. References:			
a. Are references (to other character sets, dictionaries, descriptive texts etc.) provided?			Yes
b. Are published examples of use (such as samples from newspapers, magazines, or other sources) of proposed characters attached?			Yes
7. Special encoding issues:			
Does the proposal address other aspects of character data processing (if applicable) such as input, presentation, sorting, searching, indexing, transliteration etc. (if yes please enclose information)?			No

8. Additional Information:

Submitters are invited to provide any additional information about Properties of the proposed Character(s) or Script that will assist in correct understanding of and correct linguistic processing of the proposed character(s) or script. Examples of such properties are: Casing information, Numeric information, Currency information, Display behaviour information such as line breaks, widths etc., Combining behaviour, Spacing behaviour, Directional behaviour, Default Collation behaviour, relevance in Mark Up contexts, Compatibility equivalence and other Unicode normalization related information. See the Unicode standard at <http://www.unicode.org> for such information on other scripts. Also see Unicode Character Database (<http://www.unicode.org/reports/tr44/>) and associated Unicode Technical Reports for information needed for consideration by the Unicode Technical Committee for inclusion in the Unicode Standard.

¹ Form number: N4502-F (Original 1994-10-14; Revised 1995-01, 1995-04, 1996-04, 1996-08, 1999-03, 2001-05, 2001-09, 2003-11, 2005-01, 2005-09, 2005-10, 2007-03, 2008-05, 2009-11, 2011-03, 2012-01)

C. Technical - Justification

1. Has this proposal for addition of character(s) been submitted before?	Yes
If YES explain see L2/24-244 (L-2438) N5277 / L-24-02n	
2. Has contact been made to members of the user community (for example: National Body, user groups of the script or characters, other experts, etc.)?	Yes
If YES, with whom? Leibniz-Archiv, Forschungsstelle der Leibniz-Edition, Niedersächsische Landesbibliothek (GWLb), Hanover, Göttingen Academy of Science and Humanities in Lower Saxony (DE), Philium research group of CNRS (UMR 7219, laboratoire SPHERE) / Université de Paris VII; general: scholars, researchers, authors and editors working in the field of science history and upon editions of historic text corpora (e.g. of G. W. Leibniz, but also many others)	
If YES, available relevant documents: L-2409, L-2410	
3. Information on the user community for the proposed characters (for example: size, demographics, information technology use, or publishing use) is included?	Yes
Reference:	
4. The context of use for the proposed characters (type of use; common or rare)	Common
Reference: mainly specialist usage, scholarly, worldwide	
5. Are the proposed characters in current use by the user community?	Yes
If YES, where? Reference: mainly Europe, Americas; other countries	
6. After giving due considerations to the principles in the P&P document must the proposed characters be entirely in the BMP?	No
If YES, is a rationale provided?	
If YES, reference:	
7. Should the proposed characters be kept together in a contiguous range (rather than being scattered)?	Yes
8. Can any of the proposed characters be considered a presentation form of an existing character or character sequence?	No
If YES, is a rationale for its inclusion provided?	
If YES, reference:	
9. Can any of the proposed characters be encoded using a composed character sequence of either existing characters or other proposed characters?	No
If YES, is a rationale for its inclusion provided?	
If YES, reference:	
10. Can any of the proposed character(s) be considered to be similar (in appearance or function) to, or could be confused with, an existing character?	No
If YES, is a rationale for its inclusion provided?	
If YES, reference:	
11. Does the proposal include use of combining characters and/or use of composite sequences?	No
If YES, is a rationale for such use provided?	
If YES, reference:	
Is a list of composite sequences and their corresponding glyph images (graphic symbols) provided?	No
If YES, reference:	
12. Does the proposal contain characters with any special properties such as control function or similar semantics?	No
If YES, describe in detail (include attachment if necessary)	
13. Does the proposal contain any Ideographic compatibility characters?	No
If YES, are the equivalent corresponding unified ideographic characters identified?	
If YES, reference:	