L2/25-124

ISO/IEC JTC1/SC2/WG2 Nxxxx
LUCP L-2511

Universal Multiple-Octet Coded Character Set
International Organization for Standardization
Internationale Standardisierungs-Organisation
Organisation Internationale de Normalisation
Διεθνής Οργανισμός Τυποποίησης
Международная организация по стандартизации

Doc Type: Working Group Document

Title: Proposal to encode 10 mathematical symbols

Source: Uwe Mayer, Siegmund Probst, David Rabouin, Elisabeth Rinner, Andreas Stötzner, Achim Trunk, Charlotte Wahl
Version: 2nd revised version
Previous version: proposal doc. L-2447
Status: forward to Script Encoding Working Group / WG2
Action: for expert review and encoding pipeline
Date: March 25, 2025
Requester's reference: LUCP L-2511

1. Background

With this document we deliver an updated and extended version of the fifth of separate proposals which follow our initial proposal for encoding of 228 historic scientific characters. A number of manuscript samples have been added, to better illustrate the appearance of the characters in printed texts.
For more background information about the Philiumm project (headed by Prof. David Rabouin, Paris) and the related research work, please visit the Philiumm website or see the doc. no. N5277.

2. Mathematical-geometrical symbols in historic sources: ▽ ▽ ▽ ▽ ∧ ∠ ∠ ∠ ∠

The symbols proposed here are testified in historic mathematical works by G. W. Leibniz. Leibniz always aimed to achieve a detailed and diversified notation for special problems or concepts. The sets of four "sector" symbols and of five "angle" symbols represent a certain way of mathematical thinking, therefore the single characters not only bear several unique meanings respectively but they also have a certain meaning *as sets* – as is with many other series of related mathematical symbols.
We propose to make these 10 characters a part of a new 'Math Supplement' block.

3. Characters

If this proposal gets accepted, the following 10 characters will exist:


SMALL SECTOR


SMALL SECTOR WITH CHORD


SMALL SECTOR WITH DOUBLE ARC


SMALL SECTOR TRIANGLE


HYPERBOLA


ANGLE-1


ANGLE-2


ANGLE-3


ANGLE-4


ANGLE OPENING UP

The symbol for SMALL SECTOR ▽ is distinct from 2314 SECTOR by its sharp angle instead of a right angle and should therefore not get unified with the latter, for the sake of unambiguity. SMALL SECTOR ▽ is also to be the basic glyph shape for the other three sector symbols proposed, which feature the same proportion.

The symbols ANGLE-1 to ANGLE-4 ⦟ ⦠ ⦡ ⦢ constitute a fixed set of specialized angle symbols which extend the range of existing angle symbols encoded at 2220–2222 and 299B to 29AF. The fifth angle symbol ANGLE OPENING UP ⩛ corresponds to 29A1 SPHERICAL ANGLE OPENING UP. Despite its similar shape it ought not to be unifed with 2228 LOGICAL OR because in publications a distinction between the *logical* expression and a *geometry*-related angle symbol needs to be maintained. The glyphs representing the logical operators (2227–222A) are smaller and wider, in proportion similar to e.g. the + or = symbols; whereas the default basic shape of the angle symbols is larger and more slim.

4. Unicode Character Properties

```
yd01;SMALL SECTOR;Sm;0;ON;;;;;Y;;;;;
yd02;SMALL SECTOR WITH CHORD;Sm;0;ON;;;;;Y;;;;;
yd03;SMALL SECTOR WITH DOUBLE ARC;Sm;0;ON;;;;;Y;;;;;
yd04;SMALL SECTOR TRIANGLE;Sm;0;ON;;;;;Y;;;;;
yd05;HYPERBOLA;Sm;0;ON;;;;;Y;;;;;
yd06;ANGLE-1;Sm;0;ON;;;;;Y;;;;;
yd07;ANGLE-2;Sm;0;ON;;;;;Y;;;;;
yd08;ANGLE-3;Sm;0;ON;;;;;Y;;;;;
yd09;ANGLE-4;Sm;0;ON;;;;;Y;;;;;
yd10;ANGLE OPENING UP;Sm;0;ON;;;;;Y;;;;;
```

"y" stands for *unspecified codespace*. "d" refers to our internal characters classification, see N5277.

5. Bibliography

LAA – refers to: Leibniz, Gottfried Wilhelm: Sämtliche Schriften und Briefe. ('Leibniz-Akademie-Ausgabe', many volumes)
LH – refers to: Leibniz's original manuscripts, GWLB Hanover

Rinner, Elisabeth: List of glyphs in Leib.mf. PDF, Hanover 2022

Leibniz-Akademie-Ausgabe (LAA, general edition of Leibniz's writings)

LAA series VII (mathematical manuscripts, volumes 3 to 7 available online)

6. Figures and explanations

videmur obtinuisse, ut hoc pacto quadratura circuli devenerit problema solidum solubile, et construi possit, quemadmodum problemata solida omnia. Sed in eo malum est, quod una tantum est cognita a^2. Si quaedam b. aequationem ingrederetur, tunc solvi posset problema ope parabolae, deberet nimirum fieri aequatio talis posito $y = x$.

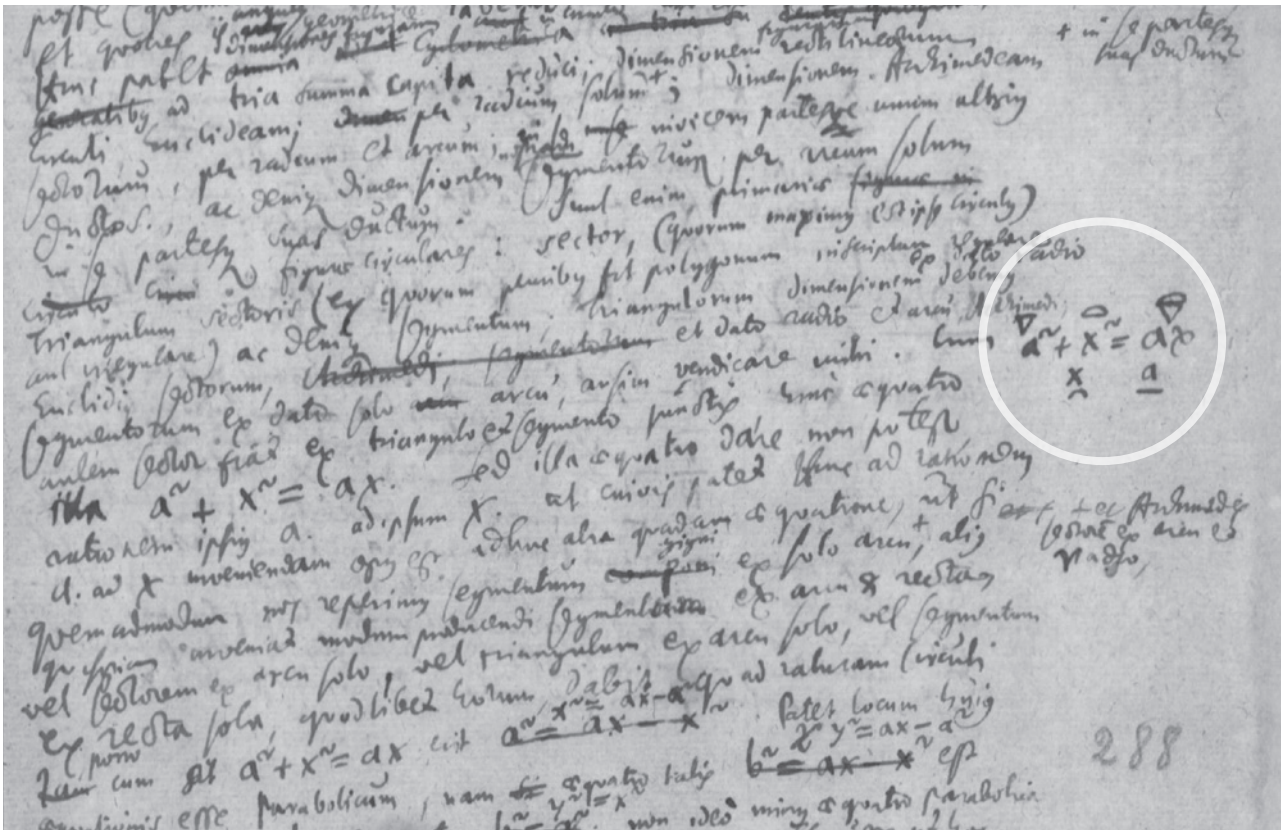
$$y^2 = ax - b^2. \quad \text{vel} \quad x^2 = [ay] - b^2.$$

haberemus solutionem saltem per parabolam, seu locum solidum. Quare si quis exhibere posset segmentum circuli aequale cuidam sectori cuius arcus est radix segmenti demto quodam quadrato cuius radix est alia a radio. Sed his non opus, sufficit prior illa aequatio:

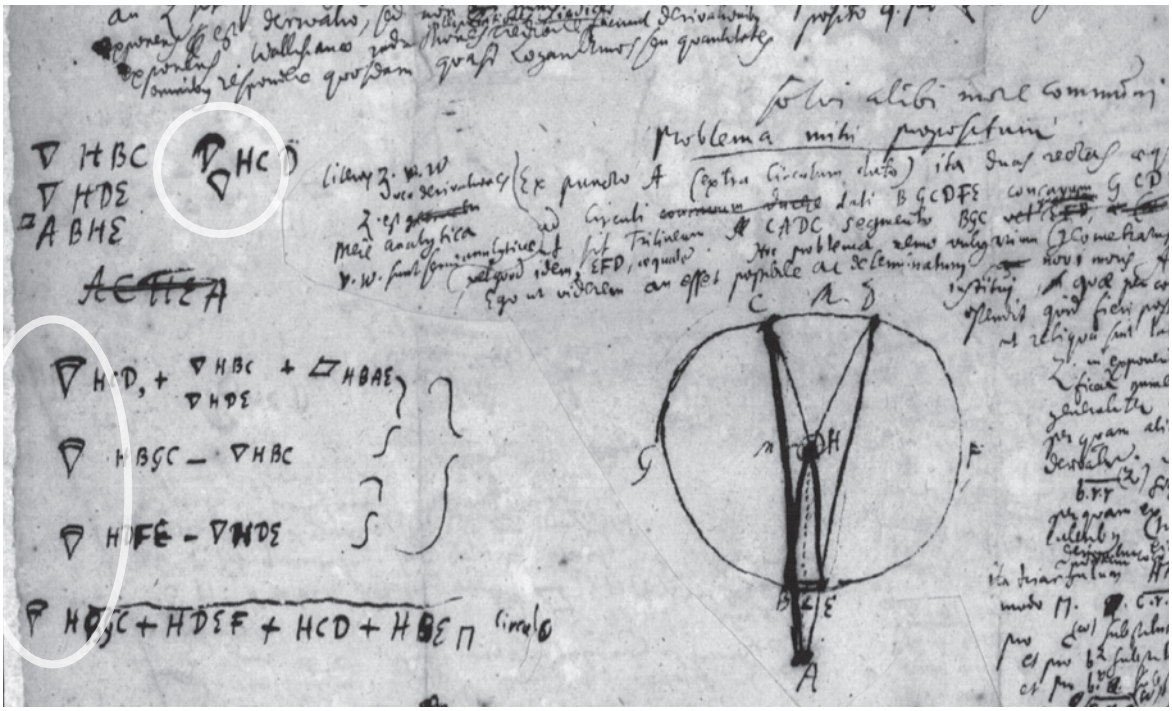
$$\frac{x^2}{\alpha} = \frac{bx}{\beta} - b^2.$$

1

$\triangledown$ SMALL SECTOR WITH CHORD – LAA VII-4 p. 192

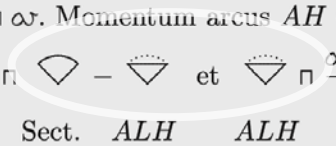
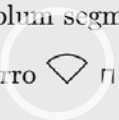


$\triangledown$ SMALL SECTOR WITH CHORD
The corresponding Ms.: LH 35 II 1, fol. 287v



▽ SMALL SECTOR, ▽ SMALL SECTOR WITH DOUBLE ARC
LH 35 I 14 fol. 88v. *The edition of this manuscript is currently in progress*.

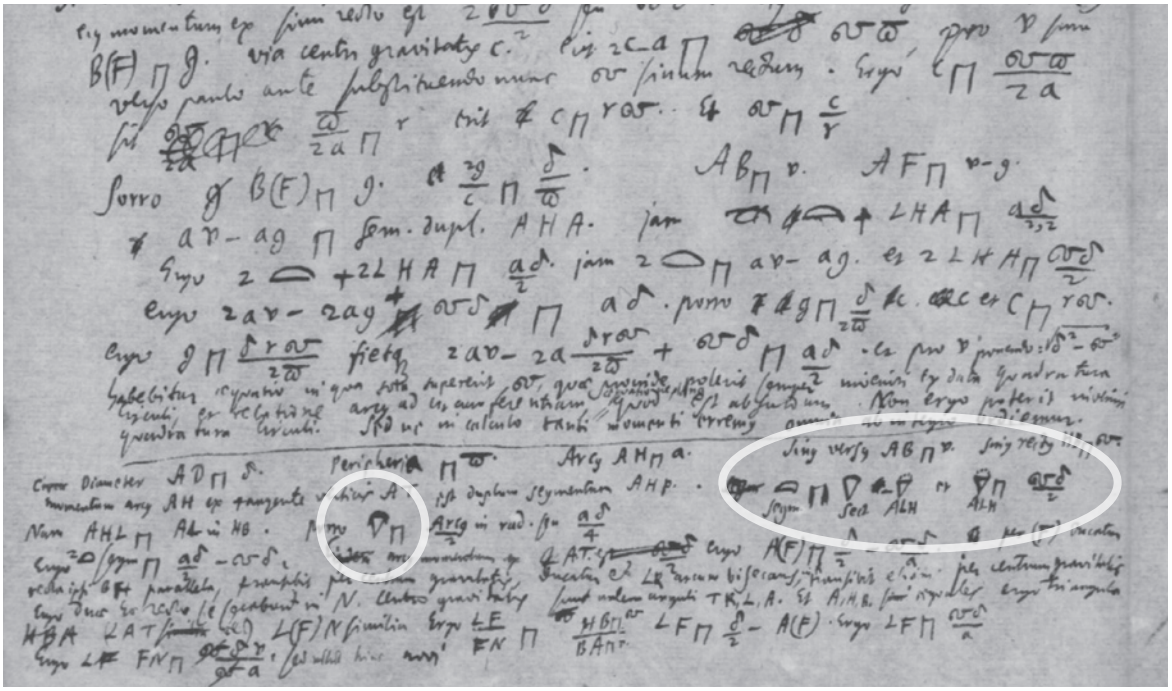
gravitatis c. erit $2ca \sqcap \omega\pi$, pro v sinu verso paulo ante substituendo nunc ω sinum rectum. Ergo $c \sqcap \frac{\omega\pi}{2a}$. Sit $\frac{\pi}{2a} \sqcap r$ erit $c \sqcap r\omega$. Et $\omega \sqcap \frac{c}{r}$. Porro $B(F) \sqcap g$. et $\frac{2g}{c} \sqcap \frac{\delta}{\pi}$. $AB \sqcap v$. $A(F) \sqcap v - g$. $av - ag \sqcap$ se[g]m. dupl. AHA. Jam $\overset{\frown}{} + LHA \sqcap \frac{a\delta}{2,2}$. Ergo $2\overset{\frown}{} + 2LHA \sqcap \frac{a\delta}{2}$. Jam $2\overset{\frown}{} \sqcap av - ag$. et $2LHA \sqcap \frac{\omega\delta}{2}$ ergo $2av - 2ag + \omega\delta \sqcap a\delta$. Porro $g \sqcap \frac{\delta}{2\pi}c$. et $c \sqcap r\omega$. Ergo $g \sqcap \frac{\delta r\omega}{2\pi}$ fietque $2av - 2a\frac{\delta r\omega}{2\pi} + \omega\delta \sqcap \frac{a\delta}{2}$. et pro v ponendo: $\sqrt{\delta^2 - \omega^2}$ habebitur aequatio in qua sola supererit ω, quae proinde poterit semper inveniri ex data Quadratura Circuli, et relatione arcus ad circumferentiam, aequatione plana quod est absurdum. Non ergo poterit inveniri quadratura circuli. Sed ne in calculo tanti momenti erremus omnia ab integro ordiemur.

Diameter $AD \sqcap \delta$. Peripheria $\sqcap \pi$. Arcus $AH \sqcap a$. Sinus versus $AB \sqcap v$. Sinus rectus $HB \sqcap \omega$. Momentum arcus AH ex tangente verticis AT est duplum segmentum AHR. $\overset{\frown}{} \sqcap$  et $\sqcap \frac{\omega\delta}{2}$. Nam $AHL \sqcap AL$ in HB. Porro  $\sqcap \frac{\text{Arcus}}{2}$

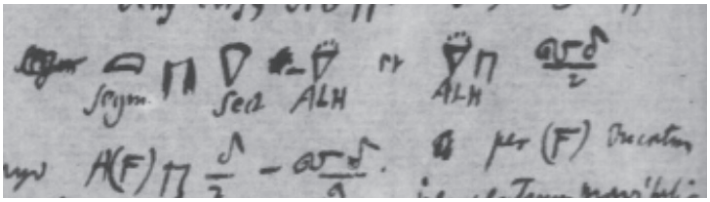
Segm. Sect. ALH ALH

in rad. seu $\frac{a\delta}{4}$. Ergo $2\overset{\frown}{}$ segm. $\sqcap \frac{a\delta}{2} - \omega\delta$, arcus momentum ex AT. Ergo $A(F) \sqcap \frac{\delta}{2} -$

▽ SMALL SECTOR and ▿ SMALL SECTOR TRIANGLE – LAA VII-5 p. 555
The too wide angle of the glyphs in this example is not prescriptive.



▽ SMALL SECTOR and ▿ SMALL SECTOR TRIANGLE
Corresponding Ms.:
LH 35 VI 5, fol.11v



Mercatoris per divisionem; quia, ita series qualescunque propositae etiam irregulares satis nec ordine procedentes, ad figuram convenientem, revocantur, qualis ista est: $\frac{b}{1} - \frac{b^3}{3} + \frac{b^2}{2}$ etc. Variae aliae coniunctiones institui possunt, ut ista:

$$\overbrace{\frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4}} + \overbrace{\frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8}} + \overbrace{\frac{1}{9} - \frac{1}{10} + \frac{1}{11} - \frac{1}{12}} \text{ [etc.]}$$

$$\frac{3}{4} - \frac{1}{6} + \frac{3}{40} - \frac{1}{42} + \frac{3}{108} - \frac{1}{110} \text{ etc.}$$

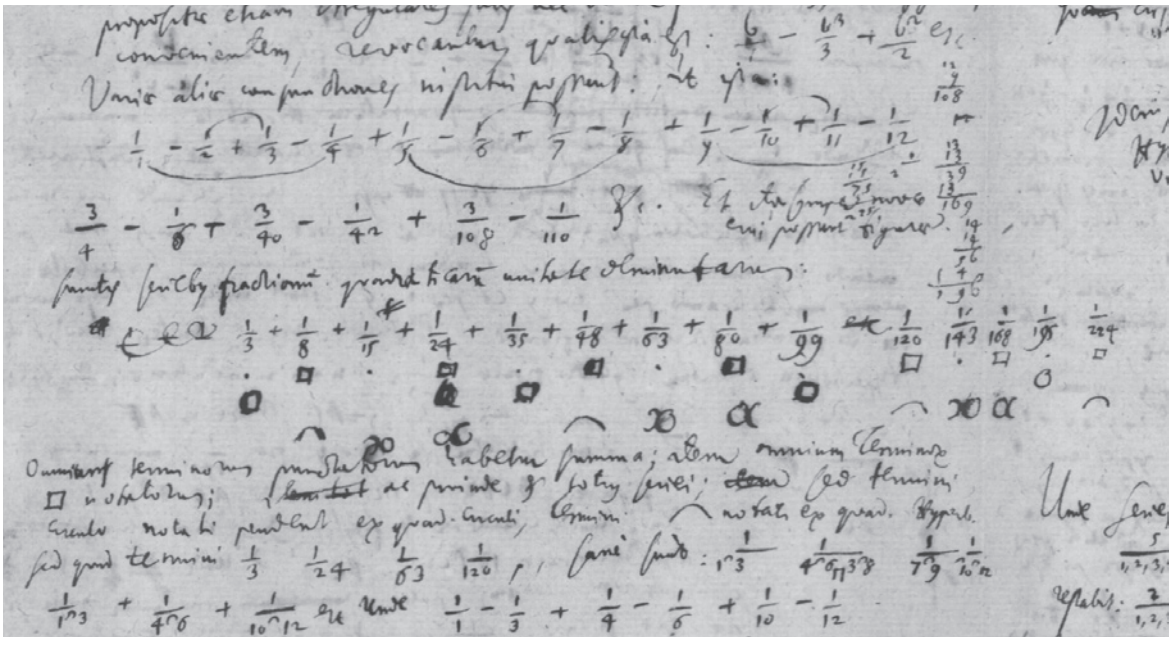
Et ita semper novae erui possunt figurae. Sumtis seriebus fractionum quadraticarum unitate deminutarum:

$$\frac{1}{3} + \frac{1}{8} + \frac{1}{15} + \frac{1}{24} + \frac{1}{35} + \frac{1}{48} + \frac{1}{63} + \frac{1}{80} + \frac{1}{99} + \frac{1}{120} + \frac{1}{143} + \frac{1}{168} + \frac{1}{195} + \frac{1}{224} \text{ [etc.]}$$

Omnium terminorum punctatorum habetur summa; item omnium terminorum □ notatorum; ac proinde et totius seriei; sed termini circulo notati pendent ex quad. circuli, termini ⌒ notati ex quad. hyperb.

Sed quid termini $\frac{1}{3}$ $\frac{1}{24}$ $\frac{1}{63}$ $\frac{1}{120}$ [etc.], sane sunt: $\frac{1}{1 \frown 3}$ $\frac{1}{4 \frown 6 \sqcap 3 \frown 8}$ $\frac{1}{7 \frown 9}$ $\frac{1}{10 \frown 12}$ [etc.]

⌒ HYPERBOLA
LAA VII-3 p. 386



The corresponding Ms.: LH 35 V 4 fol. 2v

Quoniam autem series

$\frac{1}{3}\ \frac{1}{15}\ \frac{1}{35}\ \frac{1}{63}\ \frac{1}{99}\ \frac{1}{143}$ etc. a me inventa est; et series

$\frac{1}{3}\ \ \frac{1}{35}\ \ \frac{1}{99}$ etc. pendet ex quadratura circuli, itaque series

$\frac{1}{15}\ \ \frac{1}{63}\ \ \frac{1}{143}$ [etc.] etiam pendebit ex quadratura circuli.

$\frac{1}{3}\ \ \frac{1}{35}\ \ \frac{1}{99}$ [etc.] resoluta dant:

$\frac{1}{1 \frown 3}\ \ \frac{1}{5 \frown 7}\ \ \frac{1}{9 \frown 11}$ etc., cuius seriei origo est

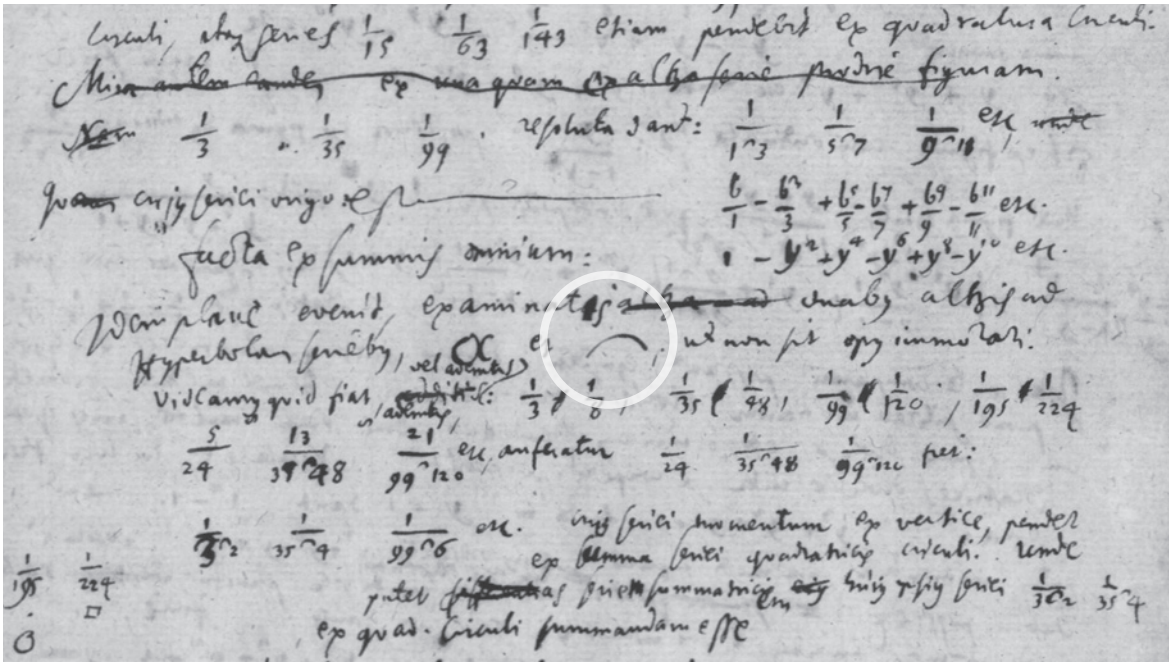
$\frac{b}{1} - \frac{b^3}{3} + \frac{b^5}{5} - \frac{b^7}{7} + \frac{b^9}{9} - \frac{b^{11}}{11}$ etc. facta ex summis omnium:

$1 - y^2 + y^4 - y^6 + y^8 - y^{10}$ etc.

Idem plane evenit, examinatis duabus alteris ad hyperbolam seriebus, $\propto$ et $\frown$; ut non sit opus immorari. Videamus quid fiat, ademtis:

3 f. $\frac{y^2}{1+y^2}$. *(1)* Eodem modo sumatur series, alia per saltus tertianos, quam ita notavi $\propto . \frac{1}{3}\ \frac{1}{35}\ \frac{1}{99}$ *(2)* Quoniam autem *(a)* constat seriem *(3)* series *L* 7–389,6 etc. *erg. Hrsg. fünfmal* 7 f. circuli. *(1)* Miror autem eandem ex una quam ex altera serie prodire figuram. Nam *(2)* $\frac{1}{2}$ *L* 9 etc., *(1)* unde

$\frown$ HYPERBOLA
LAA VII-3 p. 388



The corresponding Ms.: LH 35 V 4 fol. 3r

13 Zu Fig. 3: Nach Aussage (4) soll D ein beliebiger Punkt auf dem Quadranten AO sein. Leibniz hat in seiner Handzeichnung den Bogen AD jedoch gleich 60° gewählt, wodurch die Allgemeinheit verloren gegangen ist. Leibniz hat dies, wie die Zusätze neben der Figur zeigen, später bemerkt. Er hat aber keine neue Zeichnung angefertigt, sondern hat sich damit begnügt, den allgemeinen Fall mittels Einzeichnen der Linie $P\varpi\varphi$, der Verlagerung der Linie $A\beta\alpha$ sowie vieler zusätzlicher Winkelmarkierungen darzustellen. Hierbei bedeuten $\angle = 25°$; $\measuredangle = 50°$; $\angle = 65°$ und $\measuredangle = 40°$. — Die Handzeichnung ist bis auf einige wenige Winkelangaben korrekt. 14 ~~AN~~: s. dazu N. 29 S. 523 Z. 22 – S. 524 Z. 8 .
15 modo: Eine ähnlich unbestimmte Haltung bezüglich der Existenz des Höhenschnittpunkts im Dreieck nimmt Leibniz *LSB* VII, 1 N. 2 S. 4 ein.

∡ ANGLE-1, ⦟ ANGLE-2, ∡ ANGLE-3, ⦟ ANGLE-4 – LAA VII-4 p. 409

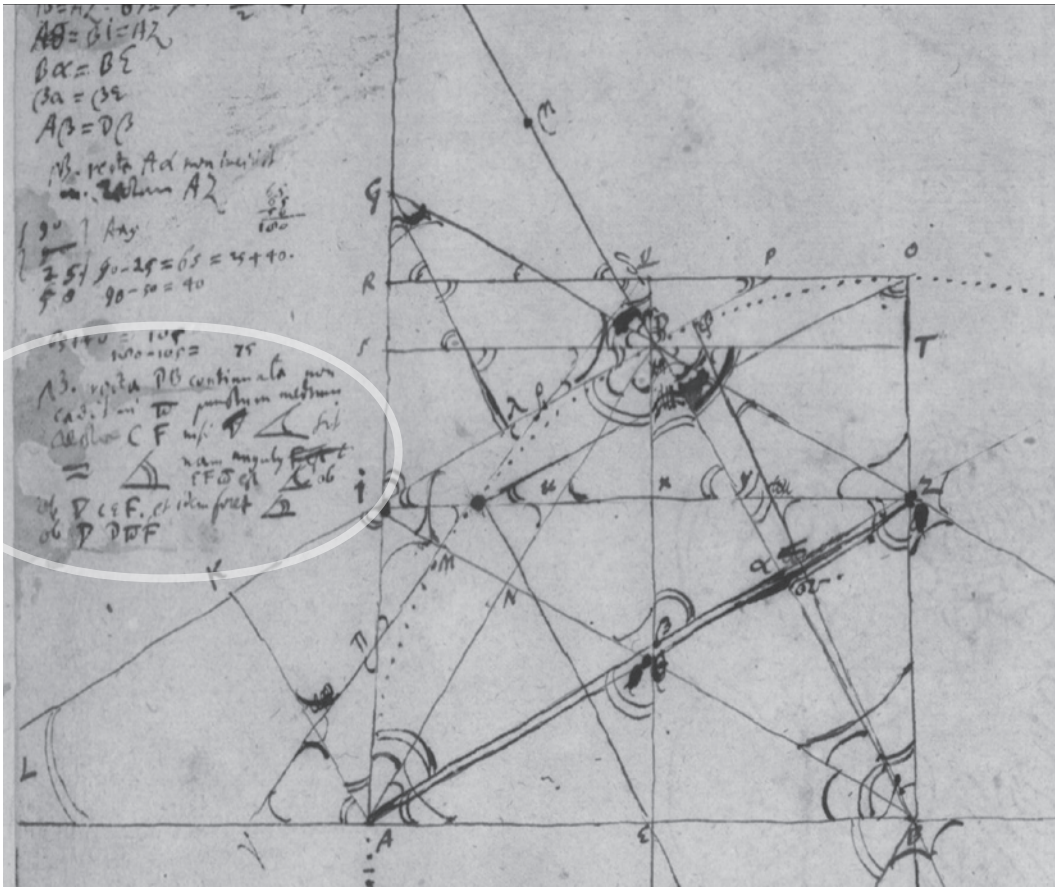
90	$90 - 25 = 65 = 25 + 40$	65
25 Ang.	$90 - 50 = 40$	65
50	$65 + 40 = 105$	50
	$180 - 105 = 75$	180

NB. recta DB continuata non cadit in ϖ punctum medium rectae CF nisi $\angle$ sit = $\measuredangle$ nam angulus $EF\varpi$ est $\angle$ ob $\bigtriangledown CEF$. et idem foret $\measuredangle$ ob $\bigtriangledown D\varpi F$.

[*Teil 2*]

Determinatio punctorum, sive quantitas linearum in fig. 3.

⦟ ANGLE-2, ∡ ANGLE-3 – LAA VII-4 p. 410



Corresponding Ms.: LH 35 II 1, fol. 143r

In circulo AB ducta applicata seu sinu CD iunctisque chordis AD. DB erit $\bigtriangledown^{lo}$ ADB simile ADC. quia $\measuredangle ACD = \measuredangle ADB$. rectus recto et $\measuredangle DAB = \measuredangle DAC$. ergo $\measuredangle ADC = \measuredangle DBA$. Eodem modo $\bigtriangledown^{lum}$ DCB simile utrique.

Ergo $\frac{AB}{AD} = \frac{AD}{AC}$. Ergo $AB \frown AC = AD \frown AD$. seu rectangulum sub diametro et sinu verso aequatur quadrato chordae.

∨ ANGLE OPENING UP
LAA VII-4 p. 377

Si dividantur omnia per ab vel a est genus quoddam solidi hyperboloeiformis, quod quadrari potest.

Erit $\frac{a-1}{a-1}\ \frac{4a-4}{a-2}\ \frac{9a-9}{a-3} = a+a+a$ etc. summa a^2. Ecce planum hyperboliforme quadrabile.

Ergo ista rectangula ita crescent: $\frac{b^2a}{a-b} = a^2\ \ \frac{2b^2a}{a-2b} = a^2\ \ \frac{3b^2a}{a-3b}$ etc. Unde apparet solidum istud ex rectangulis factum aequari momento hyperbolico seu ungulae.

Videndum exactius.

[*Teil 2*]

In $\bigtriangledown^{lo}$ ADL radius AL in sinum $CD = AD$ sin. dimidii duplicatum, seu chordam arcus dati in LM sinum complementi arcus dimidii.

$\measuredangle TUD = LDB$. Ergo UDT et ADL ($\measuredangle^{li}$) aequales, ergo $\bigtriangledown^{la}$ UTD et ADK similia, item LMD, item HDL. MTW ang. $= LDB$. ang. $TMW = ADL$.

$\bigtriangledown DML = \bigtriangledown MTW$.

Ang. ADC dimid. ang. ALE (alter ad centrum, alter ad circumferentiam, super eodem arcu AE). Ergo et HDI $\measuredangle$ duplus ADC (quia $HD = DI$ et $HM = MI$) $= ALD$. qui est $= ALE$. quia AD arcus $= AE$.

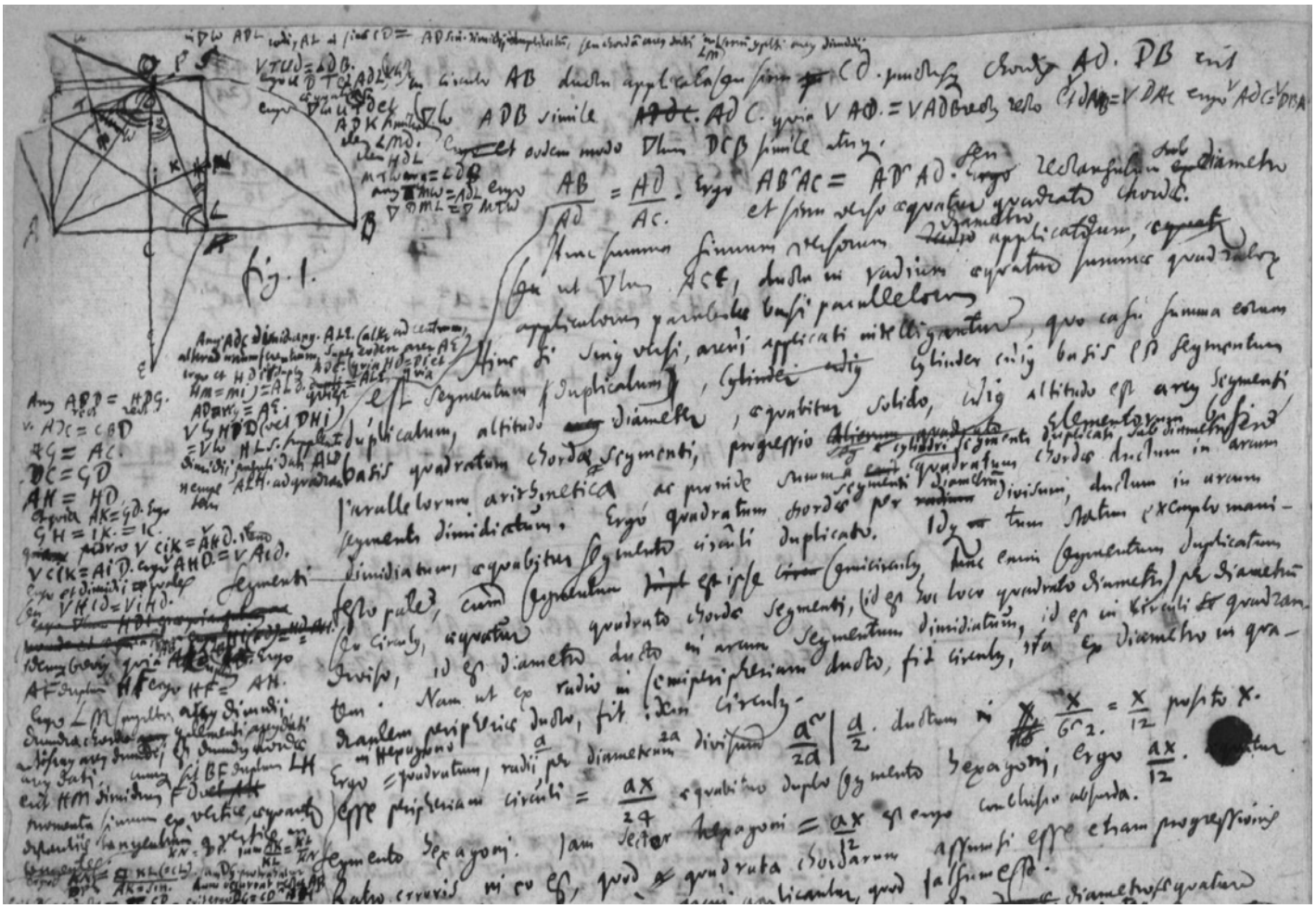
$\measuredangle^{lus}$ HID (vel DHI) $= \measuredangle^{lo}$ HLS. supplenti dimidii anguli dati ALD nempe ALH ad quadrantem.

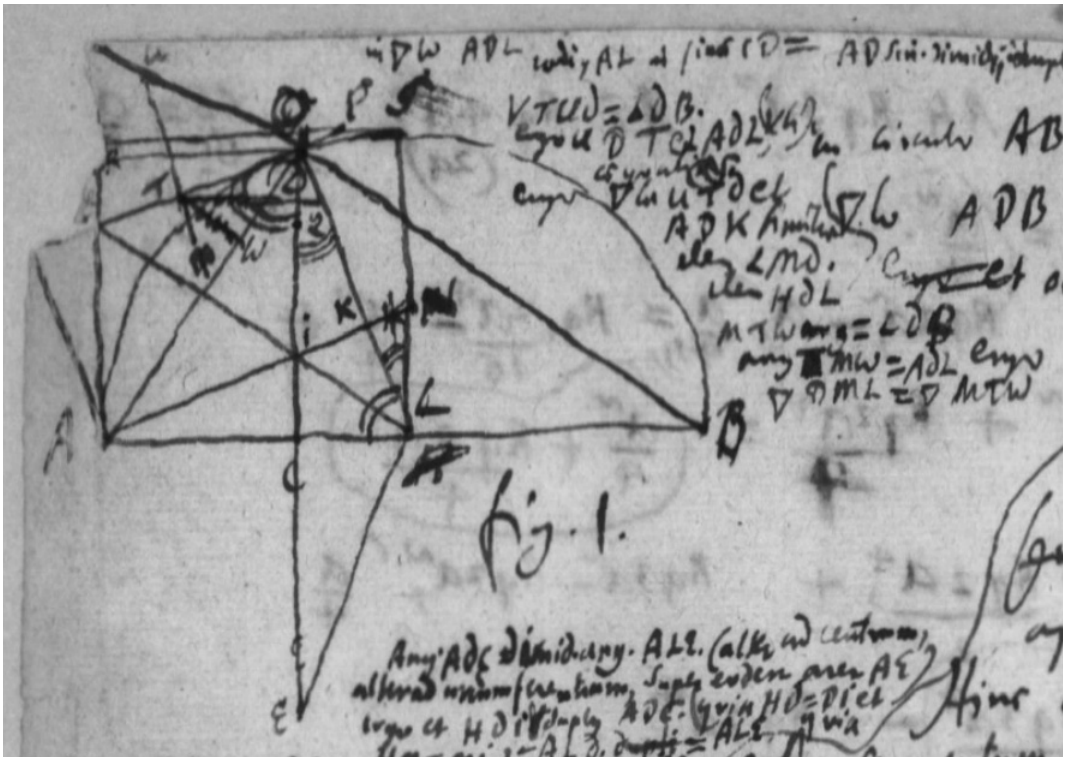
Ang. ADB rect. $= AGD$ rect. $\measuredangle ADC = CBD$. $AG = AC$. $DC = GD$. $AH = HD$. et quia $AK = GD$. ergo $GH = IK = IC$. Porro $\measuredangle CIK = \measuredangle AHD$. item $\measuredangle CIK = AID$. ergo $\measuredangle AHD = \measuredangle AID$. Ergo et dimidii aequales seu $\measuredangle HID = \measuredangle IHD$.

1 per *(1)* a^2b *(2)* ab vel a *L* 4 f. quadrabile. *(1)* Incipiatur inverso modo $\frac{\text{sinus versus b} \frown \text{chorda Rq ba}}{\text{sinus versus supplementi a} - \text{b}} \frown$ *(a)* b^2 *(b)* chorda $= \frac{b^2a}{a-b} = a^2$ *(2)* $\frac{2b^2a}{a-2b}$ *(3)* Ergo *L* 21–386,1 $\measuredangle$IHD. *(1)* Ergo $\bigtriangledown^{lum}$ HDI est aequiangulum, ac proinde et aequilaterum ergo HI (FD) = HD = AH. *(2)* Idemque *L*

∨ ANGLE OPENING UP
LAA VII-4 p. 385

Ms.: see next page





LH 35 II 1, fol. 268v

ISO/IEC JTC 1/SC 2/WG 2
PROPOSAL SUMMARY FORM TO ACCOMPANY SUBMISSIONS
FOR ADDITIONS TO THE REPERTOIRE OF ISO/IEC 10646[1]
Please fill all the sections A, B and C below.
Please read Principles and Procedures Document (P & P) from http://std.dkuug.dk/JTC1/SC2/WG2/docs/principles.html **for guidelines and details before filling this form.**
Please ensure you are using the latest Form from http://std.dkuug.dk/JTC1/SC2/WG2/docs/summaryform.html**.**
See also http://std.dkuug.dk/JTC1/SC2/WG2/docs/roadmaps.html **for latest *Roadmaps*.**

A. Administrative

1. **Title:** Proposal to encode 10 mathematic symbols
2. Requester's name: Uwe Mayer, Siegmund Probst, David Rabouin, Elisabeth Rinner, Andreas Stötzner, Achim Trunk, Charlotte Wahl
3. Requester type (Member body/Liaison/Individual contribution): Individual (work group)
4. Submission date: 2025-03.25
5. Requester's reference (if applicable): LUCP L-2511
6. Choose one of the following:
   This is a complete proposal: Yes
   (or) More information will be provided later:

B. Technical – General

1. Choose one of the following:
   a. This proposal is for a new script (set of characters): No
      Proposed name of script:
   b. The proposal is for addition of character(s) to an existing block: No
      Name of the existing block:
2. Number of characters in proposal: 10
3. Proposed category (select one from below - see section 2.2 of P&P document):
   A-Contemporary ; B.1-Specialized (small collection) Yes ; B.2-Specialized (large collection) ;
   C-Major extinct ; D-Attested extinct ; E-Minor extinct ;
   F-Archaic Hieroglyphic or Ideographic ; G-Obscure or questionable usage symbols
4. Is a repertoire including character names provided? Yes
   a. If YES, are the names in accordance with the "character naming guidelines" in Annex L of P&P document? Yes
   b. Are the character shapes attached in a legible form suitable for review? Yes
5. Fonts related:
   a. Who will provide the appropriate computerized font to the Project Editor of 10646 for publishing the standard?
      Andreas Stötzner
   b. Identify the party granting a license for use of the font by the editors (include address, e-mail, ftp-site, etc.):
      Andreas Stötzner Gestaltung, Klauflügelweg 21, 88400 Biberach/R., Germany, as@signographie.de
6. References:
   a. Are references (to other character sets, dictionaries, descriptive texts etc.) provided? Yes
   b. Are published examples of use (such as samples from newspapers, magazines, or other sources) of proposed characters attached? Yes
7. Special encoding issues:
   Does the proposal address other aspects of character data processing (if applicable) such as input, presentation, sorting, searching, indexing, transliteration etc. (if yes please enclose information)? No

8. Additional Information:

Submitters are invited to provide any additional information about Properties of the proposed Character(s) or Script that will assist in correct understanding of and correct linguistic processing of the proposed character(s) or script. Examples of such properties are: Casing information, Numeric information, Currency information, Display behaviour information such as line breaks, widths etc., Combining behaviour, Spacing behaviour, Directional behaviour, Default Collation behaviour, relevance in Mark Up contexts, Compatibility equivalence and other Unicode normalization related information. See the Unicode standard at http://www.unicode.org for such information on other scripts. Also see Unicode Character Database (http://www.unicode.org/reports/tr44/) and associated Unicode Technical Reports for information needed for consideration by the Unicode Technical Committee for inclusion in the Unicode Standard.

[1] Form number: N4502-F (Original 1994-10-14; Revised 1995-01, 1995-04, 1996-04, 1996-08, 1999-03, 2001-05, 2001-09, 2003-11, 2005-01, 2005-09, 2005-10, 2007-03, 2008-05, 2009-11, 2011-03, 2012-01)

C. Technical - Justification

1. Has this proposal for addition of character(s) been submitted before? Yes

   If YES explain *see L-2447,* N5277 / L-24-02n

2. Has contact been made to members of the user community (for example: National Body, user groups of the script or characters, other experts, etc.)? Yes

   If YES, with whom? Leibniz-Archiv, Forschungsstelle der Leibniz-Edition, Niedersächsische Landesbibliothek (GWLB), Hanover, Göttingen Academy of Science and Humanities in Lower Saxony (DE), Philiumm research group of CNRS (UMR 7219, laboratoire SPHERE) / Université de Paris VII; general: scholars, researchers, authors and editors working in the field of science history and upon editions of historic text corpora (e.g. of G. W. Leibniz, but also many others)

   If YES, available relevant documents: L-2409, L-2410

3. Information on the user community for the proposed characters (for example: size, demographics, information technology use, or publishing use) is included? Yes

   Reference:

4. The context of use for the proposed characters (type of use; common or rare) Common

   Reference: mainly specialist usage, scholarly, worldwide

5. Are the proposed characters in current use by the user community? Yes

   If YES, where? Reference: mainly Europe, Americas; other countries

6. After giving due considerations to the principles in the P&P document must the proposed characters be entirely in the BMP? No

   If YES, is a rationale provided?

   If YES, reference:

7. Should the proposed characters be kept together in a contiguous range (rather than being scattered)? Yes

8. Can any of the proposed characters be considered a presentation form of an existing character or character sequence? No

   If YES, is a rationale for its inclusion provided?

   If YES, reference:

9. Can any of the proposed characters be encoded using a composed character sequence of either existing characters or other proposed characters? No

   If YES, is a rationale for its inclusion provided?

   If YES, reference:

10. Can any of the proposed character(s) be considered to be similar (in appearance or function) to, or could be confused with, an existing character? Yes

    If YES, is a rationale for its inclusion provided? *Yes*

    If YES, reference: *see p. 2*

11. Does the proposal include use of combining characters and/or use of composite sequences? No

    If YES, is a rationale for such use provided?

    If YES, reference:

    Is a list of composite sequences and their corresponding glyph images (graphic symbols) provided? No

    If YES, reference:

12. Does the proposal contain characters with any special properties such as control function or similar semantics? No

    If YES, describe in detail (include attachment if necessary)

13. Does the proposal contain any Ideographic compatibility characters? No

    If YES, are the equivalent corresponding unified ideographic characters identified?

    If YES, reference: