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## Title: Proposal to encode Leibnizian ambiguity signs

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### Background

With this document we deliver the sixth of separate proposals which follow our initial proposal for encoding of 228 historic scientific characters (N5277 / L-2402n), forwarded on February 19, 2024. For more background information about the Philiumm project (headed by Prof. David Rabouin, Paris) and the related research work, please visit the Philiumm website or see the doc. no. N5277. The preceeding proposals authored by our research group are:

L-2432 Proposal to add 7 historic alchemical symbols L-2438 Proposal to encode 11 cossic characters (L2/24-244) L-2442 Proposal to encode 5 historic mathematical operators L-2444 Proposal to encode 17 geometric shapes L-2447 Proposal to encode 10 mathematical symbols

This proposal requests the encoding of 59 ambiguity operator signs that are testified in works of Gottfried Wilhelm Leibniz (1646–1716), in editions of his works, and in literature from the field of history of mathematics. This document includes an in-depth explanation about the systematics and semantics of these signs. This explanation (author: Elisabeth Rinner) has been issued previously as doc. L-2404 (February 15, 2024).

The ambiguity characters are related to the well-known  $\pm$  and  $\mp$  characters (00B1, 2213), both by their graphical structure and historically. For editorial work the ambiguity signs are important for e.g. ascribing dates to manuscript sources which lack an original *datum*. The signs also inform about Leibniz's way of systematic thinking about how to notate certain logical concepts. We propose an encoding scheme of complete sets of ambiguity signs because incomplete sets

would be useless for editorial purposes.

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Ambiguity signs in one of Leibniz's manuscripts. The signs are based on the + and – symbols. (LH 35 XII 1, 217v)

### 1. Introductory remarks on Leibnizian Ambiguity signs

Early in his career, Leibniz wrote several texts in which he designed and systematically examined systems of symbols for analytical calculations. Complex systems of ambiguity signs, with which more than two cases are distinguished, represent an elementary and novel component of this *Méthode de l'universalité*. As part of the *Ars Characteristica*, the treatment of this method belongs to that branch of philosophy that is "the art of forming and arranging characters so that they agree with thoughts" (Mugnai 2018, abstract).

However, Leibniz's interest is not only theoretical. Rather, the design of higher ambiguity signs is closely linked to his occupation with the mathematics of conic sections. There he has to consider sub-cases of cases, but would like to write only one equation to treat them all together, since often the equations do not differ except for the signs of the terms. The use of double signs, which allows to represent two cases simultaneously, is already part of common practice in mathematics. The characters  $\pm$  and  $\mp$ , which are still in use today, are used for this purpose.

According to current knowledge, Leibniz designed six different systems over the course of time—as long as transitional forms and preliminary considerations are ignored.

One reason why a system of ambiguity signs is abandoned by Leibniz is the consideration that a large number of specific new printing types are required if a system does not rely on the traditional set of printing types. Leibniz's further penetration of the topic also

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led to improved, simpler or, in some cases, even more complex characters. The draft of the first system, for example, provides for special characters to express the product of the two double signs A-01  $\ddagger$  and A-02  $\ddagger$ . For these, Leibniz envisages the ligature A-07  $\ddagger$  and A-08  $\ddagger$  of these two symbols with the LEIBNIZIAN PRODUCT SIGN he typically uses. Only later does he take advantage of the fact that the mathematical meaning can also be expressed using existing symbols.

Some systems also take into account the relationship between several ambiguity signs in the same expression. Ambiguity signs can be *homogeneous* or *corresponding* and therefore dependent on one another, as well as *heterogeneous* and therefore independent of one another.

Likewise, it was only in his 5<sup>th</sup> system that Leibniz gave up structuring ambiguity signs according to the distinction between cases and sub-cases as they arise in the calculation process. Even though, from the perspective of modern mathematics, it makes no difference with regard to calculations whether the ambiguity sign (mp)m (i. e. a sign which has the subcases mp in the first case and m in the second case, with p as abbreviation for plus and m for minus) or m(pm) (i. e. a sign which has m in the first case and the sub-cases pm in the second case) is used, they do refer to two fundamentally different conceptions of the mathematical situation.

Design questions also play a role when considering the layout of systems of ambiguity signs, which lead Leibniz to the discussion of different positioning of lines and thus to variants that are compared to the systems ultimately favored.

Particularly in Leibniz's drafts, the ambiguity signs that occur can contribute to the dating of the texts, as a sequence of systems can be observed.

### 2. Overview of systems and character names for the UCS

Some systems of ambiguity signs are designed in such a way that they can be extended to distinguish any number of cases. In systems that use specific new characters and do not use the existing character set of a typesetting box, the surviving texts only contain characters that distinguish a maximum of four different cases. Usually, not all possible combinations of p and m occur in the texts. However, the systematics described or reconstructed on the basis of the surviving texts often allows to reconstruct the full set of ambiguity signs that belong to a system.

The overview in the appendix therefore contains only systems that use special new characters. For them, a list of all possible cases is provided. A representation of their glyphs is given, provided their use is documented in the texts written by Leibniz. In the overview, the meaning of the ambiguity signs is also stated in an abbreviated form.

For the encoding of Leibniz's ambiguity signs in the Unicode standard, we propose a name consisting of the components "AMBIGUITY SIGN", an identifier for the system ("A" for system 1, "B" for system 2, and "C" for system 5) which is followed by a hyphen, and a sequential number, with a leading zero being added to single-digit numbers. The character-specific parts of the proposed names are also included in the overview.

The characteristics of the systems are briefly described below with references to the overview.

## 2. 1 System 1

System 1 is based on the signs + for p and – for m that are still in use today, with A-01  $\ddagger$  being understood as a combination of these signs. The additional bar in A-02  $\ddagger$  represents negation, so the sign stands for mp.

When looking at the layout of the triple signs A-03 + $\ddagger$ , A-04 + $\ddagger$ , A-05 - $\ddagger$  and A-06 - $\ddagger$ , it can be seen that Leibniz takes the structure of possible distinctions of cases and sub-cases

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into account. In the texts, only signs for which a distinction between two sub-cases arises in the second case are described and documented. This second case with both sub-cases is represented in the right part of the sign in analogy to the associated double signs A-01  $\neq$  and A-02  $\neq$ : the two upper crossbars suspended from the vertical bar again refer to the combination of *p* and *m*, while the third, lower bar appearing in A-04  $\neq$  and A-06  $\neq$  represents the negation of this part of the ambiguity sign. The value of the first case is on the left side (*p* or *m*). This part is connected to the second crossbar from the top in the right part of the ambiguity sign.

A special feature of the 1<sup>st</sup> system are the ambiguity signs A-07  $\oplus$  and A-08  $\oplus$  that stand for products of the double signs of the system.

### 2. 2 System 2

System 2 develops from the same combination of + and – that forms the sign A-01  $\ddagger$ . Unlike in the first system, the negation of the sign is not expressed by a third crossbar with the same width, but by a longer, horizontal bar that is placed at the bottom of the vertical bar (e. g. in B-01  $\pm$ ). Such negations of the complete ambiguity sign can be applied to all of them, with B-05  $\pm$ , B-06  $\pm$ , B-07  $\pm$ , and B-10  $\pm$ <sup>+</sup> being examples.

As before, triple signs are composed of signs and double signs, with the first case on the left (subdivided or not) and the second case (not subdivided or subdivided) on the right. During this transition from double sign to triple sign, the negation bar of B-01  $\pm$  slides upwards, so to speak, so that the vertical bar in the partial sign that is given in one of Leibniz's texts, which rather coincidentally has the same design as A-02  $\pm$ , now protrudes at the bottom, and the width of the crossbar is adjusted to that of others in the sign. In contrast to the first system, both parts of the triple sign are composed by connecting the horizontal bar of the single sign part to the top bar of the double sign part. The ambiguity signs B-02  $\pm$ , B-03  $\pm$ , B-04  $\pm$ , B-05  $\pm$ , B-06  $\pm$ , B-07  $\pm$ , B-08  $\pm$ , B-09  $\pm$  and B-10  $\pm$  are of this kind.

The principle of composition is meant to be continued for distinguishing further ambiguities (i. e. the type of combination of cases that are distinguished). B-11  $\ddagger$  and B-12  $\ddagger$  which represent the negation of p(mp) and m(mp) make it clear that the negation bar is "moved up" again in these partial signs, and that its size corresponds to the size of all other crossbars.

As Leibniz discussed questions about the suitability of different positions of the crossbars when designing this system, the six ambiguity signs B-13  $\ddagger$ , B-14  $\ddagger$ , B-15  $\ddagger$ , B-16  $\ddagger$ , B-17  $\ddagger$ , and B-18  $\ddagger$  have come down to us. They represent variants of ambiguity signs of the standard form.

## 2. 3 System 3

Leibniz builds system 3 from the ambiguity signs A-01  $\pm$  and B-01  $\pm$ , which are used for *pm* and *mp* in the 2<sup>nd</sup> system. This means a reduction of the number of characters required, while still any complex ambiguity as well as all dependencies between ambiguity signs (i. e. whether they are *homogeneous*, *corresponding* or *heterogeneous*) can be expressed. To do this, numbers are added to the left and right of A-01  $\pm$  and B-01  $\pm$  according to certain given rules. Levels of case distinctions can also be expressed by building nested expressions according to rules. The entire expression is marked by a bracket with *vinculum* (i. e. they are connected by an overline).

As a ligature of the brackets "(" and ")" with the *vinculum* is needed, LEFT VIRGULA PARANTHESIS and RIGHT VIRGULA PARANTHESIS are included in the proposal to encode these expressions. The overview at the end of the document does not

contain any characters that are specifically assigned to this system since the glyphs of the ambiguity signs used in this system match with those of signs from the first two systems.

## 2. 4 System 4

System 4 has no new characters at all and instead uses lowercase letters of the Greek alphabet. Ambiguity is expressed by strings of certain pairs of letters such as  $\alpha$  and  $\omega$ ,  $\beta$  and  $\psi$ , or  $\gamma$  and  $\chi$ , where the two letters are equidistant from the beginning or end of the alphabet, and the letter from the beginning stands for *p* and the other for *m*. As in the 3<sup>rd</sup> system, the letters are written one after the other, following the order of the cases, and marked by brackets and a *vinculum*.

By using several pairs of letters it is possible to express the relationship between the ambiguity signs that occur in an expression, because the same pair of letters is used for interdependent ambiguity signs and different pairs of letters for independent ones. The level of cases can be represented by structuring these sequences with commas.

## 2. 5 System 5

The principles of the standard form of system 5 and all its predecessors were reconstructed on the basis of ambiguity signs that can be found in Leibniz's manuscripts.

In its final version (see 3. 5 subgroup "Standard" in the overview), the 5<sup>th</sup> system probably has the simplest structure in the design of the glyphs. The set of *n*-fold ambiguity signs consists of almost all possible *n*-combinations of *p* and *m*. All the signs with the meaning pp...p (string with *n* characters) and mm...m (string with *n* characters) are omitted as they have the same meaning as + and -. The level of cases is not represented in this system. Likewise, dependencies that occur between several ambiguity signs of the same expression are not represented.

The double signs C-16  $\ddagger$  and C-17  $\ddagger$ , the triple signs C-18  $\ddagger$ , C-19  $\ddagger$ , C-20  $\ddagger$ , C-21  $\ddagger$ , C-22  $\ddagger$ , and C-23  $\ddagger$  as well as the quadruple signs C-24  $\ddagger$ , C-25  $\ddagger$ , C-26  $\ddagger$ , C-27  $\ddagger$ , C-28  $\ddagger$ , C-29  $\ddagger$ , C-30  $\ddagger$ , and C-31  $\ddagger$  belong to this group.

Their design follows a uniform principle. On a vertical bar, horizontal bars of equal length are positioned at equal distances depending on the cases distinguished in the sign. If a bar represents p, it is bisected by the vertical bar. If it stands for m, it starts on the left at the same distance from the vertical bar as the p-bars, but already ends at the vertical bar.

This approach was derived from the  $2^{nd}$  system, and there is a total of four stages in the development of the standard version of system 5 of which only few examples of ambiguity signs have been preserved. The representation of the level of case distinction is a common feature of all these systems.

- The "Transition Form 2 → 5" (subgroup 3. 1) preserves the division into left and right part from the 2<sup>nd</sup> system as a means to represent two cases on the top level of case distinction. What is new, however, is the reduction of the representation of the cases on the second level, where, among other things, a crossbar shortened to the half width appears for the first time. The ambiguity sign C-01 <sup>+</sup> thas been handed down in this group.
- In the ambiguity signs of the group "prae-pro-proto-5" (subgroup 3. 2), instead of being divided into left and right halves to distinguish the two cases, there is a subdivision of the sign into an upper and lower section which is arranged along a vertical bar. In the case of the two known ambiguity signs C-02 \\$ and C-03 \$\$, a subordinate case distinction occurs in the first (upper) case, where *p* and *m* are expressed by a long and a short horizontal bar, respectively, which are positioned in the middle of the vertical bar and to its left, respectively. A connection of these two

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horizontal bars by a short vertical bar at their left which ends above the upper horizontal bar illustrates that they belong to the same group of sub-cases.

- From the group "pro-proto-5" (subgroup 3. 3), only the ambiguity signs C-04 \$\\$\$ and C-05 \$\\$\$ have come down to us. For both, signs that represent the same type of ambiguity in the previous group "prae-pro-proto-5" (subgroup 3. 2) are also known. Compared to them, the upper halves of the short vertical bars which illustrate in the previous variant that the horizontal bars that are connected by them belong to the same group of sub-cases are omitted, so that the resulting glyphs are further reduced compared to their predecessors.
- The group "proto-5" (subgroup 3. 4) already shows very close similarities to the standard form. However, Leibniz continues to distinguish the levels of case distinction in the triple signs, with the first case or its two sub-cases being shown in the upper part of the signs, the second case or its sub-cases in the lower section. C-09 ‡ and C-10 ‡, which stand for *p(mp)* and *(pm)p* respectively, differ only in the positions of the short crossbar that represents *m*. They have the same meaning as long as the levels of case distinction are ignored. The triple signs C-08 ‡ and C-11 ‡ also belong to this group, as well as the double signs C-06 ‡ and C-07 ‡.

In this system there are also composed forms: C-12  $\pm$  represents a composition of *mp* and *p(mp)* according to a rule, the negation is C-13  $\pm$ . There are also combinations based on this: in C-14  $\mp$ , C-13  $\pm$  occurs as the second case of an ambiguity sign whose other case is *p*, while in C-15  $\mp$  it is the first case, again in combination with *p*.

Thus, it is giving up the representation of the level of case distinctions and the choice of equal distances that ultimately constitute the final step towards the standard form. At the same time, this reduces the number of ambiguity signs to be taken into account. While in its standard form the continuation of the system for distinguishing more cases is known, it is not clear from the surviving texts for the transitional and preliminary forms.

### 2. 6 System 6

System 6 which can be derived from several manuscripts shares its basic idea with system 4. In all examples known from these texts, p is expressed by 1. For m, Leibniz uses 3 in one text and 2 in all others.

By relying entirely on character types which are included in the usual typesetting box, this system does not have to be taken into account in the proposal.

## **References and additional literature**

Mugnai 2018 Massimo Mugnai, *Ars Characteristica, Logical Calculus, and Natural Languages*, in: Maria Rosa Antognazza (ed.): *The Oxford Handbook of Leibniz*, Oxford University Press. Oxford 2018, p. 177-207 (published online as <a href="https://doi.org/10.1093/oxfordhb/9780199744725.013.20">https://doi.org/10.1093/oxfordhb/9780199744725.013.20</a>).

Probst & Trunk 2019 Siegmund Probst and Achim Trunk: *Einleitung*, in: Gottfried Wilhelm Leibniz: *Sämtliche Schriften und Briefe*, Vol. VII, 7.

 Trunk 2016-2017 Achim Trunk, Sechs Systeme: Leibniz und seine signa ambigua, in: Wenchao Li (ed.): Für unser Glück oder das Glück anderer, Vorträge des X. Internationalen Leibniz-Kongresses. Hildesheim 2016-2017, vol. 4.

# Overview of Leibniz's systems of ambiguity signs

The columns in this overview contain the following information:

- 1 number within the system
- 2 mathematical meaning
- 3 specific part of proposed character name (if it is part of the proposal)
- 4 representative glyph
- 5 ID in the proto standard as defined by the Leibniz-Edition (LE)
- 6 number and glyph in the current font of the Leibniz-Edition (if existing). The current glyphs can deviate from their actual layout.

The following signs are used to express the meaning of the signs:

р	plus
m	minus
()	group of sub-cases
<i>non</i> []	negation
•	multiplication of signs
0	composition of signs

# 1 System 1

## 1.1 Double Signs

nr.	meaning	character name ID	repr. glyph	ID in proto standard of LE	glyphs used in the Leibniz Edition (LE)
1	Sys. 1 pm (= sys. 2 pm)	A-01	+	T-01 = T-19	12: +
2	Sys. 1 mp (= sys. 2 (parts) mp)	A-02	ŧ	T-20 = T-02	13: ≢

## 1. 2 Triple Signs

1	Sys. 1 p(pm)	A-03	+‡	T-03	
2	Sys. 1 p(mp)	A-04	±+ ++	T-04	
3	Sys. 1 m(pm)	A-05	+	T-05	
4	Sys. 1 m(mp)	A-06	#	T-06	

## **1.3 Multiplication Forms**

1	Sys. 1 pm · pm	A-07	Ħ	T-07	
2	Sys. 1 mp · pm	A-08	<b>₽†</b>	T-08	

# 2 System 2

# 2.1 Double Signs

## 2.1.1 Standard Forms

1	Sys. 2 pm (= sys. 1 pm)	(A-01)	+	T-01 = T-19	6: †
2	Sys. 2 mp	B-01	#1	T-12	7: ±

## 2. 1. 2 Standard Forms (Parts)

1	Sys. 2 (parts) pm				_
2	Sys. 2 (parts) mp (= sys. 1 mp)	(A-02)	#	T-20 = T-02	230: ≢

# 2. 2 Triple Signs

# 2. 2. 1 Standard Forms

2. 2. 1 a) Type \_ ( \_ \_ )

1	Sys. 2 p(pm)	B-02	*‡	R-118	118: +
2	Sys. 2 p(mp)	B-03	*‡	T-10	120: †
3	Sys. 2 m(pm)			_	
4	Sys. 2 m(mp)	B-04	#	T-09	
5	Sys. 2 <i>non</i> [p(pm)]	B-05	±+-	T-15	222: 1
6	Sys. 2 <i>non</i> [p(mp)]	B-06	1 <u>+</u>	R-119	119: 🔨
7	Sys. 2 <i>non</i> [m(pm)]	B-07	++	R-84	84: ±
8	Sys. 2 <i>non</i> [m(mp)]				

## 2. 2. 1 b) Type ( \_ \_ ) \_

9	Sys. 2 (pm)p	B-08	<del>†</del> +	T-17	231: ++
10	Sys. 2 (mp)p		_	_	
11	Sys. 2 (pm)m	B-09	ŧ	R-233	233: +

12	Sys. 2 (mp)m			—	
13	Sys. 2 <i>non</i> [(pm)p]	B-10	±+	R-234	234: ±*
14	Sys. 2 <i>non</i> [(mp)p]			_	
15	Sys. 2 <i>non</i> [(pm)m]			_	
16	Sys. 2 <i>non</i> [(mp)m]			_	

## 2. 2. 2 Standard Forms (Parts)

# 2. 2. 2 a) Type \_ ( \_ \_ )

1	Sys. 2 (parts) p(pm)			_	
2	Sys. 2 (parts) p(mp)				
3	Sys. 2 (parts) m(pm)				—
4	Sys. 2 (parts) m(mp)			_	
5	Sys. 2 (parts) <i>non</i> [p(pm)]				
6	Sys. 2 (parts) <i>non</i> [p(mp)]	B-11	1 1 1	T-25	224: *‡
7	Sys. 2 (parts) <i>non</i> [m(pm)]				_
8	Sys. 2 (parts) <i>non</i> [m(mp)]	B-12	+	R-226	226: 👎

## 2. 2. 2 b) Type ( \_ \_ ) \_

9	Sys. 2 (parts) (pm)p	—	 	
10	Sys. 2 (parts) (mp)p	—	 	
11	Sys. 2 (parts) (pm)m	—	 	
12	Sys. 2 (parts) (mp)m		 	
13	Sys. 2 (parts) non[(pm)p]		 	
14	Sys. 2 (parts) <i>non</i> [(mp)p]		 	
15	Sys. 2 (parts) <i>non</i> [(pm)m]		 	
16	Sys. 2 (parts) <i>non</i> [(mp)m]		 	

## 2.2.3 Variants

1	Sys. 2 p(pm)	B-13	*#	T-13	228: ++
2	Sys. 2 p(mp)	B-14	+# +	T-14	223: *‡
6	Sys. 2 <i>non</i> [p(mp)]	B-15	1 1 1	T-16	225: 1
10	Sys. 2 (mp)p	B-16	‡+ +	T-18	229: ‡*
6	Sys. 2 (part) non[p(mp)]	B-17	+# +	R-232	232: 1
6	Sys. 2 (part) non[p(mp)]	B-18	*‡	R-227	227: *

# 3 System 5

# **3.** 1 Subgroup "Transition Form $2 \rightarrow 5$ "

Sys. Transition Form 2 to 5 p(mp)	C-01	++	T-75	

# 3. 2 Subgroup "prae-pro-proto-5"

 Sys. prae-pro-proto-5 (pm)p	C-02	¥ +	Т-72	
 Sys. prae-pro-proto-5(mp)p	C-03	±+	T-73	_

# 3. 3 Subgroup "pro-proto-5"

 Sys. pro-proto-5 (pm)p	C-04	<b>4</b> +	R-220	220: \$
Sys. pro-proto-5 (mp)p	C-05	4+	T-74	221: \$

# 3. 4 Subgroup "proto-5"

## 3.4.1 Double Signs

1	Sys. proto-5 pm	C-06	+	T-46	
2	Sys. proto-5 mp	C-07	1	T-71	

## 3.4.2 Triple Signs

1	Sys. proto-5 p(pm)	C-08		T-69	
2	Sys. proto-5 p(mp)	C-09	++	T-47	
3	Sys. proto-5 m(pm)		—		

4	Sys. proto-5 m(mp)				
5	Sys. proto-5 (pm)p	C-10	#	T-42	—
6	Sys. proto-5 (mp)p	C-11	++	T-70	—
7	Sys. proto-5 (pm)m				—
8	Sys. proto-5 (mp)m				—

# 3.4.3 Composed Forms

	Sys. proto-5 mp $\circ$ p(mp)	C-12	ŧ	T-41	
_	Sys. proto-5 <i>non</i> [mp $\circ$ p(mp)]	C-13	+	T-48	_
	Sys. proto-5 p <i>non</i> [mp $\circ$ p(mp)]	C-14	Ŧ	T-44	
_	Sys. proto-5 <i>non</i> [mp $\circ$ p(mp)]p	C-15	1	T-43	

# 3. 5 Subgroup "Standard"

## 3. 5. 1 Double Signs

1	Sys. 5 pm	C-16	+	T-55	8: †
2	Sys. 5 mp	C-17	+	T-56	38: +

## 3. 5. 2 Triple Signs

1	Sys. 5 ppm	C-18	丰	T-57	<b>99:</b> ≢
2	Sys. 5 pmp	C-19	#	T-58	39: ≢
3	Sys. 5 mpp	C-20	#	T-45	85: ≢
4	Sys. 5 pmm	C-21	ŧ	T-59	<b>40: </b>
5	Sys. 5 mpm	C-22	+	T-60	42: ‡
6	Sys. 5 mmp	C-23	1	T-61	41: ₹

## 3. 5. 3 Quadruple Signs

1	Sys. 5 pppm	C-24	ŧ	T-66	46: ≢
2	Sys. 5 ppmp	C-25	ŧ	T-65	45: ≢
3	Sys. 5 pmpp	C-26	ŧ	T-64	44: ≢
4	Sys. 5 mppp	C-27	#	T-63	43: ≢
5	Sys. 5 ppmm	C-28	ŧ	T-67	47: ≢

6	Sys. 5 pmpm				
7	Sys. 5 mppm	C-29	╫	R-49	
8	Sys. 5 pmmp		_		
9	Sys. 5 mpmp				
10	Sys. 5 mmpp	C-30	π <del>ll</del>	T-68	
11	Sys. 5 pmmm		_		
12	Sys. 5 mpmm	C-31	नीत	T-62	48: ≢
13	Sys. 5 mmpm				
14	Sys. 5 mmmp				

Introduction and tables by Elisabeth Rinner

## 3. Characters proposed for encoding

If this proposal gets accepted, the following characters will exist:

+	AMBIGUITY SIGN A-01	<b>†</b>	AMBIGUITY SIGN C-05
ŧ	AMBIGUITY SIGN A-02	‡	AMBIGUITY SIGN C-06
+‡	AMBIGUITY SIGN A-03	1	AMBIGUITY SIGN C-07
+‡	AMBIGUITY SIGN A-04	ŧ	AMBIGUITY SIGN C-08
+	AMBIGUITY SIGN A-05	‡	AMBIGUITY SIGN C-09
-‡	AMBIGUITY SIGN A-06	‡	AMBIGUITY SIGN C-10
Ħ	AMBIGUITY SIGN A-07	‡	AMBIGUITY SIGN C-11
ŧŧ	AMBIGUITY SIGN A-08	ŧ	AMBIGUITY SIGN C-12
土	AMBIGUITY SIGN B-01	+	AMBIGUITY SIGN C-13
++	AMBIGUITY SIGN B-02	ŧ	AMBIGUITY SIGN C-14
+‡	AMBIGUITY SIGN B-03	丰	AMBIGUITY SIGN C-15
1	AMBIGUITY SIGN B-04	+	AMBIGUITY SIGN C-16
* <u>‡</u>	AMBIGUITY SIGN B-05	+	AMBIGUITY SIGN C-17
1圭	AMBIGUITY SIGN B-06	<b>‡</b>	AMBIGUITY SIGN C-18
土	AMBIGUITY SIGN B-07	<b>‡</b>	AMBIGUITY SIGN C-19
<del>†</del> +	AMBIGUITY SIGN B-08	+	AMBIGUITY SIGN C-20
+-	AMBIGUITY SIGN B-09	ŧ	AMBIGUITY SIGN C-21
<u></u> ±+	AMBIGUITY SIGN B-10	+	AMBIGUITY SIGN C-22
† <u>†</u>	AMBIGUITY SIGN B-11	1	AMBIGUITY SIGN C-23
ŧ	AMBIGUITY SIGN B-12	丰	AMBIGUITY SIGN C-24
++	AMBIGUITY SIGN B-13	圭	AMBIGUITY SIGN C-25
*‡	AMBIGUITY SIGN B-14	1	AMBIGUITY SIGN C-26
挂	AMBIGUITY SIGN B-15	1	AMBIGUITY SIGN C-27
<b>‡</b> +	AMBIGUITY SIGN B-16	圭	AMBIGUITY SIGN C-28
*#	AMBIGUITY SIGN B-17	<b>‡</b>	AMBIGUITY SIGN C-29
+丰	AMBIGUITY SIGN B-18	1	AMBIGUITY SIGN C-30
++	AMBIGUITY SIGN C-01	1	AMBIGUITY SIGN C-31
¥ +	AMBIGUITY SIGN C-02	8	PLUSMINUS LOOP SIGN
Ц +	AMBIGUITY SIGN C-03	8	MINUSPLUS LOOP SIGN
\$	AMBIGUITY SIGN C-04		

Achim Trunk (GWLB Hanover) described six different systems, invented by Leibniz. System 3 deploys the same characters as system 2, mostly. The fourth system employes Greek letters and the sixth system uses ordinary numbers, so basically three systems remain (1., 2. and 5.) which consist of special graphic symbols. The technical numbering of the characters in this proposal (A-xx, B-xx, C-xx, see previous page) relates to (sub-)systems 1, 2 and 5.

## 4. The ambiguity signs in samples from Leibniz's manuscripts



First system (basic signs, compound signs, transitional and secondary forms)



Second system (basic signs, compound signs, transitional form)



Third system (basic signs, compound signs)



Fifth system (basic signs, compound signs)

≠ a <sup>2</sup> y <sup>4</sup> + 2 aq ≠ q <sup>2</sup>	+# # 20ar y3	+ + 29°af y = - 9af = + 29°a =	+= 20239 Y	+ 2239	ריז י
		+ a'q = 2*a*			

A notation of an algebraic problem by Leibniz, using symbols of the 2nd system. (after A. Trunk)

### 5. Figures and explanations



Example of ambiguity signs, 1st system. LAA VII-7, p. XXVI

$$[2]xq \pm 2x^{2} \stackrel{\sim}{\sim} \frac{x - f}{q \pm 2x}.$$

$$d^{2} \stackrel{\sim}{\sim} \underbrace{a \not \pm \frac{a}{q} x^{\cancel{p}}}_{y^{2}} = \underbrace{a^{2} x^{\cancel{p}} \bigoplus_{\cancel{q} = 2}^{\sqrt{p}} x^{\cancel{p} = 3}}_{y^{4}} \underbrace{\frac{2a^{2} x^{\cancel{p} = 2}}{q}}_{y^{4}} + 4x^{\cancel{p} q^{2}} \bigoplus_{\cancel{q} = 4}^{\sqrt{p} = 4} x^{\cancel{p} = 2} x^{\cancel{p} = 2} \underbrace{\frac{a^{2} x^{\cancel{p} = 2}}{q}}_{x \stackrel{(n)}{=} x^{2} + f^{2} - 2xf} \underbrace{\frac{a^{2} x^{\cancel{p} = 2}}{q}}_{x \stackrel{(n)}{=} x^{2} + f^{2} - 2xf} \underbrace{\frac{a^{2} x^{\cancel{p} = 2}}{q}}_{x \stackrel{(n)}{=} x^{2} + f^{2} - 2xf} \underbrace{\frac{a^{2} x^{\cancel{p} = 2}}{q}}_{x \stackrel{(n)}{=} x^{2} + f^{2} - 2xf} \underbrace{\frac{a^{2} x^{\cancel{p} = 2}}{q}}_{x \stackrel{(n)}{=} x^{2} + f^{2} - 2xf} \underbrace{\frac{a^{2} x^{\cancel{p} = 2}}{q}}_{x \stackrel{(n)}{=} x^{2} + f^{2} - 2xf} \underbrace{\frac{a^{2} x^{\cancel{p} = 2}}{q}}_{x \stackrel{(n)}{=} x^{2} + f^{2} - 2xf} \underbrace{\frac{a^{2} x^{\cancel{p} = 2}}{q}}_{x \stackrel{(n)}{=} x^{2} + f^{2} - 2xf} \underbrace{\frac{a^{2} x^{\cancel{p} = 2}}{q}}_{x \stackrel{(n)}{=} x^{2} + f^{2} - 2xf} \underbrace{\frac{a^{2} x^{\cancel{p} = 2}}{q}}_{x \stackrel{(n)}{=} x^{2} + f^{2} - 2xf} \underbrace{\frac{a^{2} x^{\cancel{p} = 2}}{q}}_{x \stackrel{(n)}{=} x^{2} + f^{2} - 2xf} \underbrace{\frac{a^{2} x^{\cancel{p} = 2}}{q}}_{x \stackrel{(n)}{=} x^{2} + f^{2} - 2xf} \underbrace{\frac{a^{2} x^{\cancel{p} = 2}}{q}}_{x \stackrel{(n)}{=} x^{2} + f^{2} - 2xf} \underbrace{\frac{a^{2} x^{\cancel{p} = 2}}{q}}_{x \stackrel{(n)}{=} x^{\cancel{p} = 2}}$$



Leibniz-Akademie-Ausgabe (LAA, general edition of Leibniz's writings) LAA series VII (mathematical manuscripts, volumes 3 to 7 available online) LAA series VII volume 3 LAA series VII volume 5 LAA series VII volume 7

$$\left. \begin{array}{cccc} + 4g & + 8ag & + 4ag^2 & - 2c^2g^2 \\ & - 2c^2 & - 4ae^2 & - 2c^2e^2 \\ & + 6g^2 & - 4gc^2 & + g^4 \\ & + 2e^2 & + 4g^3 & + 2g^2e^2 \\ & & + 4ge^2 & + e^4 \end{array} \right\} =$$

0

Examinato ergo Canone, per exempla circuli, et parabolae, pergem<br/> $\langle us \rangle \langle cum \rangle$  Calculo generali. Habuimus paulo ante valorem ipsi<br/>us g. indagemus eum adhuc semel ope terminorum tertiorum, collatorum, seu ope multiplicantium secundae dimensionis incognitos. Fiet

$$= 2\frac{a}{q}g^{2}(\mp \mp)bag + \frac{16a^{[2]}}{\pm 2\frac{a}{q} + 6}, \uparrow a^{2} = \frac{\pm 2\frac{a}{q}e^{2} \pm 2\frac{a}{q}c^{2} - 2e^{2} + 2c^{2} - 4a^{2}, \uparrow \frac{a^{2}}{h^{2}}, \\ = \frac{((\uparrow \mp \mp)) = 2q^{2}f, -qf^{2} \pm 2q^{2}a + a^{2}q \pm d^{2}a}{\pm a + 2q \pm \frac{q^{2}}{a}}$$

20 Kontrollansatz zur quadratischen Ergänzung:  $\sqrt{\pm 2\frac{a}{q} + 6} g \uparrow \frac{(\pm \mp)}{\sqrt{\pm 2\frac{c}{q} + 6}}$ 

15 f. } = 0 (1) Ponendo jam $\mathbf{x}^2 = \mathbf{z}^2 \frac{\mathbf{h}}{\mathbf{a}}$ (2) Examinato L

15

2

20 ... = ... : Die Koeffizienten, die Leibniz vergleicht, bezieht er wie oben aus den Gleichungen in N. 5 S. 35 sowie auf S. 48 Z. 3–12. Erneut vergisst er den Faktor  $\frac{a^2}{q^2} \neq 2\frac{a}{q} + 1$ . Zudem nimmt er die

### ≠ AMBIGUITY SIGN B-04; a character belonging to the 2nd system. LAA VII-7, p. 52

$$\underbrace{N.7} \quad \text{AEQUATIO EX INTERSECTIONE ORIENS, Ende Dezember 1673 - Juni 1674} 53$$
seu extracta utrobique Radice
$$\frac{g\sqrt{\pm 2\frac{a}{q} + 6} \quad (= \mp) - \frac{4a}{\sqrt{\pm 2\frac{a}{q} + 6}}}{\frac{h}{a}} = \sqrt{\dots} \text{sive}$$

$$g = \sqrt{\frac{\pm 2\frac{a}{q}e^2 \pm 2\frac{a}{q}c^2 - 2e^2 + 2c^2 - 4a^2, \quad ((\pm \mp \mp)) = 2q^2f - qf^2 \pm 2q^2a + a^2q \pm d^2a}{\frac{1}{q}c^2} - \frac{h^2}{e^2}}{\frac{1}{q}e^2 + 2\frac{a}{q}e^2 + 2e^2 + 2c^2 - 4a^2, \quad ((\pm \mp \mp)) = 2q^2f - qf^2 \pm 2q^2a + a^2q \pm d^2a}{\frac{1}{q}c^2} - \frac{h^2}{e^2}}{\frac{1}{q}e^2 + 2\frac{a}{q}e^2 + 2e^2 + 2c^2 - 4a^2, \quad ((\pm \mp \mp)) = 2q^2f - qf^2 \pm 2q^2a + a^2q \pm d^2a}{\frac{1}{q}c^2} - \frac{h^2}{e^2}}{\frac{1}{q}e^2 + 2\frac{a}{q}e^2 + 2\frac{a}{q}e^2 - 2e^2 + 2c^2 - 4a^2, \quad ((\pm \mp \mp)) = 2q^2f - qf^2 \pm 2q^2a + a^2q \pm d^2a}{\frac{1}{q}c^2} - \frac{h^2}{e^2}}{\frac{1}{q}e^2 + 2\frac{a}{q}e^2 + 2\frac{a}{q}e^2 - 2e^2 + 2c^2 - 4a^2, \quad ((\pm \mp \mp)) = 2q^2f - qf^2 \pm 2q^2a + a^2q \pm d^2a}{\frac{1}{q}e^2} - \frac{h^2}{e^2}}{\frac{1}{q}e^2 + 2\frac{a}{q}e^2} - \frac{h^2}{e^2}}{\frac{1}{q}e^2 + 2\frac{a}{q}e^2} - \frac{h^2}{e^2} - \frac{h^2}{e^2}}{\frac{1}{q}e^2} - \frac{h^2}{e^2}}{\frac{1}{q}e^2} - \frac{h^2}{e^2}}{\frac{1}{q}e^2} - \frac{h^2}{e^2} - \frac{h^2}{e^2}}{\frac{1}{q}e^2} - \frac{h^2}{e^2}}{$$

Unde evanescit incognita g. valore ejus jam aliter supra dato. Ubi erat:

$$g = \frac{\frac{((+\ddagger)) \neq a ((+\ddagger)) q}{\ddagger a + 2q \neq \frac{q^2}{a}} \cap 2d \cap \frac{\mu^2}{a^2}(\theta) \cap h^2 \neq 4\frac{a^2}{q} - 4a \xrightarrow{} \frac{1}{2} \left( + \frac{q^2}{a} - \frac{q}{q} - \frac{q}{a} - \frac{q}{q} - \frac{q}{a} \right) \xrightarrow{} \frac{1}{\mp (\mp \mp) 4} \frac{a}{q} \left( = -\frac{\pi}{\mp} \right) 4 \xrightarrow{} \frac{1}{2} \left( = -\frac{\pi}{2} - \frac{q}{q} - \frac{q}{q} - \frac{q}{q} - \frac{q}{q} \right)$$

Atque ita novam habemus aequationem inter hos duos valores, cujus aequationis ope

## ≢ AMBIGUITY SIGN B-04; LAA VII-7, p. 53

 $\mathbf{5}$ 

er in der kurzen Notiz N.8, die er vielleicht noch im Dezember 1673, vielleicht auch erst im Mai 1674 niederschreibt. Hier erläutert er vier neue Doppelvorzeichen, mit deren Hilfe sich jeweils drei Fälle unterscheiden lassen: das Symbol ++, welches für "+ oder  $\neq$  "(sprich: "im einen Fall +, im anderen entweder + oder - "):steht,  $-\ddagger \epsilon$ ls sein Gegenteil sowie die auf gleiche Weise durch Zusammenschieben eines + oder - mit einem einfachen Doppelvorzeichen gebildeten Symbole + und - Zusammen mit den beiden Grundzeichen bilden diese vier zusammengesetzten Dopperverzeichen (oder signes composés, wie Leibniz solche Zeichen später nennt) ein erstes System aus einfachen und komplexen signa ambigua. Ein praktischer Einsatz der zusammengesetzten Zeichen dieses ersten Systems ist allerdings nicht bekannt. Zwar verwendet er in N. 7, das sich auf demselben Papierbogen wie N. 8 findet, tatsächlich zusammerigesetzte Vorzeichen womöglich zum ersten Mal überhaupt in seiner mathematischen Praxis (ein anderer Kandidat hierfür ist eine Nebenbetrachtung in N. 5). Und als deren Bausteine fungieren die einfachen Zeichen  $\neq$  und  $\neq$ , die Grundzeichen des ersten Systems also. Die komplexen Zeichen werden jedoch nach geringfügig anderen Regeln gebildet, welche Leibniz erst 1

A-01 A-02 A-04 A-05 A-03 A-06

Example of ambiguity signs, 1st system. Introduction to LAA series VII volume 7, p. XXVI

A-02	<i>B-13</i>	<b>B</b> -01
•	•	•
•	•	•
•	•	•

Das zweite System, welches Leibniz in N.10 darstellt, übernimmt zunächst die einfachen Doppelvorzeichen = und = aus dem ersten System und wendet für die Bildung zusammengesetzter Symbole wie <sup>+</sup> a is + und <sup>‡</sup> nur geringfügig abgewandelte Regeln an. Noch während der Arbeit am Konzopt ersetzt Leibniz jedoch das negierte einfache Zeichen  $\equiv$  curch ein neues Zeichen,  $\pm$ , das sich aus dem Symbol  $\neq$  ergibt, indem man an seinen Fuß einen (meist etwas langer gezogenen) Querbalken anfügt. Bereits in N. 7 negiert er zusammengesetzte Zeichen auf diese Weise; in der  $M\acute{e}thede$  erhebt er sie zum allgemeinen Bildungsprinzip negierter Zeichen. Um aber das Zeichen <sup>+</sup>I, die Negation von <sup>++</sup>, von dem aus + und ± zusammengesetzten Doppelvorzeichen zu unterscheiden, wird bei letzterem der Längsstrich über den unteren Querbalkon hinaus verlängert, so dass das Zeichen '† entsteht: Dessen Negation wiederum ist '‡. Dieses Symbol kann seinerseits zum Bestandteil eines noch weiter zusammengesetzten Zeichens werden; dies deutet Leibniz an, indem er den Längsbalken erneut verlängert und so den Bauste n ቹ erzeugt. Ein entsprechendes Symbol schreibt er jedoch nicht einmal beispielshalber auf. B-13 B-14 B-01 B-15 B-05 B-11

Example of ambiguity signs, 1st and 2nd system. LAA VII-7, p. XXVIII

sich aus + und  $\neq$  zusammen und bedeutet "im einen Fall +, im anderen Fall entweder + oder -". In seiner Praxis setzt Leibniz die zusammengesetzten Zeichen (*signes composés*) des ersten Systems allerdings niemals ein. Das Beispiel:



Example of ambiguity signs, 1st system. LAA VII-7, p. XLI

hierfür hält er in der *Méthode de l'universalité* I (N. 10), verfasst wohl im Mai oder Juni 1674, fest. Aus + und  $\ddagger$  etwa bildet er das Symbol '†, velches in Worten ausgedrückt bedeutet: "im einen Fall +, im anderen Fall entweder + oder -". Auch hier gibt es also zwei Hierarchicebenen. Ist die Reihenfolge der beiden Fälle vertauscht, schreibt Leibniz dies als †\*. Das Symbol '‡ dagegen stellt die Negation von '† dar, bedeutet also "immer dann -, wenn '† für + steht, und immer dann +, wenn jenes Zeichen für -B-08 B-13 B-05 B-13 B-13

Example of ambiguity signs, 2nd system. LAA VII-7, p. XLI



Example of ambiguity signs, 2nd system. LAA VII-1, p. 326-327. See following figure for details.

quadrato-quadratus, quam quadratus simul :	sint dati, quod fiet, si aequationem hanc
v $(\ddagger 2l + 10l + 4q)$ reddemus talem, ut v	pro arbitrio sumta, q inveniri queat: Fiet
25 $\pm 2l7 + 10lv \neq 4qv \cap \ddagger 2lq - 2q^2 - l^2$ . Ponatu	If $v \neg z - \beta$ , fie : $\pm 2 z \neq 2 + 10 z - 10 \beta$
* $4qz \pm dq\beta$ r $(2q^2 - 2q^2 - l^2)$ . Pone $(2q^2 - 1q^2) = 1$ $3''''=2lz + 1(lz + 4)(lz \pm \frac{4ql}{(2q^2 - 1)})$ r $2lq - 2q^2$ .	ol $\beta \sqcap -l^2$ , fiet $\beta \urcorner \frac{1}{2} - 10$ et restabit:
A-01, B-14 B-13 B-01	B-05 B-13
B-05 B-05 A-01	

Sunt autem x et d, et l ac per consequens etiam  $\mathbb{D}$  et  $\odot$  numeri dati, quare si  $4\mathbb{D}l + \mathbb{O}^4$ evenit esse numerum quadratum, tunc effici potest ut tam quadratus quam quadratoquadratus sint dati. Illud vero manifestum est, quia in aequatione  $\mathfrak{P}$ . neque x. neque d. id quadratum ascendunt, hinc semper effici posse, ut alterutra earum sit data, item ut



Example of ambiguity signs, 2nd system. LAA VII-1, p. 327 (top), 329

Soit maintenant une certaine grandeur affectée du signe + par exemple + a, c'est à dire : o + a. car puisque + aussi bien que - signifie une Relation entre deux, et qu'il n'y a qu'une seule grandeur a, l'autre sera o ou rien : supposons donc que la dite grandeur + a doit estre adjoutée à une autre b, le produit sera  $b + \pm a < ou b$  plus  $\pm a > c$ 'est à dire  $b \neq a$ , car le signe + ne change point les autres signes : mais à present supposons que la dite grandeur  $\pm a$  doit estre soubstraite d'une autre b, 29 recto. le produit sera  $b - \pm a$ , ou b moins  $\pm a$ , et | par ce que cela arrive bien souvent, je trouve à propos d'employer un seul signe, ± au lieu de ces deux — et  $\pm$  joints ensemble, et le produit susdit sera  $b \pm a$ , et ± vaudra - + et generalement j'observeray cette regle, qu'un signe anchigu insistant sur un - aura une signification contraire à celle qu'il auroit sans cela, ou que le signe avec le - < au bas du caractere > signifie moins le < même > signe sanc -. Par exemple 1/2 (que nous expliquerons cy après : ) signifiera -- + Par consequent si dans une meme formule ou Equation ces deux signes opposés se trouvent à la fois, comme par exemple  $\pm a \pm b \sqcap c$ , et que cette formule vienne a estre expliquée ou appliquée à un certain cas particulier, ou + signifie par exemple +, alor ± s'expliquera aussi et signifiera -, et si + signifie — dans le cas particulier dont nous avons besoin, ± signifiera +

B-14

Example of ambiguity signs, 2nd system. Couturat 1903 (1961), p. 126

B-01



B-01

fait voir que ces deix signes ambigus  $^{+}$  et  $^{+}$  signifient ou tous deux +, ou que l'un signifiant  $^{\pm}$ , l'autro signifie  $^{\pm}$ , je les exprime en mettant + au devant, en tous deux  $^{+}$  et  $^{+}$ , au lieu de  $^{+}$  et  $^{\pm}$  dont nous aurons besoin dans une autre rencontre.

- On voit en fin par la; la grande difference qu'il y a entre le signe ≠, et tous les autres.
  <sup>5</sup> Car le signe simple ≠ peut subsister tout seul, sans changement, par ce qu'il ne dit point de relation a aucun autre; mais tous les autres contiennent quelque relation à un autre signe provenant d'une meme equation ambigue, et pour cela je les appelle Correspondants. Par exemple si nous avons deux signes ambigus simples, ≠ et ± provenans de l'equation ‡a±y □ b, et si dans la suite du calcul le signe ≠ evanouit, comme il arrive en cet exemple,
- ou nous trouvons en fin cette equation,  $y \sqcap \pm b + a$ , alors si nous nous determinons à abandonner entierement la premiere equation, avec tout ce qui en est provenu, hormis cette nouvelle trouvée, dont nous pretendons nous servir à l'avenir dans le calcul qui reste à faire; nous pourrons sans scrupule changer le signe  $\pm$  en  $\pm$ , et nous servir de cette

Example of ambiguity sign B-16, 2nd system. LAA VII-7, p. 126

B-15

	<i>B-14</i>	B-05 B-14	B-15 B-03	B-17	B-11	
		DE LA N	ÉTHOPE DE L'UNIVER	SALITE	131	i i i i i i i i i i i i i i i i i i i
	diavantage += at lieu de la dist et pour q	e vers embas la lig a d : '± en '‡ au lie ance que je hisse uoy je fais '± au	gne perpendiculaire eu de 哇. 法 à fin au e, < entre le trait h lieu de 坤, et 瑋 at	du caractere, o ssi qu'on voye aussé, et les p a lieu de '≢ o	et de faire F e la raison remiers > 1 = je dis	Phil., V, 10, f
	qu'on déc tion de to	ouvre par ce moy	ven à la promière ve ais qu'outre cette c	eue l'origine et commodité il y	t composi- y a même	
B-05	sion de de	eux signes de diff	ferente signification, composition d'un	car posons qu autre; si on	en faisoit	
B-03 ••••:	eiors >	> '# en haussant : s du signe '# <	simplement le trait quand il entreroit	d'embas on ne aussi dans une	le discer- composi-	
	$\therefore  \text{fion} > pa$ $\neq a n  \text{lie}$	ur ce que en le hau u de +≢ donc voi le +≢ ≪ c'est à di	ssant simplement, n la deux $= < de $	ous aurions eu lifferente signi = c'est à dire à	< aussi > ification > + ou = :	
B-14 ••••	l'autre fai contraire	it de 🛬, c'est à à + : co qui n'est	dire > de + ou $\pm$ pas le même	c'est à dire di	u + et du	····· B-13
	Quand vaut + o	je dis $<$ par exe ou $\pm$ cela se doit	emple > que + v entendre avec une	aut $+$ ou $\pm$ , relation entre	et que += . e ces l'eux	····· B-14
	signes an biguité o alors += s	u generalité à u cera expliqué par	de sorte que si da n cas particulier, + et vice versa <	the st explication the st explicit of the st explic	n de l'am- né par —, trois equa-	
	tions sus bien BC	dites $<$ de la 5 <sup>r</sup> tout a la fois s	<sup>ae</sup> figure > il n'y oient affectées par	a pas une, ou $->$ . Mais	AB aussi si + est	
	expliqué j	par +, il n'est pa	s necessaire que +±	soit expliqué	par — par	

Example of ambiguity signs, 2nd system. Couturat 1903 (1961), p. 131

A-01

Necesse est ergo dividi posse aut per  $a^2 \ddagger \frac{y^4}{x^2}$ , aut per  $a^2 \ddagger \frac{y^2}{x}$ . Sin ordinetur secundum y, necesse est si dividi potest dividi posse per  $y^4 \ddagger a^2x^2$ , vel  $y^3 \ddagger a^2x$  vel denique si ordinatur secundum x, fiet:  $x^2 \frac{+y^3x^2}{x^2y^2 + a^{4/2}}x \frac{-y^6}{a^2y^2 + a^4}$  quo casu solus ex prioribus divisoribus tentandis restat:  $x \ddagger \frac{y^3}{a^2}$ . Multiplicetur per x + b. fiet:  $x^2 \ddagger \frac{y^3}{a^2}x \ddagger \frac{y^3b}{a^2}$ . Unde conferendo: +b...  $b \mapsto \frac{\ddagger \frac{y^3}{y^2 + a^2}}{y^2 + a^2}$  et fiet:  $\frac{\ddagger \frac{y^2}{x}}{a^2} \ddagger \frac{y^2}{y^2 + a^2} \sqcap \frac{y^2}{y^2 + a^2}$ , sive  $\ddagger y^2 (\ddagger a^2 \pm a^2) \sqcap a^2$ . Quod est absurdum. Ergo: nullum habet aequatio inventa divisorem rationalem. Aequatione ergo ad tangentes ordinata fiet:  $6y^6 - 3a^2xy^3 - 2a^2x^2y^2 \sqcap + 2a^4xl + a^2y^3xl$ , et fiet :  $l \sqcap \frac{6y^6 - 3a^2xy^3 - 2a^2x^2y^2}{2a^4x + 2a^2y^2x}$ .

Ambiguity signs, 2nd system. LAA VII-3, p. 567 see also LH 35 V 2, f. 4v (next page)

Minitrate & AB has marchate Calent when my letting reputer abfolite FACTU PB2 AB grank et a pint at m AB ad BC, ite t in californ when when ACD ACC, AH. aquan's green ACD hink 3 Acher A when he and on my fee hanguly she IT AH ] PH (1) y n Hich and Spipe hat my attack Anc at Abe agrading of fronting when ABC BC ef PM H BC - AH . OI DEN B. AB BO -DE BC. - AB EC + DE EC juin  $\frac{B(1)}{B(1)} = \frac{B(1)}{B(1)} = \frac{B(1)}{B(1)$ 2 3 BC -A B EC + 30 BC, we wile , BC - AB, BS, BT AB, ES, BS + 30 BC, AB, BS AB, AB BG AR EL 3R\_ BCB - AB, g, B. T. AB, EC, at + 3B BC, AB, D. at his multiplicate o BC It en squalis well m BS sepitrate & AB ? BS BC B - AB 2000 BC fier: BC BC at + 30 BCAB, at ion figure CE H B BS A try CE H Bar BC BC - AB, at B, BC H AB Bat + 30 BC ABat BC unde BC appelentery at y3X - at y2X - at X H P Rinder BC on ret freining apparent BCC - ABrach valren fier ; per: respect of eye sind pope and per a + & 6'on malm / Q pet: a, vel Ilm y' = ax goopminily Simprily Centendy tanx recepte of Owin Sui of Dincher The gres cafe for necepte It conferend ; fill: 'ya XX x fa Č# 主型 П y yan xt + arys hei no value anyo ad Sabet aqua molishe unte peret 3-22×242 + asys Xl, a foi: In 6 y6 - 3 a x y In y a y X - a y X - atx 1 0 4 aryon his early goe deberet the Valent as orum let times neaf " falf . Aul de v Con de' diad ambrhear · Im FB-AB ad AH IT FB & BE. En FBA BEE Ha BE BS FBN BE a) a Isme AHM a, BE AB. M BE ABA CEN BE C. 88 BET ad BE TH fui: BE QUAR COL - AB w A HM BE BE AD BE THE AB A A HM AB BE - BEAB + BEB + CEAB CIBBE , hime Lais - PM B M A B BE, -, BE-CE, AB-B prin (ABBE - BEAB) + BEB + CEAB CIBBE ; hime Ram 2 PM BIT ABCBE, 2 PM, BE TH BE AB. es PM BE + AB ABO ANT BE ABa eyo PM H2BE BE2 + ABa D. BE ABa BEC + a ABBE 2 pmpn azBEet forse forite when and Berning AH PMI BE epi et as well ente alter > he Tin che als gua 6n3: 62 reacht InCentr ne m prime ML M Bi Dene IT BS I BEN hod CAN and ED BSH AS min Sta HLayvar resamply A Bin. you done Hingowhin com 41 A MM

Manuscript by G.W.Leibniz, LH 35 V 2, f. 4v

BUB - HB, BC, P. 11 HD, CY BC fres: BBG A eng CEN P + 30 BC, AB, a2 CEM lan Jupa CL BCunte AB Ba4 + 30 BC ABat elle BGay AB net faciliz Pindata Init rope and per a \$ rel: Hing 6' on m free breti y ya Π . L Alyvahore crys as Tangently ni hin alem whe swildren unte peret 6 y - 3 a x y 'n + asys XL a for: 11+ 201 + 2 yt . que argen his callen efte veben ragy y 1 21 in. ad Atalo

see next page au lieu de  $\pm$ ; et  $\pm$  au lieu de  $\pm$ . Et à fin aussi qu'on voye la raison de la distance que je laisse entre le treit haussé, et les premiers, et pour quoy je fais  $\pm$  au lieu de  $\pm$ , et  $\pm$  au lieu de  $\pm$  or  $\pm$  je dis qu'on découvre par ce moyen à la premiere veue l'origine et composition de tous ces signes, mais qu'outre cette commodité il y a même quelque necessité de faire de la sorte, pour eviter l'equivocation, ou confusion de deux signes de differente signification, car posons que le signe  $\pm$  doive entrer dans la composition d'un entre si m en faiseit alore  $\pm$  en haussent simplement le trait d'embas on ne le *B-11 B-17 B-18 B-05* ns une composition par ce que

en le haussant simplement, nous aurions eu aussi  $\ddagger$  au lieu de  $\ddagger$  donc voila deux  $\ddagger$  de differente signification l'un fait de  $\ddagger$ , c'est à dire du contraire à  $\ddagger$  c'est à dire à + ou ‡: l'autre fait de  $\ddagger$ , c'est a dire de + ou  $\ddagger$  c'est à dire du + et du contraire à  $\ddagger$ : ce qui n'est pas le même.

Quand je dis par exemple que  $\dagger$  vaut + ou  $\ddagger$ , et que  $\ddagger$  vaut + ou  $\ddagger$  cela se doit entendre avec une relation entre ces deux signes ambigus composez; de sorte que si dans l'application de l'ambiguité ou generalité à un cas particulier,  $\ddagger$  est expliqué par -, alors  $\ddagger$  sera expliqué par + et vice versa car entre ces trois equations susdites de la 5<sup>me</sup> figure il n'y a pas une, ou AB aussi bien que BC, tout a la fois soient affectées par -. Mais si  $\ddagger$  est expliqué par +, il n'est pas necessaire que  $\ddagger$  soit expliqué par - par ce que dans une de ces equations particulieres, AB, aussi bien que BC, sont affectées par +. Par consequent si l'un de ces deux signes composés est expliqué par + l'autre sera expliqué par  $\ddagger$  et vice versa (: avec la caution pourtant, que nous y apporterons plus bas:) de sorte que l'ambiguité decomposée qu'elle est, deviendra simple. Et par ce que la liste des Equations particulieres

 $\begin{array}{ccc} AC & \sqcap & + & AB + & BC \\ & & - & \\ & + & + & AB + \\ & + & AB - \\ \end{array} \right\} \pm \begin{array}{c} BC \\ BC \\ BC \end{array} \right\} \text{ qui peuvent estre entendues} \end{array}$ 

sous la Generale

 $^{\dagger}AB$ 

### *B-12*

2 entre... premiers erg. L 3 de  $\ddagger$  ou  $\ddagger L$  ändert Hrsg. 8–10 signe  $\ddagger |$  qvand... composition erg. | par ce qve (1) si on haussoit le signe (2) en ... eu | aussi erg. |  $\ddagger au$  ... deux  $\ddagger |$  de differente signification erg. | l'un (a) faisoit de (aa)  $\ddagger$ , l'autre de + ou  $\ddagger$  (bb)  $\ddagger$ , l'autre de  $\ddagger$ , c'est à dire de + ou  $\ddagger$  (b) fait de  $\ddagger$ , c'est à dire (aa) de + ou  $\ddagger$  (bb) du contraire L 13 (1) On voit par la, a (2) Qvand je dis | par exemple erg. | L 14f. dans (1) l'explication (2) l'application L 16f. car ... susdites | de la 5<sup>me</sup> figure erg. | il ... bien | qve erg. Hrsg. | BC ... par - erg. L

 $\ddagger BC$ ,

Ambiguity signs, 2nd system. LAA VII-7, p. 125 see also LH 4 V 10, f. 31 (next page)

la liste des Equations ponticulites + AC TI + AB + BC 1 Prolongues d'avantage versembas la ligne 2 11++ 14-1 strong in notion. the cubre le trait has pe art les pri production , but quals down interior tigentin Provident of the signe ++ igh à dire a autre ; mais +00 ou +: l'autre Se nous army deup riging ambig ± provenans se frequation 7 6, ch fi Jany la Juite Ju saf copli wong & l'un cus expliqués pau + + es viccorsa Par con ique par + + es viceversa aurbars, que novas y apporterens plus bas.) mema equation asubigues et si dans la du calcul tous les autres evanouisfent "se composer qu'elle Et par ce qué de Jorte que l'ambiguité (A, de viendra rimple un seul qui reste, alors selus qui Teste,

#### LH 4 V 10, f. 31v-32r

prolonguer d'avantage versembas la highe perpendiculaire du caractère, et de faire += au hieu de +=; et = += au heu de += structure a graveste : au hiru de += ; r1 and al other and Et à fin aupri qu' on voye la raison de la distance que le laisté, et pour qu'es je fais to # entre le brait haupe et les premions distance que le laist, et pour quoy je fais to # entre le trait hausse set les premiors t = au lieu de + et + au lieu de = ou + est je dis gimer qu'on miteretere de composition par le moyen à la premiere veue l'inigine et composition le tous ces signes, mais qu'ontre cette commodité il y a même quella necessité de faire de la forte il y a même quella necessité de faire de la forte june de differente signification, car posins signes de differente signification, car posins un autre, si on en faisoit + chiseerneroit passition le trait d'embas on en faisoit + chiseerneroit passition du signe + care quella de la forte de la forte il y a neces en faisoit + composition in autre, si on en faisoit + chiseerneroit passition le trait d'embas on the le hiseerneroit passition du signe + care quella forte de la faiser San Nant Simp (emend, d'en baymand il entrevoit and on Saufant implication + par l'aufi nous différence signification fait de 1 'en de du signe + du Fou =: l'autre de deupo = ('un finde + et du - i'ep à dire du + ou and a math for HI NOM STY the contraine à = : ce qui m'est pas le même par exemple grand je dis que ++ vanit + ou = 1 or que une velation tonbre de ces deux signes ambigues compages 3, de soute que ces deux signes ambigues compages 3, de soute que vant + ou = 1 et que and the second + 11. +

IV. Ambig.

$$\begin{array}{ccc} a \sqcap \overline{(3^{\ddagger\dagger})} \, b \, \overline{(3^{\ddagger\dagger})} \, c & \text{signifie} & a \sqcap + b & + c, & \text{c'est à dire,} & a \sqcap + b + c \\ & \overline{(3^{\ddagger})} \left\{ \begin{array}{c} + b \\ - b \end{array} (\overline{3^{\ddagger}}) \left\{ \begin{array}{c} - c \\ + c \end{array} \right. & \begin{array}{c} \text{ou} \, \overline{(3^{\ddagger})} \, b \, \overline{(3^{\ddagger})} \, c \end{array} \right. \end{array}$$

a estant ou la somme, ou la difference de b. c. cela fait voir clairement la raison de la fabrique des signes, et il faut remarquer seulement que de + ou  $(3\pm)$ , on a fait tout expres  $(3^{+})$  au lieu de  $(3^{+})$  par ce que  $(3^{+})$  signifie le signe opposé à  $(3^{+})$ .

Si nous eussions eu

146 TABLE DES SIGNES DE LA MÉTHODE DE L'UNIVERSALITÉ, Mitte 1674	J. 12			
traits, ou-, horsmis un qui se pourra placer ou l'on voudra, par exemple $\ddagger (3+1)(2\pm) a$				

fait  $\ddagger (3^{++})(2^{++}) - a$  ou  $\ddagger (3^{++})(2^{++}) a$  et  $\ddagger (3^{++}) a$ , fait  $\ddagger (3^{++}) a$ .

Si les signes qui se multiplient, ou qui se divisent sont correspondants seulement: leur nature particuliere qui se reconnoit par la forme du Caractere, fera juger du produit. Par exemple

 $(2^{++}) b \cap (2^{+}) a$ , fait  $(2^{++}) ab$  $(2^{+})\,a^{~}(2^{+})\,b,$  fait  $(2^{+})\,ab$  $(2^{++})a \cap (2^{++})b \cap (2^{+})c$ , fait + abc $(3^{++}) a \uparrow (3^{++}) b$ , fait  $(3^{+}) ab$ 

10

5

Exemple d'une extraction d'une Racine Quarrée

Soit une Equation  $2ax \neq \frac{a}{a}x^2$ ,  $\sqcap y^2$ , et la question est comment il faut exprimer la valeur de x, conformement à cette Equation, or je dis, que

$$+x \sqcap \dagger \sqrt{\frac{aq^2 \dagger y^2 q}{a}} \ \pm \ q$$

dont voicy la preuve. En transposant nous aurons  $+x \ddagger q \sqcap \ddagger \sqrt{\frac{aq^2 \ddagger y^2 q}{a}}$ , dont le quarré 15 sera,  $+x^2 \ddagger 2xq + q^2 \sqcap \frac{aq^2 \ddagger y^2q}{a}$ . Multipliant tout par *a*, ce sera:  $+ax^2 \ddagger 2aqx + gq^2 \sqcap$  $\mathcal{P}q^{\mathcal{Z}} \neq y^2 q$ , ostons  $aq^2$  de deux costez, et divisons tout, par  $\neq q$ ,  $\frac{+ax^2}{\ddagger q} \frac{\ddagger 2aqx}{\ddagger q} \sqcap \frac{\ddagger y^2 q}{\ddagger q}$ , et

nous aurons:  $\ddagger \frac{a}{a}x^2 + 2ax \sqcap y^2$  qui est l'equation donnée.

La consideration de cette operation peut servir d'exemple à la pluspart de nos preceptes.

#### LAA VII-7, p. 146

La cheshore de l'univerfable It the say survey and Thereas. nous enseigne de trouver par un seul cabail des formuly analytiques of des confinding Jujets ou Germetriques generales, pour des 12 2 73.40 sans cela anovis Jifferents dort chacun INS apart besom d'un celt Eller 61 Kes In Strumper Sunt ructory am bigu grandcurs, E leg Nynes qui copriment letre relation 513 res ambigues significant bantost des grounden -deul Les Leu Jemp m Tantos ordinning autres. Ol'egard des in fimin unt a servent à marquer qu am bigu Les Sig te tel signe signifi yranden affecter d'un 1 0 ande. ( - dany un Alle. + Jung un certam cas, wignes an Iquation 5 run drever des une liste des Equations particuliere her manin "List and inter all dreper quant toutes les situations fullibles the raidor in point ambula buri d'un certum le calcul : ct on an Sent Am bigus, no der si Sec. Harris 1. les Real the state of comme aves cette deriver ?? on Equations and plusieurs ambiguitez ya and the second inter ontanter l'une or l'antre, on peut re dans des Parentheses closes p. n Buranl thing. les A er d'anney fortes de afin de les di fen 开水市 mene des nombres les marquer dont Magne signe depend, comme grand St sela ca necessarie sur mail si ly f Jem 6 lubles cf ple + r1 = Jort por open ignite ex + t I on pent aux de l'antre. 4 Carce qu'aby il n'y -16 art A VEREPORT cela augoit, To fail 143. Jung July ra Car Je en Je

### LH 4 V 10, f. 39v

This manuscript by Leibniz shows the use of ambiguity signs of the 1st, 2nd and 3rd systems. (see also the 2 following pages)

Table des Mignig de la 1º Univer lat chethode de par en hant ropus de fermes ap A FRESES Am biguile discerner ( signifie a T+6 +c 1856 - ( Consequence le la li Amb. + 0 signific 6 A c'epadireb a nb. moins = 0 Kanifie que ca la of dans une difference entre b, es c c'es i duit , on Equation ambigue, l'appelle deux Signes Opposez Si, mite une grandeur affectée de l'un, vauf autant que qui to womposent timbers Zero moins la même grandeur affectée de l'autre Komme e Equation par ce que on herement syne. Par expemple = el = sannel it pour cette rai Son CS 0 j'ay accoutume de les marques en vorte, qu'il l'antre alica n'y ait point d'autre difference entre 'rupo, soit marque d'un you celle ; que l'un d'eux Sur quil PH Biguite 1+0(==) 24 a lors (2+1+ + C 10. ule paj Jigmifiera Electer 17ŧ a sec le men for pent faire Si les signes c'ep à dire, A M par le moy WH UINGUL = 4-6 mud pourround fairs 1 A estant ou la somme, ou la difference de b. c. Si une memt granden of repeter alors Cela fait vous clairement la maison de la fabrique nourrows faire des signes, it il faut remarquer seulement an how de ¥) 3 VC +y Y, +3C+1 que de + ou(3+), on a fait tout ixpres (st lication e Scmblably par ce que ( =+ = ) signifie tim nu lin de (++) H de How Deup l' Signe oppose a maisti Drux Jignes. origine de differentes ambiguitez par des parentheles Cront distingu par des entreses mananeg acoup 12 Jugee s'il y avoit de a = +67 nous ferions d 62 **П(** roisigne, je la marquer ETh + K +6,7 +ab ain.

LH 4 V 10, f. 40r

I, les synes que se multiplient, on qu'ilivifent sout correspondants sentement leur nutur wi de recommits par la forme du Particuliere' Conardere, fera juger du produit 17 cm pile +dit (3= (= =)a, +=16 += )a (27 22 d'une cotraction de Racine fovarree de Soit une Equation 20x + ax", TY", 11 la question pp comment il fant coprimer la value de X. conformement d'acte Equation, 2000 je dis que 1 +X = [] = Jag + y + q = q En transposant nous nervous don't voisy la preuve. ¥ yq dunt le quarre Ara +X =9.17 = ag +x" = 2xq + q" I aq" = yq, Multipliant tont para, 1 032 +ax + 209x + 99" 199 oftens age to drup coster, et divisions tout, par =9 it yous aurons; to +ax2 + 249× qui est l'equation donnes. + a x + 2ax T y Law fideration to citte operation peut servir l'exemple à la pluspart de nos preceptes Operations compusees In formation des Definitions, invention Consi sent Dany des proprietez, et effection des Demandes. des Figures de bury les P droite ne sul Hons communey mer der de fi Floure en Hur c'ep. ris. siturs sey theoremis, it so confractions trouver unes de quela propleme proprise. Exemple de la Reduction des Figures riflerentes en barmonie, estayé dang les Comiques.

Spomple d'une proprieté commune a' toutes les conigues, demonspiec generalement par la methode de l'universatité ESTERY DE LA METHODE Serre AY UNIVERSALITE remple, d'un fronting les contaires et resolu par bande les contaires et resolu par belle contaires de la Consigne donnée et de l'Hyperbole, l'antre par le moyen de la conigne de la Consigne donnée et de l'Hyperbole, l'antre par le moyen de la conigne

LH 4 V 10, f. 40v



Ambiguity signs, 5th system. LAA VII-1, p. 618

Ac proinde nisi forte in nihilo minores ita incidatur, erit problemati, particulariter quidem, satisfactum tamen. Et supererunt quatuor minimum casus, ob explicationes signorum +((+)) a se invicem independentes, modo ut dixi nihilo minores non obstent, et error calculi abfuerit.

Nunc secundum inventos valores literas quaesitas retrogrado ordine explicemus: erit 10 ex 55.  $v \stackrel{(60)}{\neg} \cdot ((+))$  I et ex 52. erit  $\vartheta \stackrel{(61)}{r} + ((+))$  2. Momentibusque e. s. n. pro arbitrio, erit ex 41. 1  $\stackrel{('2)}{\neg} + ((+))$  e et ex 42. p  $\stackrel{(63)}{\neg} ((+))$  n. r  $\stackrel{(64)}{\neg} + ((+))$  is. Sed hinc iam absurdum orietur, in aequationibus 35, 36. aliisque fiet enim v. g.  $\frac{4}{m} \cap 0$ . adeoque suppositio 58. et quae ex *C-12 C-16 C-16* 

Ambiguity signs, 5th system. LAA VII-1, p. 619

Redeundum ergo ad aeq. 57. videndumque an non formula  $+2\gamma^3 + \gamma$  aequari possit <sup>15</sup> quadrato, hac enim ratione absolutum erit problema. Sit ergo  $2\gamma^3 - \gamma \stackrel{(65)}{\sqcap} + \gamma^2\lambda^2$ . fiet:  $2\gamma^2 - 1$  $\binom{56}{\sqcap} + \gamma\lambda^3$ . erit  $\gamma^2 + \frac{\lambda^2}{2} + \frac{\lambda^4}{16} \stackrel{(67)}{\sqcap} \frac{\lambda^4}{16} + \frac{1}{2}$  sive  $\frac{1}{2} + \frac{\lambda}{4} \stackrel{(68)}{\sqcap} \frac{\sqrt{\lambda^4 + 8}}{4}$ . C-12 C-13 C-14

Ambiguity signs, 5th system. LAA VII-1, p. 619

C-27,	C-26,	C-25,	C-24

	EINLEITUNG XXXIII	
	sind sodann vier Fälle gemeinsam zu betrachten, und auch hier bedient sich Leibniz neuer Doppelvorzeichen, nämlich der Symbole $\ddagger$ , $\ddagger$ , $\ddagger$ und $\ddagger$ . Er erläutert die neuen Symbole nicht; ihre Verwendung scheint für ihn entweder selbstverständlich oder selbst- erklärend zu sein. Der Übergang zur Verwendung der neuen Symbole ist also, soweit es die zusammengesetzten Symbole anbelangt, Weihnachten 1674 offenkundig bereits voll- zogen. Dass die älteren zusammengesetzten Symbole nach Dezember 1674 noch einmal eingesetzt werden, lässt sich nicht belegen.	
	Leibniz führt das fünfte System nicht in einer weiteren programmatischen Schrift	
	ein. Doch hefern manche Stucke Hinweise auf seine Genese. So finden sich in dem auf Dezember 1674 datierten Stück <i>De descriptionious curverum</i> (N. 44) nicht nur die ein	C-16 C-17
C 05	faction Doppelvorzeichen des funften Systems, 7 und 7, sondern mit den Symbolen 7	····· <i>C-04</i>
C-05 ····	ball darauf kanonisierten Formen des fünften Systems dadurch, dass sie jeweils zwei	
	der Ouerhalken mit Hilfe einer weiteren Linie verhinden. Dieser Verhindungsstrich glie-	
	dert die Zeichen. Die durch ihn verbundenen beiden Querbalken bilden zusammen den	
	(donnelde tigen) ersten Fall, der untere Querbalken den (eindeutigen) zweiten Fall. Das	
C-05	Symbol ± hedeutet also, im ersten Unterfall des ersten Falles –, im zweiten Unterfall	
	des ersten Falles $+$ : im zweiten Fall $+$ ". In seinen Exzerpten aus Mariottes $Du choc$	
	des corps (VIII, 2 N, 50), die ebenfalls aus dem Dezember 1674 stammen dürften, kann	
	sogar unmittelbar verfolgt werden, wie Leibniz das fünfte aus dem zweiten System ab- leitet. Er hält fest, die Notation müsse neu gestaltet werden, startet mit dem Zeichen	••••• C-01
B-13 ••••	•••••• $\ddagger$ , ersetzt es zunächst durch die Übergangsform $\ddagger$ und gelangt schließlich zur Form $\ddagger$ .	····· <i>C-08</i>
C-01	Die Übergangsform $\ddagger$ findet sich ausschließlich in diesem Stück und an dieser Stelle, sie spiegelt Leibniz' Emfall wider, ein Minus durch einen halben Querbalken auszudrücken. Diese Darstellungsweise wird — gemeinsam mit der geradlinigen Anordnung der Fälle an einem senkrechten Balken — für das fünfte System charakteristisch. Im selben Stück identifiziert Leibniz auch die Symbole des vierten Systems mit jenen des fünften: ( $\alpha\alpha\omega$ )	
C-18••••	setzt er mit $\ddagger$ gleich, ( $\alpha\omega\alpha$ ) n it $\ddagger$ .	····· C-18
	Die Form ≠ entspricht dem später bevorzugten Symbol ≢ bis auf eine Besonderheit: Bei ihr ist der untelere Querbalken näher an den unteren als den oberen gerückt, woge- gen das Symbol ≢ gleiche Abstände der Querbalken aufweist. Doch sind die Form ‡ und analog gestaltete Zeichen, etwa Symbol ‡, nicht lediglich Ausdruck eines Übergangssta- diums, sondern Leibniz setzt sie bisweilen auch in der Praxis ein, etwa in VII, 1 N. 96 von April 1676. Tatsächlich lassen sich die beiden Symbole ‡ und ≢ zwei unterschiedlichen	C-18
	C-18 C-19 C-11 C-08 C-18	

Ambiguity signs, 2nd and 5th system. LAA VII-7, introduction, p. XXXIII

10

(31) Ponamus jam contra directricem esse non AD, sed AE, constantem WL, quam vocabimus  $\lambda$ . Crementum ordinatarum EG, esse GW; ipsam  $EH \sqcap l$ . primum investigemus hoc modo:  $2ax \ddagger \frac{2a}{q}x^2 \sqcap 2yl$ . sive  $l \sqcap \frac{ax \ddagger \frac{a}{q}x^2}{y} \sqcap \frac{2ax \ddagger \frac{a}{q}x^2 - ax}{y}$  sive  $\frac{y^2 - ax}{y}$ . Jam ut x inveniatur, erit  $x^2 \ddagger \frac{2q\phi}{\phi}x + q^2 \sqcap q^2 \ddagger y^2$ , adeoque fiet  $\ddagger x \ddagger q \sqcap \sqrt{q^2 \ddagger y^2}$ , et  $x \sqcap \ddagger q \ddagger \sqrt{q^2 \ddagger y^2}$  edeoque  $l \sqcap \frac{y^2 \ddagger qa \ddagger a\sqrt{q^2 \ddagger y^2}}{y} \sqcap EH$ . Ergo GW erit  $\prod \frac{\lambda, \uparrow y^2 \ddagger qa \ddagger a\sqrt{q^2 \ddagger y^2}}{y, \uparrow q \ddagger \sqrt{q^2 \ddagger y^2}}$ ; et  $\frac{GB \uparrow WL^2}{GW} \sqcap \frac{y \uparrow \lambda^{\frac{1}{2}}, \uparrow y, \uparrow q \ddagger \sqrt{q^2 \ddagger y^2}}{\lambda_n \uparrow y^2 \ddagger y^2}$ , cujus seriei itidem habetur summa, ex datis omnibus  $\sqrt{q^2 \ddagger y^2}$ .

Quae theoremata vel ideo annotanda duxi, quod semel elapsa non facile rursus in mentem venirent, et non nisi per multas ambages deprehensa sint. Et haec quidem de Trianguli characteristici usu ad dimensiones curvilineorum nunc sufficiant.

Ambiguity sign C-21, 5th system. LAA VII-5, p. 191

Ambiguity signs, 5th system. LAA VII-5, p. 164 see also LH 35 V 3 f. 13v (next page)

ichlen afferentes, (1) 7, (2) 7, (2) 140 T Throw PEn = \*\* = of = xf lattle d'anne er EL II = forment = Juner Presto 200 K 17 X for : ay fat 17 K, a X 17 " + ay y + a' + + + + And the second state of th the ship of the #1# a range for the easing any frage the the and and and a" fraf + f" A A A C & Cohn which a stranger of the stranger of the states and the stranger of the stranger .a.A. - 17 IVH. En EHM MHA gr drun-x e decan - dx Viax-x + ax - 3x and an alle HI AM the stry state the Hyra elfier EL. et Ealerth Delater men El es es DE un omite a, es est facture se astern vel asimme velaligues popul, and me De alin in the promanter it the Tes A. CATP, TAK + TA + AD + TA - AD - TA + AD Π TTA . In ACT ABAX アリ末語 his DE I = ar = 1 = = + x PENT The test of the second of 井, No Adoif a very wa. AA T I DI DI TOTUTA - AD TOTUTA -Sec. 446 + P EL MARE + CL. et TU+AP, p ELEC-ELMEC+CL

LH 35 V 3, f. 13v

(+) P T P T + + + AD EZ TT + EC - CL ×17.9 haller anys To in epp t af - matter in for and in Jzax -x DE TY, per: Π rayy - afr a + ar Sec. chang fr tu An. Anil 16

Proposal to encode Leibnizian ambiguity signs

N. 6

ALGEBRAISCHE STUDIEN 1675-1676

Debet ergo (†)  $6m^3$  (†)  $48m^2$  (†) 72m (†) 64 esse maior quam †  $8r^3 + 35m^2 + 150m + 238$ , 9 differentiae scilicet, ideo, ut sciamus signum + dandum parti maiori, eorum quae signo † vel + affecta sunt.

Ad duas ergo conditiones rem reduximus scilicet, tum ut  $\pm 8\pi i^3 \pm 36m^2 \pm 150m \pm 238, 9$  minor quain ( $\pm 0m^3$  ( $\pm 0m^3$ )) for a subtrahenda inter extrahendum sint maiora addendis. Cubus a  $-4m^2 + 12m - 16$ 

Ambiguity signs, 5th system. LAA VII-2, p. 54



schreibt einfach  $\ddagger$  oler  $\ddagger$ . Eine Erweiterung auf beliebig viele Fälle ist ohne weiteres möglich, ein Einsatz für vier Fälle mit Symbolen wie etwa  $\ddagger$  tatsächlich belegt. Die Vorzeichen dieses Systems verwendet er während seines weiteren Paris-Aufenthalts und darüber hinaus noch viele Jahre später. Beispiele:

Sit  $a \ddagger \frac{a}{q} \varepsilon \sqcap \omega$  fiet  $x \sqcap \ddagger \frac{q}{a} \omega \ddagger q$  (N. 69)

fiet aequatio  $\ddagger 2cz + c^2 \sqcap c^2 \ddagger 2cx + x^2$ . et extrahendo radicem:  $\sqrt{c^2} \ddagger 2cz \sqcap \ddagger c \ddagger x$ . (N. 44)

$$r = \pm b \pm c \quad (\text{VIII, 2 N. 50})$$
pro  $\pm \text{ scriberts} \neq \text{ pro } \pm \text{ scriber} \neq \text{ pro } \pm \text{ scriber} \neq \text{ pro } \pm \text{ scriber} \neq \text{ (N. 15)}$ 

$$TD = \pm TA \equiv AD \qquad EE = \pm EC \equiv CE. \quad (\text{VII, 5 N. 18})$$

$$C-27 \quad C-26 \qquad C-25 \quad C-24$$

$$B-13 \qquad C-18 \quad B-14 \qquad C-19 \quad B-05 \qquad C-23 \qquad B-15 \qquad C-22 \quad C-04 \quad C-05$$

Example of ambiguity signs, 2nd and 5th system. LAA VII-7, introduction p. XLV

 $\sqrt{az} + \frac{ab}{y} \sqcap z. \text{ Ergo } \sqrt{az} \sqcap \frac{yz - ab}{y} \text{ sive } az \sqcap \frac{y^2z^2 - 2abyz + a^2b^2}{y^2} \text{ et fiet } y^2z^2 - y^2za - 2abyz + a^2b^2 \sqcap 0.$ Inquirendum est etiam in divisores aequationum quae sunt duarum incognitarum pluniumve.  $\frac{\ddagger x \ddagger b}{a} \sqcap \frac{c}{z}. \text{ summa scilicet aut differentia } x \text{ et } b. \text{ Ergo } \ddagger x^2 \ddagger bx \sqcap ac. \text{ sive } x^2 \sqcap ac.$  $\frac{\ddagger bx \ddagger ac. \text{ Unde jam patet hoc modo semper cum } bx \text{ est affectum signo } +, \text{ alterum } \underline{ac} \text{ affectum signo } -, \text{ nisi uno casu quo utrumque affectum signo } +, \text{ ergo etiam } x^2 \text{ aequatur summae aut differentiae ipsarum } bx. ac.$ 

Ambiguity signs, 2nd system. *B-02, B-03, B-07* LAA VII-5, p. 233

B-02 B-03

1.12 DEard zax. am (48 enf vel PC PB B n'ca (3)8 P (2) minima data (3) (4) ila hu In 1. tu n' Ja (手) 116 brevi vel C-18 C-28 C-20 C-26 C-25 C-30 DFage. 11210 + 2211 y. 1122×+12118. BE a1 561 nu. + HIZI BE 1an PB 11/2 EP Age 804 10 (12) 11210+2211y 2 1122×+12110 11120+1112 4×+1112×+12210 1122 ad + 2212 ay produce + 1122 adx + 2212 axy 12/11 810 1122 X 11 de + 66 nyx 16 quatio illa lines files

Ambiguity signs of the 5th system, LH 35 XII 1, 217v.

The edition of this manuscript is in preparation.

₽ 1 ■ ŧ ŧ ₹

C-18 C-28 C-20 C-26 C-25 C-30

*C-02* C-03 date FOR BC rent esih 200 12 20 (an) FB es AE S x 6 19 20 muchi valor yipy si 4500 C er 4 24 CV CF. 26 62 6 gra ret tion 2 Ja 9.41 7 4 te wallt Reel serma lo griby hi n S wichicio 170. good + 1 en

Ambiguity signs, 5th system. LH 35 XII 1, fol. 227v

*C-04* 



Example of ambiguity signs, 2nd and 5th system. LAA VIII-2, p. 439



[Fig. 7, ohne Bezug zum Haupttext, quer zur Schreibrichtung]

#### B-02, B-06, B-03; LAA VII-3, p. 360

5

A variety of the symbols used for denoting *plus-minus* during the 17th century is displayed in Cajori I, § 210, pp. 245-247. The symbol introduced by van Schooten was developed further by Wallis and by James Bernoulli. Leibniz used the same symbol in a different context in order to denote *congruence* (see next page). However, we propose the naming terms *plusminus* and *minusplus* for the sake of clarity and simplicity.

### 246 A HISTORY OF MATHEMATICAL NOTATIONS

by De Witt.<sup>1</sup> Wallis<sup>2</sup> wrote  $\otimes$  for + or -, and  $\otimes$  for the contrary. The sign ? was used in a restricted way, by James Bernoulli;<sup>3</sup> he says, " $\otimes$  significat + in pr. e - in post. hypoth.," i.e., the symbol stood for + according to the first hypothesis, and for -, according to the second hypothesis. He used this same symbol in his Ars conjectandi (1713), page 264. Van Schooten wrote also  $\otimes$  for  $\mp$ . It should be added that  $\otimes$  appears also in the older printed Greek books as a ligature or combination of two Greek letters, the omicron o and the upsilon v. The  $\otimes$  appears also as an astronomical symbol for the constellation Taurus.

Da Cunha<sup>4</sup> introduced  $\pm$ ' and  $\pm$ ', or  $\pm$ ' and  $\mp$ ', to mean that the upper signs shall be taken simultaneously in both or the lower signs shall be taken simultaneously in both. Oliver, Wait, and Jones<sup>5</sup> denoted positive or negative N by  $\pm N$ .

211. The symbol [a] was introduced by Kronecker<sup>6</sup> to represent

### 8 PLUSMINUS LOOP SIGN, 8 MINUSPLUS LOOP SIGN; Cajori I p. 246.

In this section Cajori explains the different usage of this two symbols for "+ or –" and "- or +" by van Schooten, Bernoulli and Wallis.

Cajori also discusses in this paragraph the close visual similarity of PLUSMINUS LOOP SIGN with the Greek lowercase ligature *omicron-upsilon* (which will be dealt with in another proposal) and with the astrological *Taurus* symbol (2649). We propose that the mathematical characters shall not be unified with neither of them, because they only make sense here as a *pair* of characters with their mathematical semantics, even if we observe that in historic printed sources typesetters seem to have 'pinched' the *Taurus* out of the Astro lettercase and have flipped it if neccessary. Neither do other encoded characters with some visual resemblance, like Latin letters 0222/0223, 1D15 or Cyrillic A64B, seem appropriate for an unification.

Typographically, in fonts these two glyphs will most likely have to get a treatment different from the Greek letter ligature as well as from the *Taurus*, but rather be visually adjusted to other mathematical operator characters.

Where the First Term hath the Sign + (becaufe made by Multiplying + mo - y) The Second Term is wanting (becaufe  $-ya^3$  and  $+ya^3$  deftroy each other:) In the Third Term, yy hath - (becaufe made of +y into -y;) and b, d, have the fane Terms as in the Quadraticks, (which Sign, be it + or -, we here defign by  $\mathfrak{B}$ , and its contrary by  $\mathfrak{R}$ :) In the Fourth Term, v hath the fame Sign as before (becaufe Multiplied into +y;) but d the contrary to what it had (becaufe multiplyed into -y.) And thus far it holds conftantly, whatever be the Signs of p, q, r.

8 PLUSMINUS LOOP SIGN, 8 MINUSPLUS LOOP SIGN Wallis, Algebra, p. 210. Here an (upright or flipped) *taurus* character has been used.

This Leibniz manuscript features the use of 8 PLUSMINUS LOOP SIGN in the altermine and en your \$.22 native meaning of congruence. um de mastij 12 LH 35 I 11, f. 9v Omne adhlong agions. ad rug niquale (fi Congrum ) it simile of: AXB. B A Significat final ege Ay Cer By D. monade pt A.B + C.d rò B & C.D. C a politic grai dig A.B.Y A.Y. MI L'AND Y A.B.D nohi :3 21 19.15 8. B. A. Bas & Propolitic anity of A.B pulity douby press by A.B. nº fitney Pulle luca corners le refinentiby Sen cruitari good of es manifestion of quin fille Rolates que sabent ad anto codem FAR BRE nes mit externis affunty rifinances relatio July S. A.By D.E. M B.CSE.F. A.C.P.F pertirly puter faile per unitatione Pril A.B.C. & D.S.F. w. fig. 16 noferm 18. if proposito Significans Eoder modo A A BOSF -AtA 6419  $C \cdot Y$ A A.B.C. in Veniri punitif A. BRO 3 his fin Dem Jih A.B, +1 0.9 cil er htu B 12.4 DA A. nal UVIF ita +1 D.S. mobu De A. B. Scruta of D es - try de MA el Jitu lodon A. B.X BLUE. A.Y. it propositio significans D.F.m . X.Y. B undy A. A et B. Inste and vah'o. datis dusby witer Iva cum periri ulin dus X il Y. 45 muneta illa Du iphy A.B. desinde inter X-Y. Jih " holig un they Inoby servato 61.6M. MA 2031. B203M vod ip es dimestrut .3 M 8 Servato Site wither Se X 2061. X 206M 3A 20 X + 3 Bgoy 64 . Nihil autem prohiber fire A. B & X.Y X 20 A : 1 unde fier A. B & A. Y effe - ere 20 20 C. data, unde A.B.g.C.Y. puter chi

Proposal to encode Leibnizian ambiguity signs

## 6. Unicode Character Properties

X001;AMBIGUITY	SIGN	A-01;Sm;0;ON;;;;;N;;;;
X002;AMBIGUITY	SIGN	A-02;Sm;0;ON;;;;;N;;;;
X003;AMBIGUITY	SIGN	A-03;Sm;0;ON;;;;;N;;;;;
X004;AMBIGUITY	SIGN	A-04;Sm;0;ON;;;;;N;;;;;
X005;AMBIGUITY	SIGN	A-05;Sm;0;ON;;;;;N;;;;
X006;AMBIGUITY	SIGN	A-06;Sm;0;ON;;;;;N;;;;;
X007;AMBIGUITY	SIGN	A-07;Sm;0;ON;;;;;N;;;;;
X008;AMBIGUITY	SIGN	A-08;Sm;0;ON;;;;;N;;;;;
X009;AMBIGUITY	SIGN	B-01;Sm;0;ON;;;;;N;;;;;
X010;AMBIGUITY	SIGN	B-02;Sm;0;ON;;;;;N;;;;;
X011; AMBIGUITY	SIGN	B-03;Sm;0;ON;;;;;N;;;;;
X012; AMBIGUITY	SIGN	B-04;Sm;0;ON;;;;;N;;;;;
X013; AMBIGUITY	SIGN	B-05;Sm;0;ON;;;;;N;;;;
X014; AMBIGUITY	SIGN	B-06;Sm;0;ON;;;;;N;;;;
X015;AMBIGUITY	SIGN	B-07;Sm;0;ON;;;;;N;;;;
X016;AMBIGUITY	SIGN	B-08;Sm;0;ON;;;;;N;;;;
X017;AMBIGUITY	SIGN	B-09;Sm;0;ON;;;;;N;;;;
X018;AMBIGUITY	SIGN	B-10;Sm:0;ON;;;;N;;;;
X019; AMBIGUITY	SIGN	B-11;Sm;0:ON;::::N:::::
X020; AMBIGUITY	SIGN	B-12; Sm; 0: ON; ::::N:::::
X021; AMBIGUTTY	SIGN	B-13; Sm; 0:ON:::::N::::
X022; AMBIGUITY	SIGN	B-14;Sm;0;ON::::N::::
X023; AMBIGUITY	SIGN	B-15; Sm; 0; ON:::::N:::::
X024: AMBTGUTTY	SIGN	B-16:Sm:0:ON:::::N::::
X025; AMBIGUTTY	SIGN	B-17; Sm; 0: ON:::::N::::
X026: AMBIGUITY	SIGN	B-18:Sm:0:ON:::::N::::
X027: AMBIGUITTY	STGN	C-01:Sm:0:ON:::::N::
X028: AMBIGUITTY	STGN	C = 0.2 : Sm : 0 : ON : : : : N : · · ·
X029: AMBIGUITTV	STGN	C = 0.3 : Sm : 0 : ON : : · · · N · · · ·
X030:AMRIGUITTV	STGN	$C = 0.4 : Sm : 0 \cdot ON \cdot \cdots N \cdot \cdots$
X031 • AMRTGIITTV	STGN	$C=05 \cdot \text{Sm} \cdot 0 \cdot \text{ON} \cdot \cdots \cdot \text{N} \cdot \cdots$
X032 · AMRTCHITTV	STGN	$C = 0.6 \cdot \text{Sm} \cdot 0 \cdot \text{ON} \cdot \cdots \text{N} \cdot \cdots$
X032, AMRTCHITTV	STGN	$C = 0.7 \cdot \text{Sm} \cdot 0 \cdot \text{ON} \cdot \cdots \cdot \text{N} \cdot \cdots \cdot \text{N}$
X037 • ZWBIGIII	STGN	$C = 0.8 \cdot \text{Sm} \cdot 0 \cdot \text{ON} \cdot \cdots \cdot \text{N} \cdot \cdots \cdot \text{N}$
X035 · AMDIGUITI	GTCM DTGN	C = 00, Sm, 0, ON, j, j, N, j, j, j
X036 • AMDTCUITTY	STON	C = 0 = 0 = 0
X030; AMDIGUITI	STON	C = 10; $SIII$ ; $0$ ; $OII$ ; $i$
AUS/ AMBIGUITY	DIGN	C 12.5m; 0; 0N; ; ; ; ; N; ; ; ;
XUSO; AMBIGUITY	SIGN	C = 12; SIII; U; UN; ; ; ; ; N; ; ; ; N; ; ; ; N; ; ; ; N; ; ; ; N; ; N; ; N; ; N; ; ; N; ; N; ; ; N; ; N; ; ; N; ; ; N; ; ; N; ; N; ; ; N; ; N; ; N; ; ; N; ; N; ; ; N;
XUS9; AMBIGUITY	SIGN	C = 13; SIII; U; UN; ; ; ; ; N; ; ; ; ; N; ; ; ; ; ;
XU4U; AMBIGUITY	SIGN	C-14; Sm; 0; ON;;;;;; N;;;;;
XU41; AMBIGUITY	SIGN	C-15;Sm;U;ON;;;;;N;;;;;
XU42;AMBIGUITY	SIGN	C-10; Sm; U; ON; ; ; ; ; N; ; ; ; ;
XU43;AMBIGUITY	SIGN	C-1/;Sm;0;ON;;;;;N;;;;
XU44; AMBIGUITY	SIGN	C-18;Sm;0;ON;;;;;N;;;;
XU45;AMBIGUITY	SIGN	C-19;Sm;0;ON;;;;;N;;;;
XU46;AMBIGUITY	SIGN	C-20;Sm;0;ON;;;;;N;;;;;
X047;AMBIGUITY	SIGN	C-21;Sm;0;ON;;;;;N;;;;;
X048;AMBIGUITY	SIGN	C-22;Sm;0;ON;;;;;N;;;;;

X049;AMBIGUITY SIGN C-23;Sm;0;ON;;;;N;;;; X050;AMBIGUITY SIGN C-24;Sm;0;ON;;;;N;;;; X051;AMBIGUITY SIGN C-25;Sm;0;ON;;;;N;;;; X052;AMBIGUITY SIGN C-26;Sm;0;ON;;;;N;;;; X053;AMBIGUITY SIGN C-27;Sm;0;ON;;;;N;;;; X054;AMBIGUITY SIGN C-28;Sm;0;ON;;;;N;;;; X055;AMBIGUITY SIGN C-29;Sm;0;ON;;;;N;;;; X056;AMBIGUITY SIGN C-30;Sm;0;ON;;;;N;;;; X057;AMBIGUITY SIGN C-31;Sm;0;ON;;;;N;;;; X058;PLUSMINUS LOOP SIGN;Sm;0;ON;;;;N;;;;

"X" stands for unspecified codespace.

### 7. Bibliography

LAA – refers to: Leibniz, Gottfried Wilhelm: Sämtliche Schriften und Briefe. ('Leibniz-Akademie-Ausgabe', many volumes)

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ISO/IEC JTC 1/SC 2/WG 2 PROPOSAL SUMMARY FORM TO ACCOMPANY SUBMISSIONS				
Please fill all the sections A, B and C below. Please read Principles and Procedures Document (P & P) from <u>http://std.dkuug.dk/JTC1/SC2/WG2/docs/principles.html</u> for				
guidelines and details before filling this form. Please ensure you are using the latest Form from <u>http://std.dkuug.dk/JTC1/SC2/WG2/docs/summaryform.html</u> . See also <u>http://std.dkuug.dk/JTC1/SC2/WG2/docs/roadmaps.html</u> for latest <i>Roadmaps</i> .				
A. Administrative				
1. Title: Proposal to encode Leibnizian ambiguity signs				
2. Requester's name: Uwe Mayer, Siegmund Probst, David Rabouin, Elisabeth Rinner, Andreas Stötzner,				
Achim Trunk, Charlotte Wahl 3. Requester type (Member body/Liaison/Individual contribution): Individual (work group)				
4. Submission date: 2025-02.24.				
5. Requester's reference (if applicable): LUCP L-2504				
6. Choose one of the following: This is a complete proposal: Yes				
(or) More information will be provided later:				
B. Technical – General				
1. Choose one of the following:         a. This proposal is for a new script (set of characters):         No				
b. The proposal is for addition of character(s) to an existing block: No				
Name of the existing block:				
2. Number of characters in proposal: 59				
3. Proposed category (select one from below - see section 2.2 of P&P document):         A-Contemporary       B.1-Specialized (small collection)         Yes       B.2-Specialized (large collection)				
C-Major extinct D-Attested extinct E-Minor extinct				
4 Is a repertoire including character names provided?				
a. If YES, are the names in accordance with the "character naming guidelines" in Annex L of P&P document?				
b. Are the character shapes attached in a legible form suitable for review? Yes				
<ol> <li>Fonts related:         <ul> <li>a. Who will provide the appropriate computerized font to the Project Editor of 10646 for publishing the standard?</li> </ul> </li> </ol>				
Andreas Stötzner				
b. Identify the party granting a license for use of the font by the editors (include address, e-mail, ftp-site, etc.): Andreas Stötzner Gestaltung, Klauflügelweg 21, 88400 Biberach/R., Germany, as@signographie.de				
a. Are references (to other character sets, dictionaries, descriptive texts etc.) provided?				
b. Are published examples of use (such as samples from newspapers, magazines, or other sources) of proposed characters attached? Yes				
7. Special encoding issues: Does the proposal address other aspects of character data processing (if applicable) such as input, presentation, sorting, searching, indexing, transliteration etc. (if yes please enclose information)? <u>No</u>				
8. Additional Information:				
Submitters are invited to provide any additional information about Properties of the proposed Character(s) or Script that will assist in correct understanding of and correct linguistic processing of the proposed character(s) or script. Examples of such properties are: Casing information, Numeric information, Currency information, Display behaviour information such as line breaks, widths etc., Combining behaviour, Spacing behaviour, Directional behaviour, Default Collation behaviour, relevance in Mark Up contexts, Compatibility equivalence and other Unicode normalization related information. See the Unicode standard at <a href="http://www.unicode.org">http://www.unicode.org</a> for such information on other scripts. Also see Unicode Character Database ( <a href="http://www.unicode.org/reports/tr44/">http://www.unicode.org/reports/tr44/</a> ) and associated Unicode Technical Reports for information needed for consideration by the Unicode Technical Committee for inclusion in the Unicode Standard.				

<sup>&</sup>lt;sup>1</sup> Form number: N4502-F (Original 1994-10-14; Revised 1995-01, 1995-04, 1996-04, 1996-08, 1999-03, 2001-05, 2001-09, 2003-11, 2005-01, 2005-09, 2005-10, 2007-03, 2008-05, 2009-11, 2011-03, 2012-01)

### C. Technical - Justification

1. Has this proposal for addition of c	naracter(s) been submitted before?	Yes		
If YES explain as a part of <u>N5277 / L-24-02n</u>				
2. Has contact been made to membe	ers of the user community (for example: National Body,			
user groups of the script or cha	aracters, other experts, etc.)?	Yes		
IT YES, WITH WHOM?	Leibniz-Archiv, Forschungsstelle der Leibniz-Edition,			
	Gättingen Aerdemy of Science and Humanities in Lewer Sever	$(\mathbf{DE})$		
	Philiumm research group of CNRS (UMR 7219 Isboratoire SP	HFRE)/		
	Université de Paris VII:	IILKL) /		
	general: scholars, researchers, authors and editors working in th	e field of		
	science history and upon editions of historic text corpora (e.g.	of G. W.		
	Leibniz, but also many others)			
If YES, available releva	nt documents: L-2409, L-2410			
3. Information on the user communit	y for the proposed characters (for example:	<b>N</b> 7		
Reference:		Yes		
4. The context of use for the propose	ed characters (type of use; common or rare)	Common		
Reference:	mainly specialist usage scholarly worldwide	Johnnon		
5. Are the proposed characters in cu	rrent use by the user community?	Ves		
If YES, where? Reference:	mainly Europe. Americas: other countries	105		
6. After giving due considerations to	the principles in the P&P document must the proposed characters b	e entirely		
in the BMP?		No		
If YES, is a rationale	provided?			
If YES, reference	: e kent together in a contiguous range (rather than being scattered)?			
8 Can any of the proposed character	rs be considered a presentation form of an existing	res		
character or character sequen	ce?	No		
If YES, is a rationale	or its inclusion provided?			
If YES, reference	: rs be encoded using a composed character sequence of either			
existing characters or other pro-	posed characters?	No		
If YES, is a rationale	or its inclusion provided?			
If YES, reference	:			
10. Can any of the proposed charact	er(s) be considered to be similar (in appearance or function)	No		
If YES is a rationale	ior its inclusion provided?	INO		
If YES, reference				
11. Does the proposal include use of	combining characters and/or use of composite sequences?	No		
If YES, is a rationale for such u	ise provided?			
If YES, reference	a and their corresponding glurch images (graphic symbols) provided	2		
	s and their corresponding gryph images (graphic symbols) provided	? No		
12. Does the proposal contain chara	cters with any special properties such as			
control function or similar sema	antics?	No		
If YES, describe in de	tail (include attachment if necessary)			
13. Does the proposal contain any lo	leographic compatibility characters?	No		
If YES, are the equivalent corre	esponding unified ideographic characters identified?	110		
If YES, reference:				