Universal Multiple-Octet Coded Character Set
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Title: Proposal to encode 12 cossic characters

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Status: forward to Script Encoding Working Group / WG2
Action: for expert review and encoding pipeline
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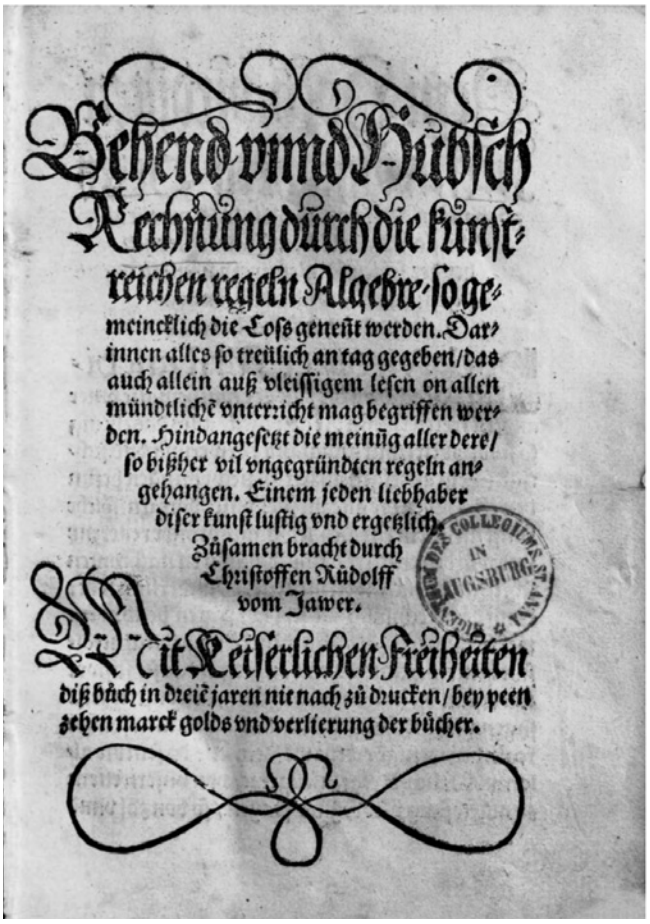
Requester's reference: LUCP L-2518

1. Background

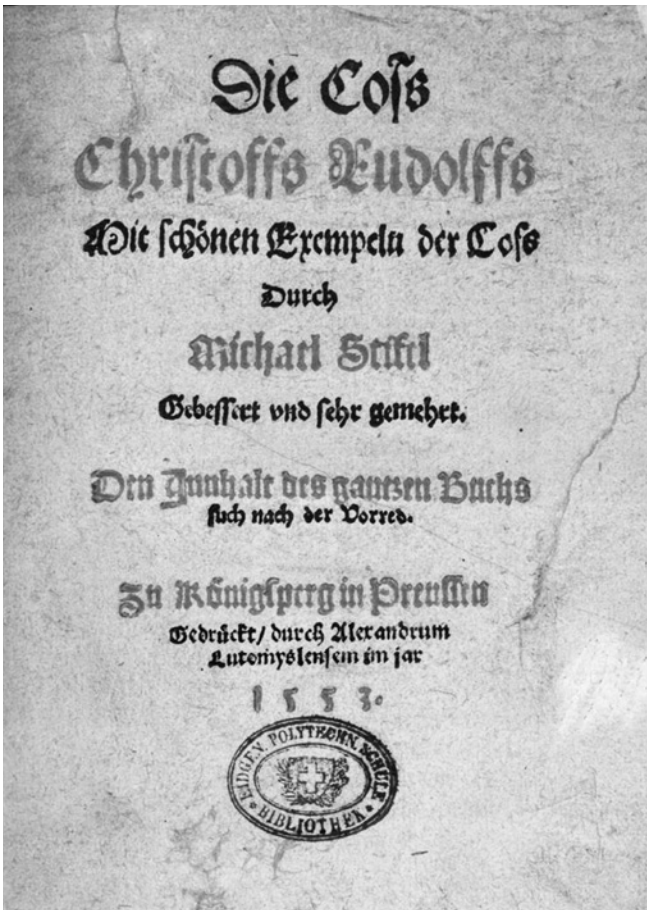
With this document we forward the 3rd version of our **Cossic characters** proposal About the previous version we received recommendations by members of UTC and SEW, in February 2025 (L-2506) and in April 2025 (L-2512). After the Zoom discussion on April 29 there was also a further discussion about one naming issue (A. Freytag / A. Stötzner), which is attached to this doc. as **appendix**, see p. 23ff.
In this new version we implement a number of alterations which follow the recommendations given.

2. About *Coss* or *cossic* characters

"Coss" (or "Coſs", historic) is a German term for written or printed treatises about Algebra.
It derives from Italian *cosa* ("thing") which was used to denote variables in calculations.
The first printed "Coss" was a book by German mathematician Christoff Rudolff (ca. 1500 – before 1543): *Behend und hübſch Rechnung durch die kunſtreichen regeln Algebre, so gemeinicklich die Coſs geneñt werden*. ("Handy and neat calculation by the artful Algebre rules, commonly so called the Coss.") The work was based on older algebra manuscripts which the author studied in Vienna. The book was released in Straßburg in 1525 and was out of stock shortly thereafter. Because it was such a desired title, Michael Stifel edited a new and extended version of Rudolff's Coss in 1553.
In the 1525 edition the character "√" was used the first time for *radix* in print. For the expression of powers (up to ninth) Rudolff used a set of special abbreviation characters. Some of them were common in writing at the time (and used for different purposes), some were rather special additions. Since this set of cossic characters appears explicitly for a longer time in mathematical literature, we see a need to have them encoded, in order to enable precise content encoding in facsimile transcriptions of the historic sources.


Behend vnnd Hübsch
Rechnung durch die kunst-
reichen regeln Algebre / so ge-
meincklich die Coss genennt werden. Dar-
innen alles so treülich an tag gegeben / das
auch allein auß vleissigem lesen on allen
mündtliche vnterricht mag begriffen wer-
den. Hindangesetzt die meinung aller dere /
so bißher vil vngegründten regeln an-
gehangen. Einem jeden liebhaber
diser kunst lustig vnd ergetzlich.
Züsamen bracht durch
Christoffen Rüdolff
vom Jawer.
Mit Keiserlichen Freiheiten
diß büch in dreie jaren nit nach zů drucken / bey peen
zehen marck golds vnd verlierung der bücher.

Title page of Rudolff's *Coss*,
edited in Straßburg 1525.
Source:
Münchner Digitalisierungszentrum


Die Coss
Christoffs Rudolffs
Mit schönen Exempeln der Coss
Durch
Michael Stifel
Gebessert vnd sehr gemehret.
Den Inhalt des gantzen Buchs
findt nach der Vorred.
Zu Königsperg in Preussen
Gedruckt / durch Alexandrum
Lutomyslensem im jar
1553.

Title page of Stifel's new edition of
Rudolff's *Coss*,
printed in Königsberg 1553.
Source:
ETH Zurich

3. Characters

The cossic characters set consists of two sub-groups. Group 1 is a range of 8 Latin abbreviation characters, derivates from Latin letters c, d, r, ſ and z. They represent the initials of the names of roots or powers. These characters are comparable to other already encoded abbreviation characters, like ℔ (*libra,* 2114), ℘ (*per,* 214C), ℻ (*denarius/penny,* 20B0), ℘ (*prae-,* A755) or ꝛ (*-rum,* A75D) which show a combination of a modified basic shape with some sort of graphic attachment, like scriptive loops directly connected to or crossing the base glyph.

We propose to encode the characters as *mathematical symbols*. In one case it has been worked out that a double encoding is required: one mathematical symbol character and one as a Latin letter character.

The aspect of case pairing is not relevant in all of the cases since no capital variants of these characters have ever been used anywhere. “Lowercase” in the proposed character names is merely to indicate the proper ‘parent’ characters. The cossic characters do not occur as abbreviations in general Latin writing but exclusively in calculation contexts. Hence their specific shapes in combination with very specific meaning should justify their encoding, even if an apparent close optical ‘neighbourhood’ to existing characters can be observed.

It is not neccessary to encode all the characters in one place. If this proposal gets accepted, the following new characters will exist:

ꝯ LOWERCASE C WITH SMALL SLASH
= cubus
• denotes cube of the unknown

ꝰ LOWERCASE C WITH RIGHT LOOP
= cubus
• denotes cube of the unknown

ꝯ LOWERCASE C WITH DESCENDER
= census
• denotes square of the unknown

ꝺ LOWERCASE D ROTUNDA WITH CROSSING LOOP
= dragma
• denotes numerus / constant
→ 1E9F ẟ latin small letter delta
→ A77A ꝺ latin small letter insular d
→ 20B0 ₰ german penny sign

ꝛ LOWERCASE R ROTUNDA WITH LOOP
= res, radix
• denotes the unknown
→ A75D ꝛ latin small letter r rotunda
→ A75D ꝝ latin small letter rum rotunda
→ A776 ꝶ latin letter small capital rum
→ 221A √ square root

ſß LATIN SMALL LIGATURE LONG S WITH DESCENDER S xi06

= sursolidum

• denotes fifth power of the unknown in historical mathematics

• glyph always resembles long s and s

→ corresponding mathematical symbol is [xi07]

→ 017F ſ latin small letter long s

→ 0073 s latin small letter s

→ 00DF ß latin small letter sharp s

→ A7D7 Ꟈ latin small letter middle scots s

MATHEMATICAL ITALIC LIGATURE LONG S WITH DESCENDER S xi07

= sursolidum

• denotes fifth power of the unknown in historical mathematics

• glyph always resembles long s and s

• in plain text the corresponding Latin letter [xi06] is preferred

LATIN SMALL LETTER LONG S WITH TOP LOOP

= sursolidum

• denotes fifth power of the unknown in historical mathematics

→ 017F ſ latin small letter long s

→ 1E9C ẜ latin small letter long s with diagonal stroke

→ 1E9D ẝ latin small letter long s with high stroke

→ xi06 latin small ligature long s with descender s

This character is proposed as a variation sequence:

(LOWERCASE KURRENT Z) *variation sequence to U+1D4CF*

This variation sequence character would introduce a new category of variation sequences related to the Mathematical Alphanumerics (block 1D500) subgroup *Script symbols* (lowercase, 1D4B6 to 1D4CF). There has been a concern wether this single char. should justify such a new series of var. sequ. characters. Therefore we give the hint that in one of our next proposals this letterlike symbol is contained:

 LOWERCASE KURRENT X SIGN

This may well be regarded as a case of the same kind and dealt with in the same way.

Further examples of the use of other *kurrent style* small letters in mathematical notation can not be demonstrated at this point. It is, however, likely that more symbols of this kind may be testified in the future.

Group 2 is a set of three *root* or *radix* symbols, related to √ (221A).
The names have been re-defined in order to match the naming scheme of 221A SQUARE ROOT.


FOURTH ROOT
→ 221A √ square root


EIGHTH ROOT
→ 221A √ square root


SIXTEENTH ROOT
→ 221A √ square root

4. The *cubus* characters

In group 1 there are two different characters for "cubus": ç and cç. Although the meaning is the same, the representative glyphs differ considerably. These typographic differences are strongly tied to certain writing or publishing traditions. Therefore we propose to encode two characters, thus being in line with a principle which has been followed in e.g. the alchemical characters block, where also (in some cases) two or three different characters bear (basically) the same meaning. This character pair situation is also evident with some other characters of the cossic set.

5. *Radix* characters

The LOWERCASE R ROTUNDA WITH LOOP ꝛ℘ occurs frequently with the meaning of "res" or "radix". The left part of the glyph is derived from the shape of the capital R, in a similar way as the left parts of the R ROTUNDA and RUM ROTUNDA characters (A75A to A75D) are derived from R. The distinctive feature of ꝛ℘ is its right half with a prominent crossing loop moving down as a descender. In this form, the character unambiguously denotes the mathematical meaning in contrast to the syllabic meaning "-rum" of A75C/A75D as well as of A776.
But, as the sources show, the small capital R with stroke ꝶ (A776, LATIN SMALL CAPITAL RUM) has also been used in the set of cossic characters. Therefore we additionally propose the addition of new anotations to this character, as follows:

A776 ꝶ LATIN LETTER SMALL CAPITAL RUM
• cossic sign for res, radix
→ A75D ꝝ latin small letter rum rotunda
→ [xxxx] ꝛ℘ lowercase r rotunda with loop

6. *Sursolidum* characters

"Sursolidum" is also represented by two different characters: ß and ſ°. This dual track situation has evolved historically by different local notation traditions. On the one hand, in an edition of historic sources it would not be tolerable to encode e.g. ß (or even ß, 00DF) instead of ſ°.

In the preceding discussions a variety of character names for ß have been considered:

MATHEMATICAL ITALIC SHARP S

SHARP S WITH HOOK

MATHEMATICAL SHARP S

SHARP S WITH DESCENDER

The background for a decision about the name has changed for two reasons: a) *two* characters are proposed now instead of one; b) a further evaluation has revealed that a definition containing the part 'SHARP S' would give a wrong interpretation of the character's nature, identity and use. A detailed discussion of this matter is to be found in the appendix at the end of this document.

Following a suggestion made by A. Freytag, we propose the names:

[xi06] LATIN SMALL LIGATURE LONG S WITH DESCENDER S
[xi07] MATHEMATICAL ITALIC LIGATURE LONG S WITH DESCENDER S

By this naming we achieve:

- compliance with established UCS naming conventions
- a structural and historical correct explanation
- a clearly understandable definition of the characters nature
- correct naming of the character's base characters
- to avoid confusion with the German ß (SHARP S)
- maintain the distinction between a plain-text character and a specific math character
- allow a different treatment of (xi07) and (xi06) in an Italic font, if there should be any need for that
- leave the door open for a possible later request for a *mathematical sharp s* (without a descender)

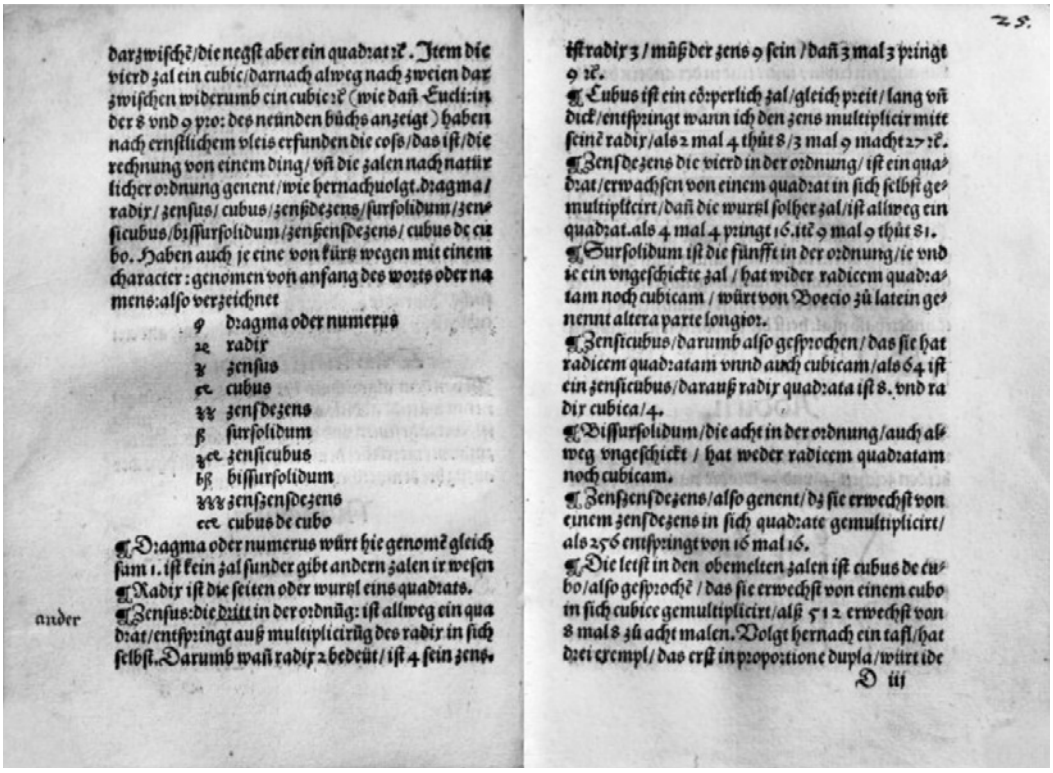
7. *Census* characters

There are also two different characters for "census/zensus": ȼ and ȝ, related either to *c* or to *z*. We propose LOWERCASE KURRENT Z as a new variation sequence on U+1D4CF. Since ȼ and ȝ are derivates of two different base letters, ȼ is proposed seperately.

8. Figures and further explanations

The *Coss* characters became a widely adopted set of characters for denoting powers and roots, in the 16th and 17th century. We show a couple of instances from printed sources and also a piece of manuscript evidence by Leibniz.

See page 19 for a synopsis of all characters belonging to the first group.



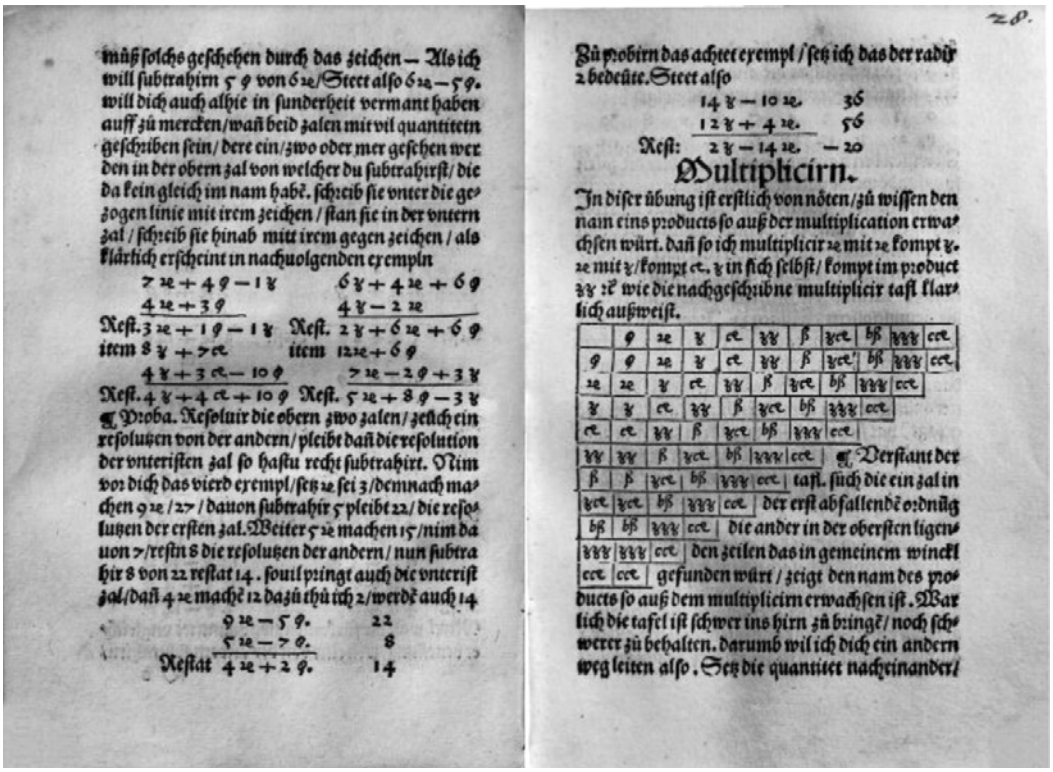
darzwischē/die negst aber ein quadrat rē. Item die vierd zal ein cubic/darnach alweg nach zweien dar zwischen widerumb ein cubic rē (wie dañ Euclides in der 8 vnd 9 pto: des neünden büchs anzeigt) haben nach ernstlichem vleis erfunden die cos/das ist/die rechnung von einem ding/ vñ die zalen nach natürlicher ordnung genent/wie hernachuolgt. Dragma/radix/zensus/cubus/zenßdezens/sursolidum/zensicubus/bissursolidum/zenßenßdezens/cubus de cubo. Haben auch je eine von kürtz wegen mit einem character: genomen von anfang des worts oder namens: also verzeichnet

9 dragma oder numerus
re radix
ȝ zensus
ce cubus
ȝȝ zenßdezens
ß sursolidum
ȝce zensicubus
bß bissursolidum
ȝȝȝ zenßzenßdezens
cce cubus de cubo

¶ Dragma oder numerus würt hie genomē gleichsam 1. ist kein zal sunder gibt andern zalen ir wesen
¶ Radix ist die seiten oder wurtzl eins quadrats.
anderer ¶ Zensus: die dritt in der ordnūg: ist allweg ein quadrat/entspringt auß multiplicirūg des radix in sich selbst. Darumb wañ radix 2 bedeūt/ist 4 sein zens.

ist radix 3/müß der zens 9 sein/dañ 3 mal 3 pringt 9 rē.
¶ Cubus ist ein cōperlich zal/gleich preit/lang vñ dick/entspringt wann ich den zens multiplicir mit seinē radix/als 2 mal 4 thůt 8/3 mal 9 macht 27 rē.
¶ Zenßdezens die vierd in der ordnung/ist ein quadrat/erwachsen von einem quadrat in sich selbst gemultiplicirt/dañ die wurtzl solcher zal/ist allweg ein quadrat. als 4 mal 4 pringt 16. itē 9 mal 9 thůt 81.
¶ Sursolidum ist die fünfft in der ordnung/ie vnd ie ein vngeschickte zal/hat wider radicem quadratam noch cubicam/würt von Boecio zů latein genennt altera parte longior.
¶ Zensicubus/darumb also gesprochen/das sie hat radicem quadratam vnnd auch cubicam/als 64 ist ein zensicubus/darauß radix quadrata ist 8. vnd radix cubica/4.
¶ Bissursolidum/die acht in der ordnung/auch allweg vngeschickt/hat weder radicem quadratam noch cubicam.
¶ Zenßenßdezens/also genent/dz sie erwechst von einem zenßdezens in sich quadrate gemultiplicirt/als 256 entspringt von 16 mal 16.
¶ Die letst in den obemelten zalen ist cubus de cubo/also gesprochē/das sie erwechst von einem cubo in sich cubice gemultiplicirt/alß 512 erwechst von 8 mal 8 zů acht malen. Volgt hernach ein tafl/hat drei exempl/das erst in proportione dupla/würt ide

D iij



můß solchs geschehen durch das zeichen — Als ich will subtrahirn 5 9 von 6 re/Stet also 6 re — 5 9. will dich auch alhie in sunderheit vermant haben auff zů mercken/waß beid zalen mit vil quantiteten geschriben sein/dere ein/zwo oder mer gesetzen werden in der obern zal von welcher du subtrahirst/die da kein gleich im nam habē. schreib sie vnter die gezogen linie mit irem zeichen/stan sie in der vntern zal/schreib sie hinab mit irem gegen zeichen/als klärlich erscheint in nachuolgenden exempln

7 re + 4 9 — 1 ȝ 6 ȝ + 4 re + 6 9
4 re + 3 9 4 ȝ — 2 re
Rest. 3 re + 1 9 — 1 ȝ Rest. 2 ȝ + 6 re + 6 9
item 8 ȝ + 7 ce item 12 re + 6 9
4 ȝ + 3 ce — 10 9 7 re — 2 9 + 3 ȝ
Rest. 4 ȝ + 4 ce + 10 9 Rest. 5 re + 8 9 — 3 ȝ

¶ Proba. Resoluir die obern zwo zalen/zeuch ein resolutzen von der andern/pleibt dañ die resolution der vnteristen zal so hastu recht subtrahirt. Nim vor dich das vierd exempl/sets re sei 3/demnach machen 9 re/27/davon subtrahir 5 pleibt 22/die resolutzen der ersten zal. Weiter 5 re machen 15/nim davon 7/restn 8 die resolutzen der andern/nun subtrahir 8 von 22 restat 14. soul pringt auch die vnterist zal/dañ 4 re machē 12 dazů thů ich 2/werdē auch 14

9 re — 5 9. 22
5 re — 7 9. 8
Restat 4 re + 2 9. 14

Zů probirn das achtet exempl/setz ich das der radix 2 bedeūte. Stet also

14 ȝ — 10 re. 36
12 ȝ + 4 re. 56
Rest: 2 ȝ — 14 re. — 20

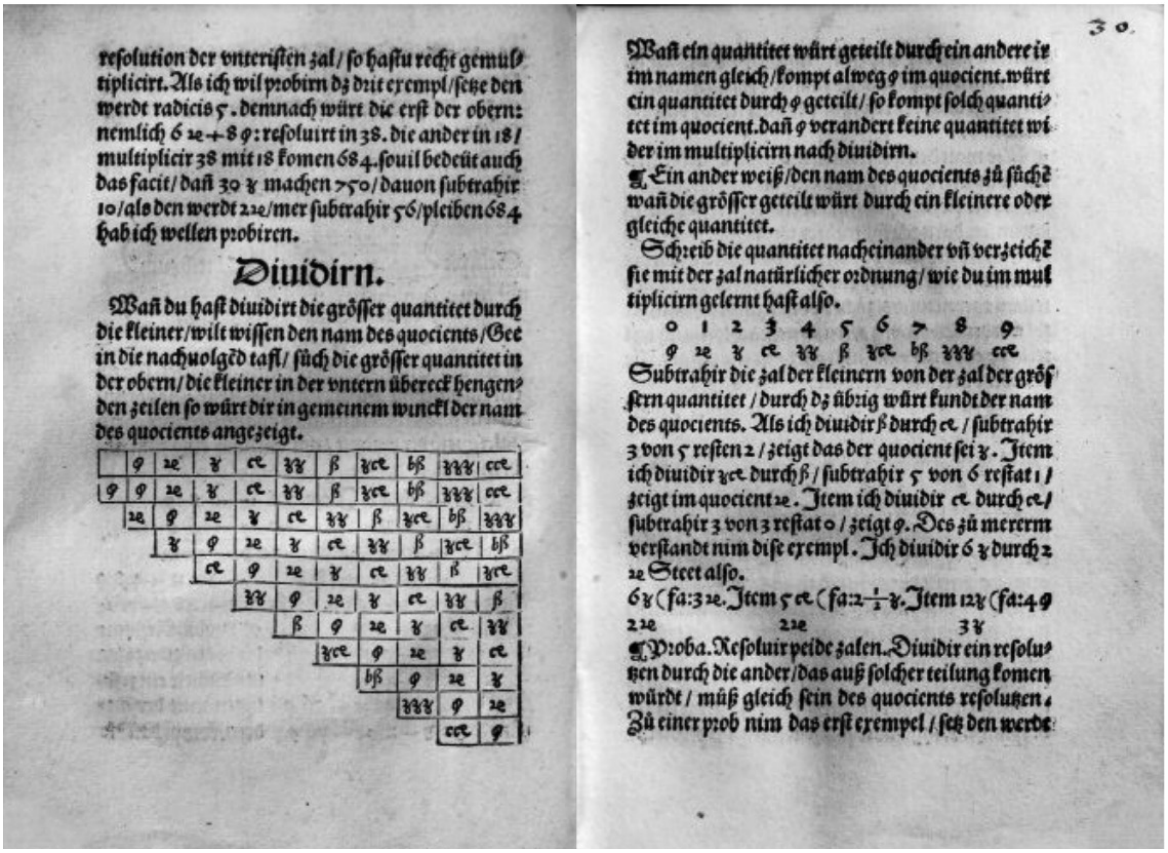
Multiplicirn.

In diser übung ist erstlich von nöten/zů wissen den nam eins products so auß der multiplication erwachsen würt. dañ so ich multiplicir re mit re kompt ȝ. re mit ȝ/kompt ce. ȝ in sich selbst/kompt im product ȝȝ rē wie die nachgeschribne multiplicir tafl klarlich außweist.

	9	re	ȝ	ce	ȝȝ	ß	ȝce	bß	ȝȝȝ	cce
9	9	re	ȝ	ce	ȝȝ	ß	ȝce	bß	ȝȝȝ	cce
re	re	ȝ	ce	ȝȝ	ß	ȝce	bß	ȝȝȝ	cce	
ȝ	ȝ	ce	ȝȝ	ß	ȝce	bß	ȝȝȝ	cce		
ce	ce	ȝȝ	ß	ȝce	bß	ȝȝȝ	cce			
ȝȝ	ȝȝ	ß	ȝce	bß	ȝȝȝ	cce				
ß	ß	ȝce	bß	ȝȝȝ	cce					
ȝce	ȝce	bß	ȝȝȝ	cce						
bß	bß	ȝȝȝ	cce							
ȝȝȝ	ȝȝȝ	cce								
cce	cce									

¶ Verstant der tafl. sůch die ein zal in der erst abfallendē ordnūg die ander in der obersten ligenden zeilen das in gemeinem winckl gefunden würt/zeigt den nam des products so auß dem multiplicirn erwachsen ist. Warlich die tafel ist schwer ins hirn zů bringē/noch schwerer zů behalten. darumb wil ich dich ein andern weg leiten also. Setz die quantitet nacheinander/

Christoff Rudolff: *Behend und hübſch Rechnung durch die kunſtreichen regeln Algebre, ſo gemeinicklich die Coſs genennt werden*. Straßburg 1525. fol. 24v-25r, 27v-28r.



resolution der vnterſten zal / ſo haſtu recht gemultiplicirt. Als ich wil probirn dz drit exempl / ſetze den werdt radicis 5. demnach würt die erſt der obern: nemlich 6 ze + 8 9: reſoluirt in 38. die ander in 18 / multiplicir 38 mit 18 komen 684. ſouil bedeüt auch das facit / daß 30 ɣ machen 750 / dauon ſubtrahir 10 / als den werdt 2ze / mer ſubtrahir 56 / pleiben 684 hab ich wellen probiren.

Diuidirn.

Waß du haſt diuidirt die gröſſer quantitet durch die kleiner / wilt wiſſen den nam des quocients / Gee in die nachuolgēd tafl / ſüch die gröſſer quantitet in der obern / die kleiner in der vntern übereck hengenden zeilen ſo würt dir in gemeinem winckl der nam des quocients angezeigt.

	9	ze	ɣ	ce	ɣɣ	ß	ɣce	bß	ɣɣɣ	cce
9	9	ze	ɣ	ce	ɣɣ	ß	ɣce	bß	ɣɣɣ	cce
ze	ze	9	ze	ɣ	ce	ɣɣ	ß	ɣce	bß	ɣɣɣ
ɣ	ɣ	ze	9	ze	ɣ	ce	ɣɣ	ß	ɣce	bß
ce	ce	ɣ	ze	9	ze	ɣ	ce	ɣɣ	ß	ɣce
ɣɣ	ɣɣ	ce	ɣ	ze	9	ze	ɣ	ce	ɣɣ	ß
ß	ß	ɣɣ	ce	ɣ	ze	9	ze	ɣ	ce	ɣɣ
ɣce	ɣce	ß	ɣɣ	ce	ɣ	ze	9	ze	ɣ	ce
bß	bß	ɣce	ß	ɣɣ	ce	ɣ	ze	9	ze	ɣ
ɣɣɣ	ɣɣɣ	bß	ɣce	ß	ɣɣ	ce	ɣ	ze	9	ze
cce	cce	ɣɣɣ	bß	ɣce	ß	ɣɣ	ce	ɣ	ze	9

30.

Waß ein quantitet würt geteilt durch ein andere in im namen gleich / kompt alweg 9 im quocient. würt ein quantitet durch 9 geteilt / ſo kompt ſolch quantitet im quocient. daß 9 verandert keine quantitet wider im multiplicirn nach diuidirn.

¶ Ein ander weiß / den nam des quocients zů ſüchē waß die gröſſer geteilt würt durch ein kleinere oder gleiche quantitet.

Schreib die quantitet nacheinander vñ verzeichē ſie mit der zal natürlicher ordnung / wie du im multiplicirn gelernt haſt alſo.

0 1 2 3 4 5 6 7 8 9
9 ze ɣ ce ɣɣ ß ɣce bß ɣɣɣ cce

Subtrahir die zal der kleinern von der zal der gröſſern quantitet / durch dz übrig würt kundt der nam des quocients. Als ich diuidir ß durch ce / ſubtrahir 3 von 5 reſten 2 / zeigt das der quocient ſei ɣ. Item ich diuidir ɣce durch ß / ſubtrahir 5 von 6 reſtat 1 / zeigt im quocient ze. Item ich diuidir ce durch ce / ſubtrahir 3 von 3 reſtat 0 / zeigt 9. Des zů mererm verſtandt nim diſe exempl. Ich diuidir 6 ɣ durch 2 ze Steet alſo.

6 ɣ (ſa: 3 ze. Item 5 ce (ſa: 2 ½ ɣ. Item 12 ɣ (ſa: 4 9
2ze 2ze 3ɣ

¶ Proba. Reſoluir peide zalen. Diuidir ein reſolutzen durch die ander / das auß ſolcher teilung komen würdt / müß gleich ſein des quocients reſolutzen. Zů einer prob nim das erſt exempel / ſetz den werdt

diuidirt werdē durch 3. Darumb laß dich alhie nit irren / was oben im fünfften capitl des erſten teyls bey der diuiſion geſagt iſt.

¶ Die erſt equation oder regl der coſs.

Waß zwo quantitetn natürlicher ordnung einander gleich werden / diuidir die kleiner in die gröſſer quantitet / der quocient zeigt an den werdt 1 ze. Als in diſen exemplN.

Die teiler

3 ze		6 9	
4 ɣ		8 ze	
5 ce		10 ɣ	
6 ɣɣ		12 ce	
7 ß	Sein gleich	14 ɣɣ	facit 1 ze. 2 9
8 ɣce		16 ß	
9 bß		18 ɣce	
10 ɣɣɣ		20 bß	
11 cce		22 ɣɣɣ	

¶ Die ander equation.

Waß zwo quantitetn einander gleich werden / zwiſchen welchen eine: natürlicher ordnung nach begriffen: geſchwigen iſt. Diuidier die kleiner in die gröſſer quantitet / radix quadrata des quocients zeigt an den werdt 1 ze. Als

54.

Die teiler

2 ɣ		8 9	
3 ce		12 ze	
4 ɣɣ		16 ɣ	
5 ß	Sein gleich	20 ce	ſa: 1 ze. 2 9
6 ɣce		24 ɣɣ	
7 bß		28 ß	
8 ɣɣɣ		32 ɣce	
9 cce		36 bß	

¶ Die dritt equation.

Waß zwo quantitetn einander gleich werden / zwiſchen welchē zwo andere: natürlicher ordnūg nach begriffen: geſchwigē ſein. Diuidir die kleiner in die gröſſer quātitet / radix cubica des quocients zeigt an den werdt 1 ze. Als

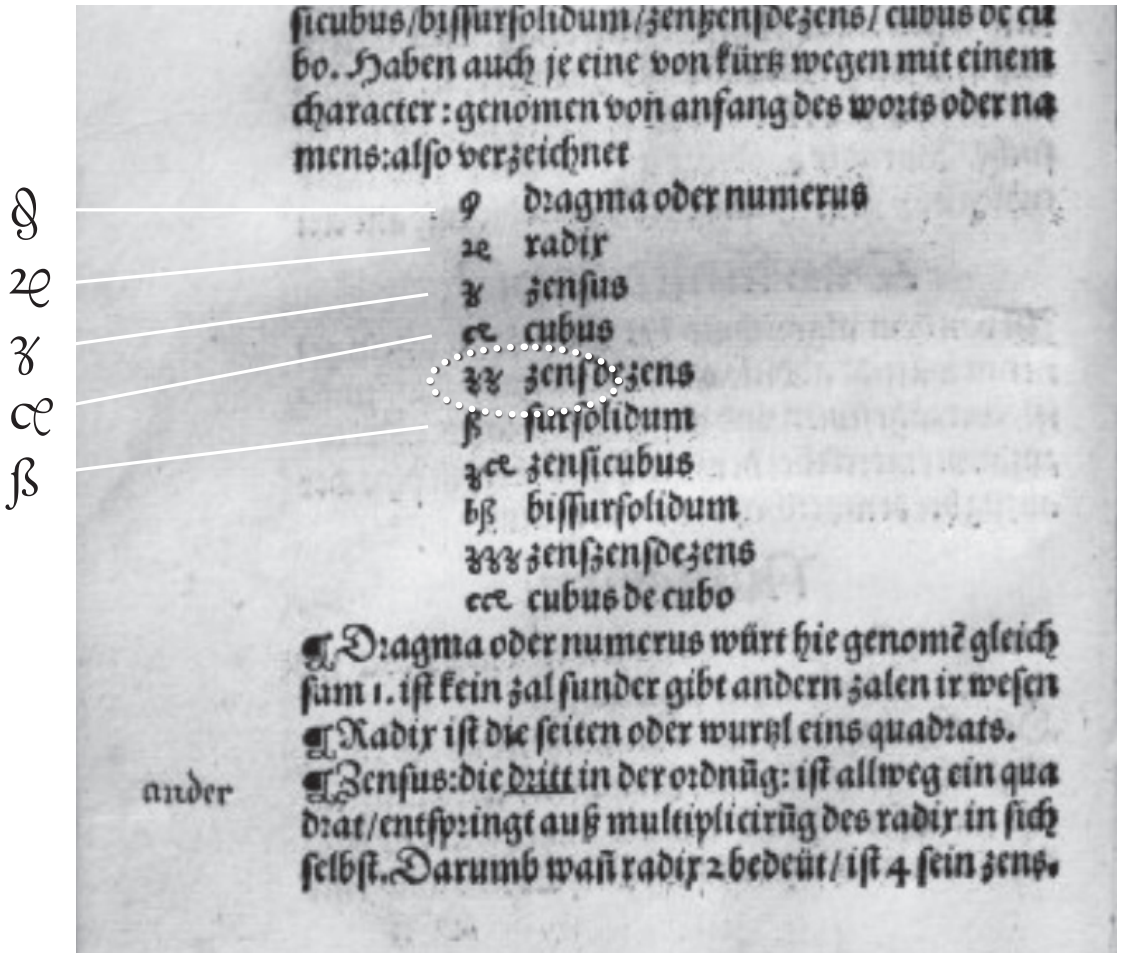
Die teiler

2 ce		16 9	
3 ɣɣ		24 ze	
4 ß		32 ɣ	
5 ɣce	Sein gleich	40 ce	ſa: 1 ze. 2 ze
6 bß		48 ɣɣ	
7 ɣɣɣ		56 ß	
8 cce		64 ɣce	

Die vierd equation

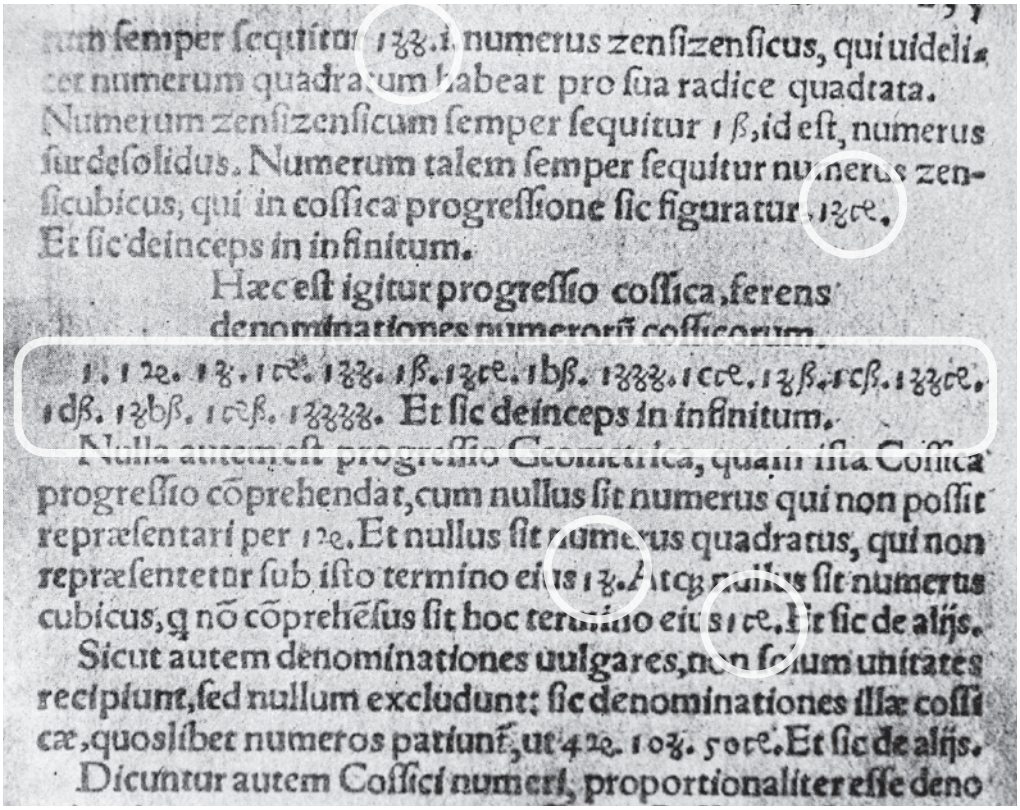
Waß zwo quantitetn einander gleich werden zwiſchen welchē / drey andere natürlicher ordnūg nach begriffen / geſchwigē ſein. Diuidir die kleiner in die gröſſer quantitet / Radix quadrata von radice quadrata des quocients / zeigt an den werdt 1 ze. Als

Christoff Rudolff: *Behend und hübſch Rechnung durch die kunſtreichen regeln Algebre, ſo gemeinicklich die Coſs genennt werden*. Straßburg 1525. fol. 29v-30r, 53v-54r.

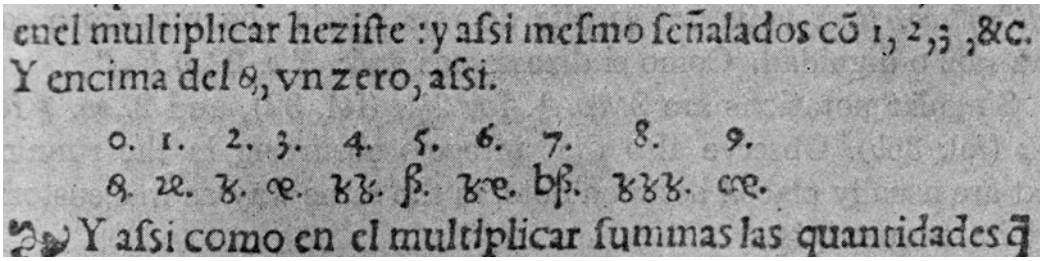


Christoff Rudolff: *Behend und hübſch Rechnung* … Straßburg 1525, part of fol. 24v. In this chapter Rudolff introduces the set of root and power symbols by samples and explanations. We can see: ȝ (LOWERCASE KURRENT Z), ꝯ LOWERCASE D ROTUNDA WITH CROSSING LOOP, ꝛe LOWERCASE R ROTUNDA WITH LOOP, ce LOWERCASE C WITH RIGHT LOOP and ß LATIN SMALL LIGATURE LONG S WITH DESCENDER S.

This print demonstrates the deliberate distinction between the cossic character ȝ and the normal fraktur z (see at). Whereas in other scenarios this two shapes could be seen as 'just' glyph variants without semantic distinction, in this case the form difference is clearly an indicator for a specific meaning. The character ȝ (LOWERCASE KURRENT Z) is denoting *zensus*. It is graphically characterized by a) a round-shaped upper part (mostly), and b) a prominent loop descender which crosses upwards. The origin of its shape is neither *Fraktur* type nor Latin script style but the German *Kurrent* writing style.

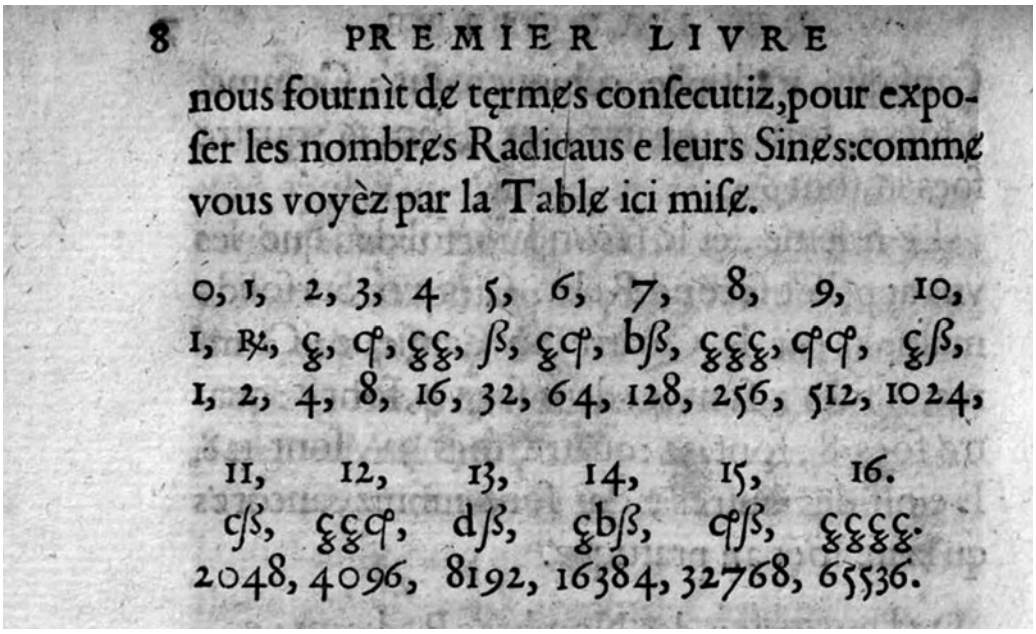


Stifel 1544 (after Cajori). This sample shows ȝ (LOWERCASE KURRENT Z), 2e LOWERCASE R ROTUNDA WITH LOOP, ce LOWERCASE C WITH RIGHT LOOP and ß LATIN SMALL LIGATURE LONG S WITH DESCENDER S.



Aurel 1552, fol. 73B (after Cajori). This sample shows ȝ (LOWERCASE KURRENT Z) (2., 4., 6., 8.), ȣ LOWERCASE D ROTUNDA WITH CROSSING LOOP (0.), 2e LOWERCASE R ROTUNDA WITH LOOP (1.), ce LOWERCASE C WITH RIGHT LOOP 3., 6., 9.), and ß LATIN SMALL LIGATURE LONG S WITH DESCENDER S (5., 7.).

These samples also show how those characters were used in combination to express the powers 4th and so on.

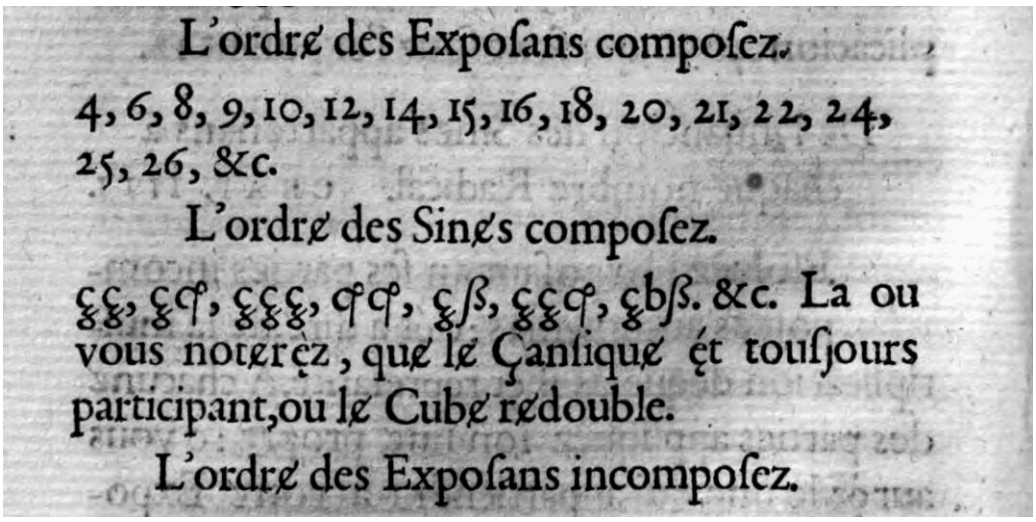


8 PREMIER LIVRE

nous fournìt de termes conſecutiz, pour expoſer les nombres Radicaus e leurs Sines: comme vous voyèz par la Table ici miſe.

0, 1, 2, 3, 4 5, 6, 7, 8, 9, 10,
1, R/, ç, c, çç, ß, çc, bß, ççç, cc, çß,
1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024,

11, 12, 13, 14, 15, 16.
cß, ççc, dß, çbß, cß, çççç.
2048, 4096, 8192, 16384, 32768, 65536.



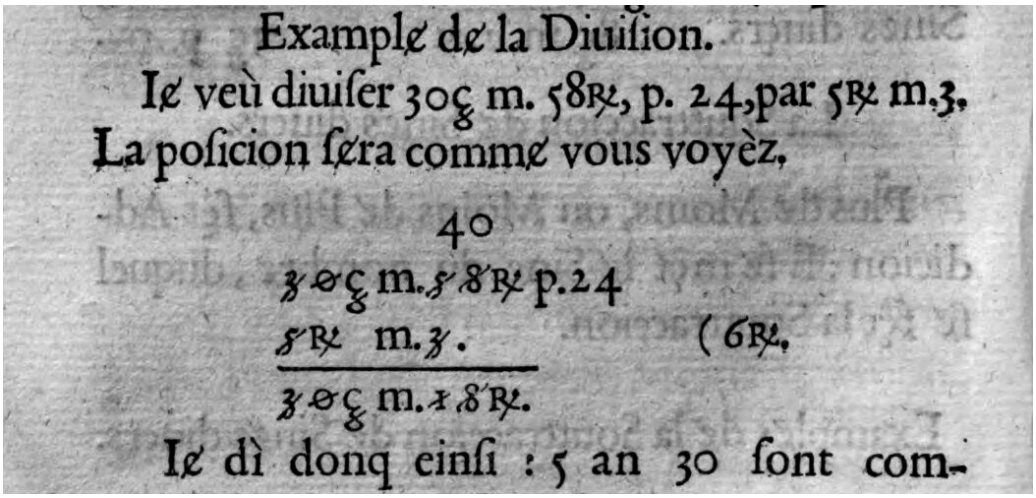
L'ordre des Expoſans compoſez.

4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, &c.

L'ordre des Sines compoſez.

çç, çc, ççç, cc, çß, ççc, çbß. &c. La ou vous noterèz, que le Çanſique ét touſjours participant, ou le Cube redouble.

L'ordre des Expoſans incompoſez.



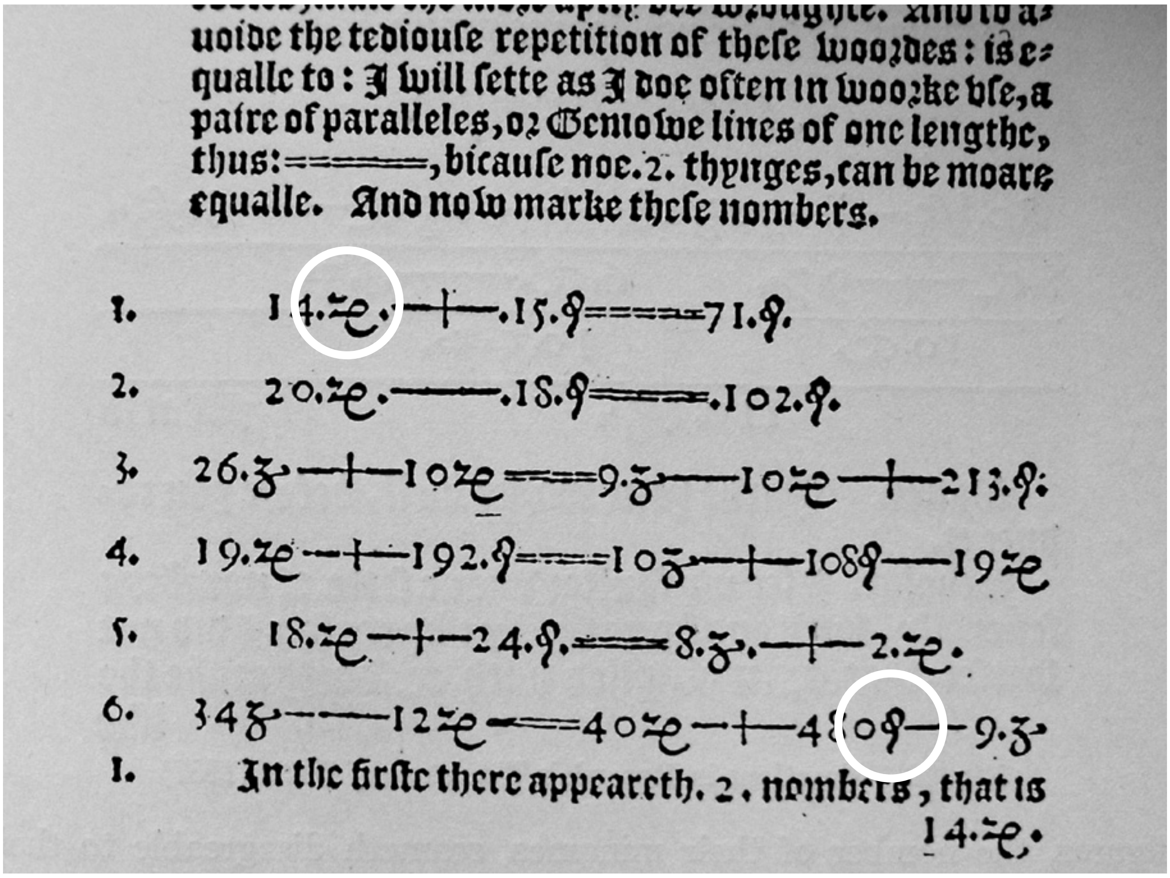
Example de la Diuiſion.

Ie veù diuiſer 30ç m. 58R/, p. 24, par 5R/ m. 3.
La poſicion ſera comme vous voyèz.

40
3 ⊘ ç m. 5 8R/ p. 24
5R/ m. 3. (6R/.
3 ⊘ ç m. 1 8R/.

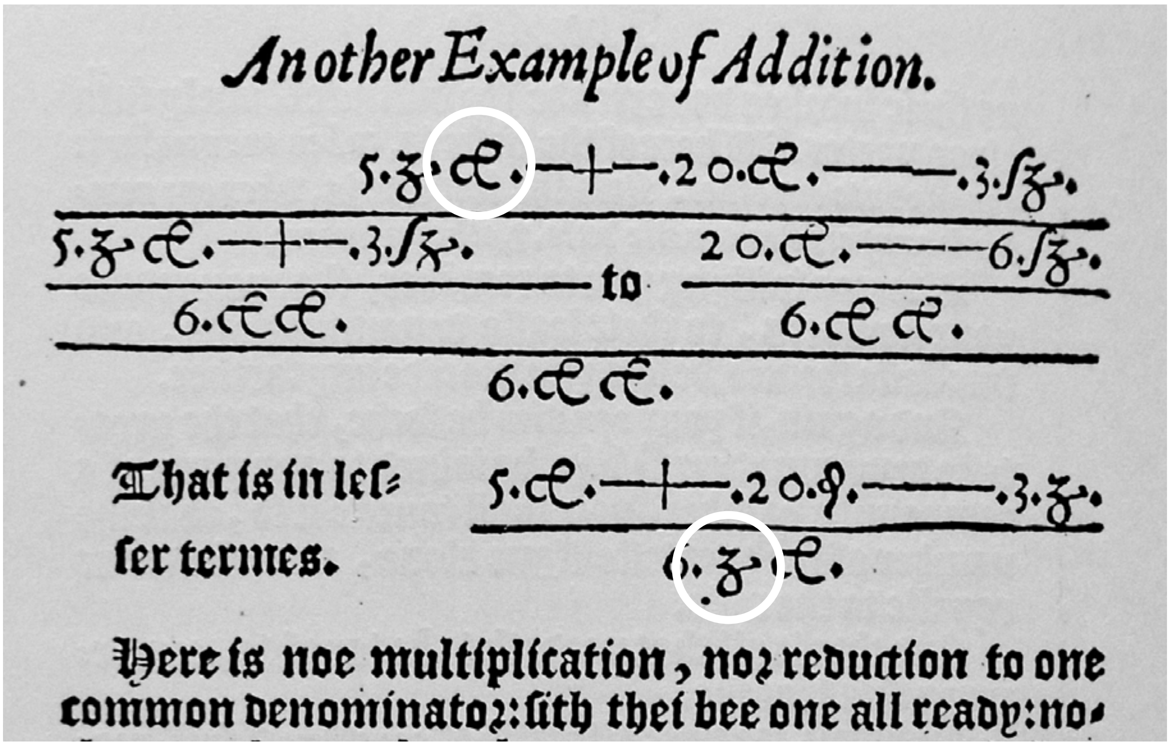
Ie dì donq einſi : 5 an 30 ſont com-

Three extracts from Peletier 1554: ç LOWERCASE C WITH DESCENDER, c LOWERCASE C WITH RIGHT LOOP and ß LATIN SMALL LIGATURE LONG S WITH DESCENDER S. These samples also demonstrate the usage of R/ (A776) as part of the cossic set, as well as the use of slashed figures (on which we elaborate in another proposal).



uoide the tediouse repetition of these woordes : is equalle to : I will sette as I doe often in woorke use, a paire of paralleles, or Gemowe lines of one lengthe, thus: =======, bicause noe .2. thynges, can be moare equalle. And now marke these nombers.

1. In the firste there appeareth. 2. nombers, that is

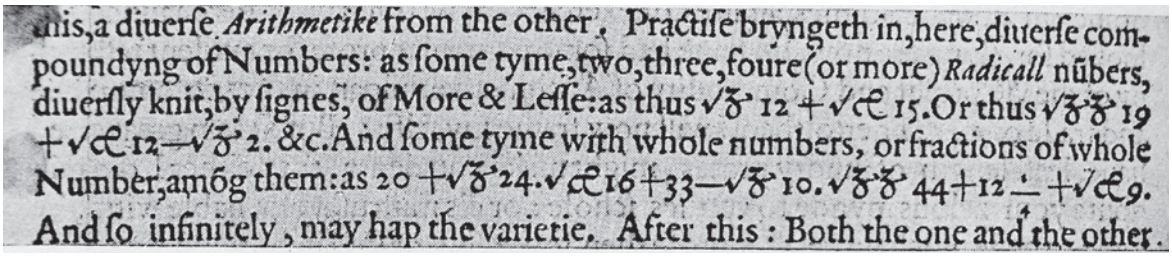


Another Example of Addition.

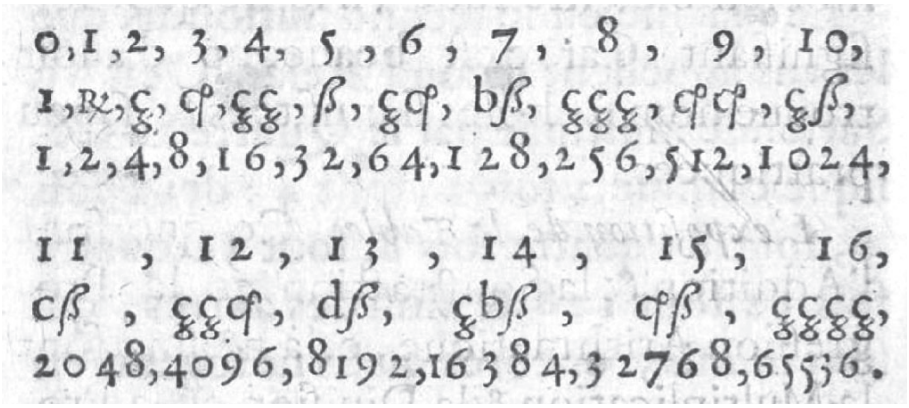
That is in lesser termes.

Here is noe multiplication, nor reduction to one common denominator: sith thei bee one all ready: no-

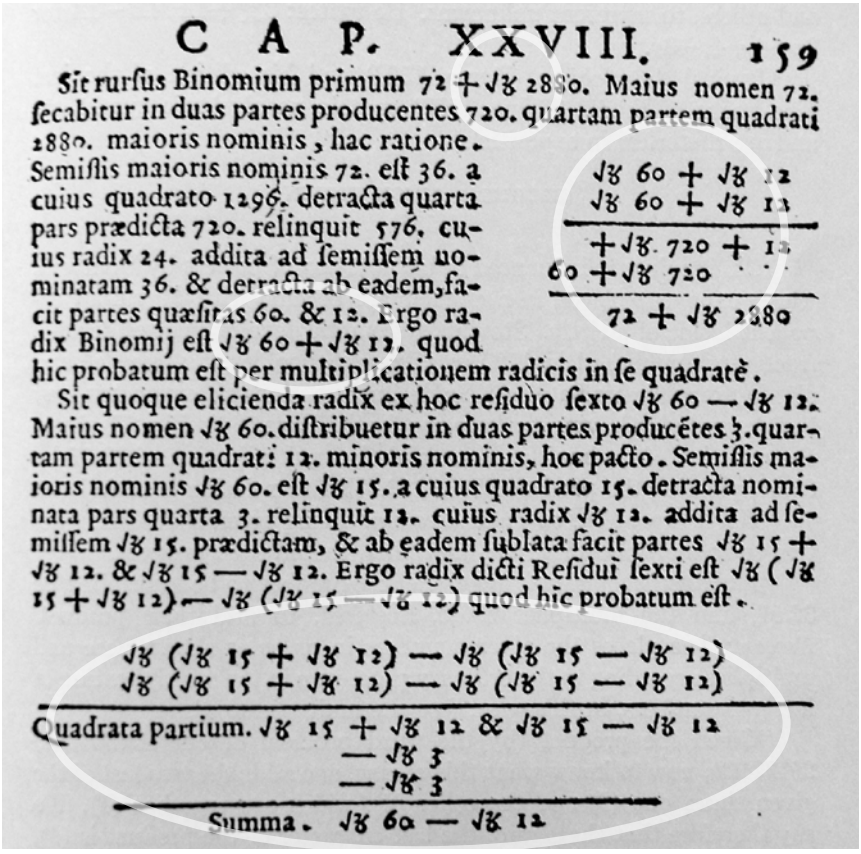
Two extracts from Recorde 1557 (after Cajori): LOWERCASE C WITH RIGHT LOOP, LOWERCASE D ROTUNDA WITH CROSSING LOOP, LOWERCASE R ROTUNDA WITH LOOP and (LOWERCASE KURRENT Z).


this, a diuerſe *Arithmetike* from the other. Practiſe bryngeth in, here, diuerſe compoundyng of Numbers: as ſome tyme, two, three, foure (or more) *Radicall* nũbers, diuerſly knit, by ſignes, of More & Leſſe: as thus √ 12 + √ 15. Or thus √ √ 19 + √ 12 — √ 2. &c. And ſome tyme with whole numbers, or fractions of whole Number, amõg them: as 20 + √ 24. √ 16 + 33 — √ 10. √ √ 44 + 12 ½ + √ 9. And ſo infinitely, may hap the varietie. After this: Both the one and the other.

Example from Dee 1570 (after Cajori): c LOWERCASE C WITH RIGHT LOOP and z (LOWERCASE KURRENT Z).



From Peletier 1620.


CAP. XXVIII. 159

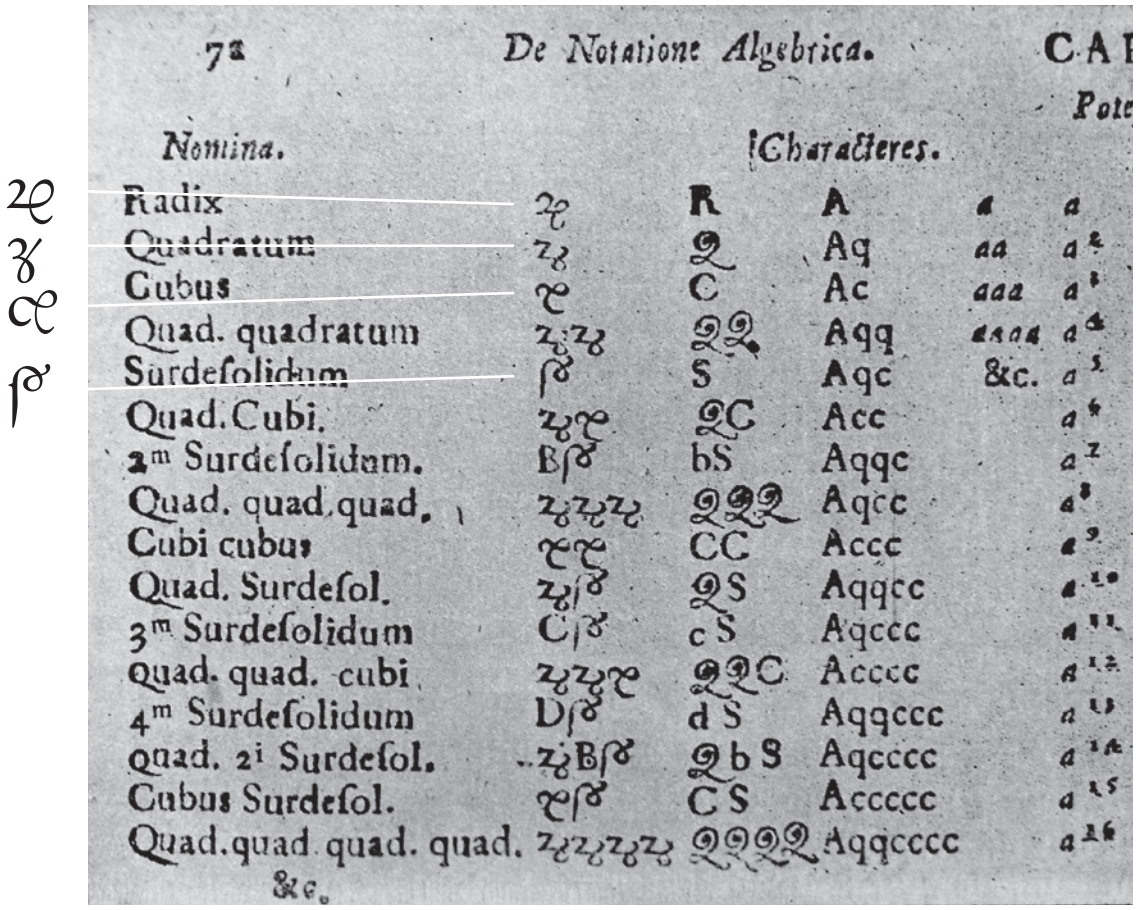
Sit rurſus Binomium primum 72 + √ 2880. Maius nomen 72. ſecabitur in duas partes producentes 720. quartam partem quadrati 2880. maioris nominis, hac ratione. Semiſſis maioris nominis 72. eſt 36. a cuius quadrato 1296. detracta quarta pars prædicta 720. relinquit 576. cuius radix 24. addita ad ſemiſſem nominatam 36. & detracta ab eadem, facit partes quæſitas 60. & 12. Ergo radix Binomij eſt √ 60 + √ 12. quod hic probatum eſt per multiplicationem radicis in ſe quadratè.

√ 60 + √ 12
√ 60 + √ 12
+ √ 720 + 12
60 + √ 720
72 + √ 2880

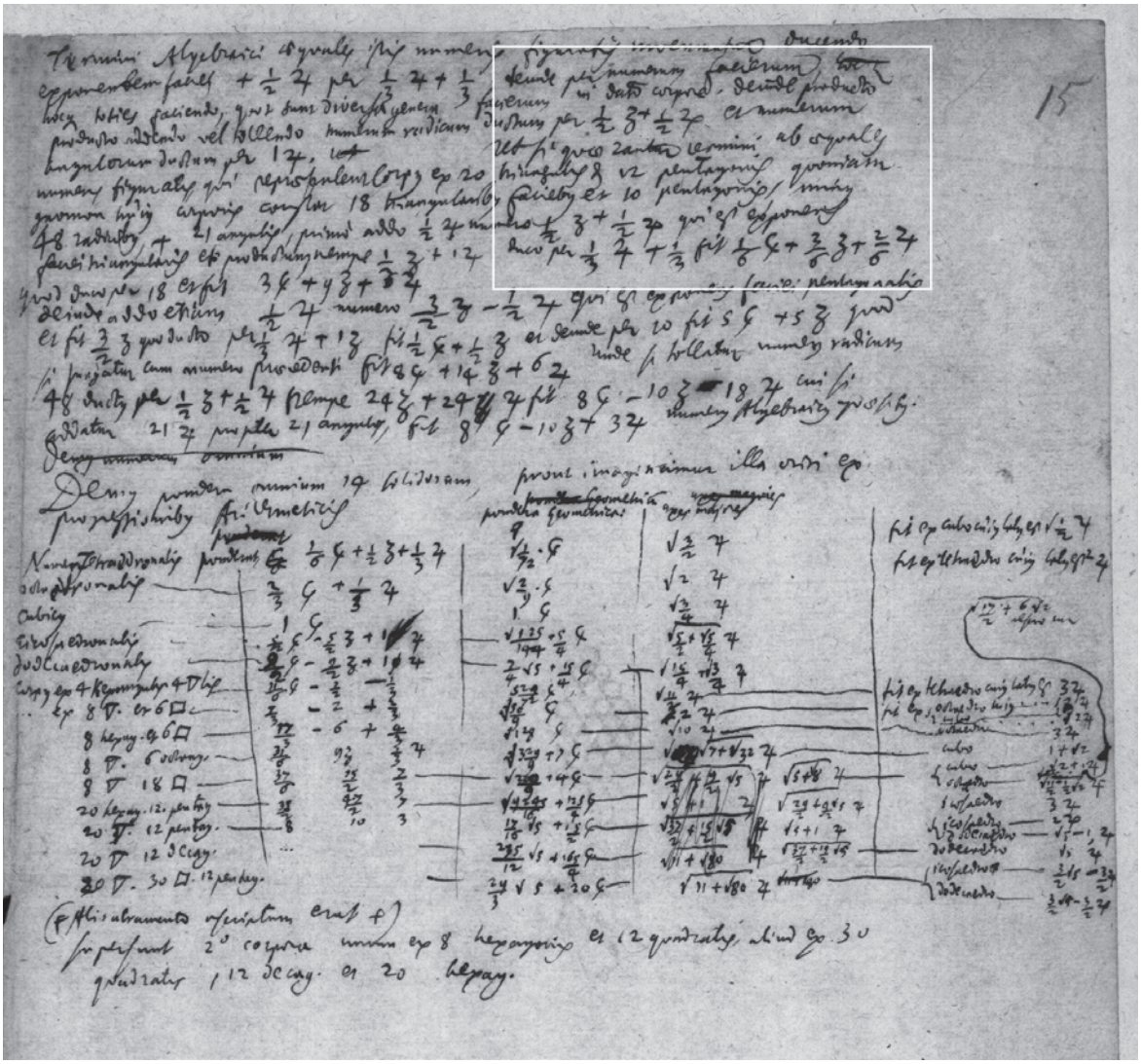
Sit quoque elicienda radix ex hoc reſiduo ſexto √ 60 — √ 12. Maius nomen √ 60. diſtribuetur in duas partes producẽtes 3. quartam partem quadrati 12. minoris nominis, hoc pacto. Semiſſis maioris nominis √ 60. eſt √ 15. a cuius quadrato 15. detracta nominata pars quarta 3. relinquit 12. cuius radix √ 12. addita ad ſemiſſem √ 15. prædictam, & ab eadem ſublata facit partes √ 15 + √ 12. & √ 15 — √ 12. Ergo radix dicti Reſidui ſexti eſt √ (√ 15 + √ 12) — √ (√ 15 — √ 12) quod hic probatum eſt.

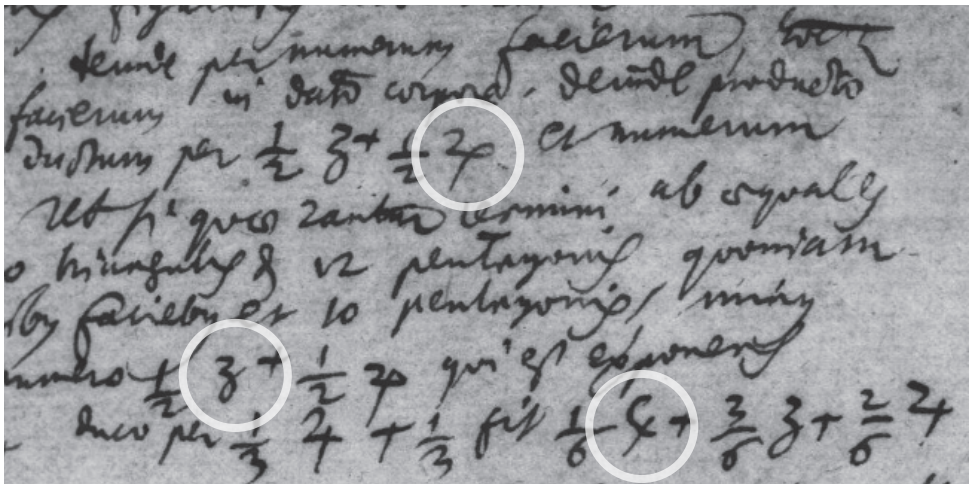
√ (√ 15 + √ 12) — √ (√ 15 — √ 12)
√ (√ 15 + √ 12) — √ (√ 15 — √ 12)
Quadrata partium. √ 15 + √ 12 & √ 15 — √ 12
— √ 3
— √ 3
Summa. √ 60 — √ 12

Clavius 1608 (after Cajori): z (LOWERCASE KURRENT Z). In this setting of Roman type style the common z character will have the usual Greek-Latin 'Zeta' shape, **z**, whereas the symbol for *zensus* retains not only the z initial (in this Latin treatise one may expect **c**ensus instead), but also the specific *kurrent* script form of the letter.

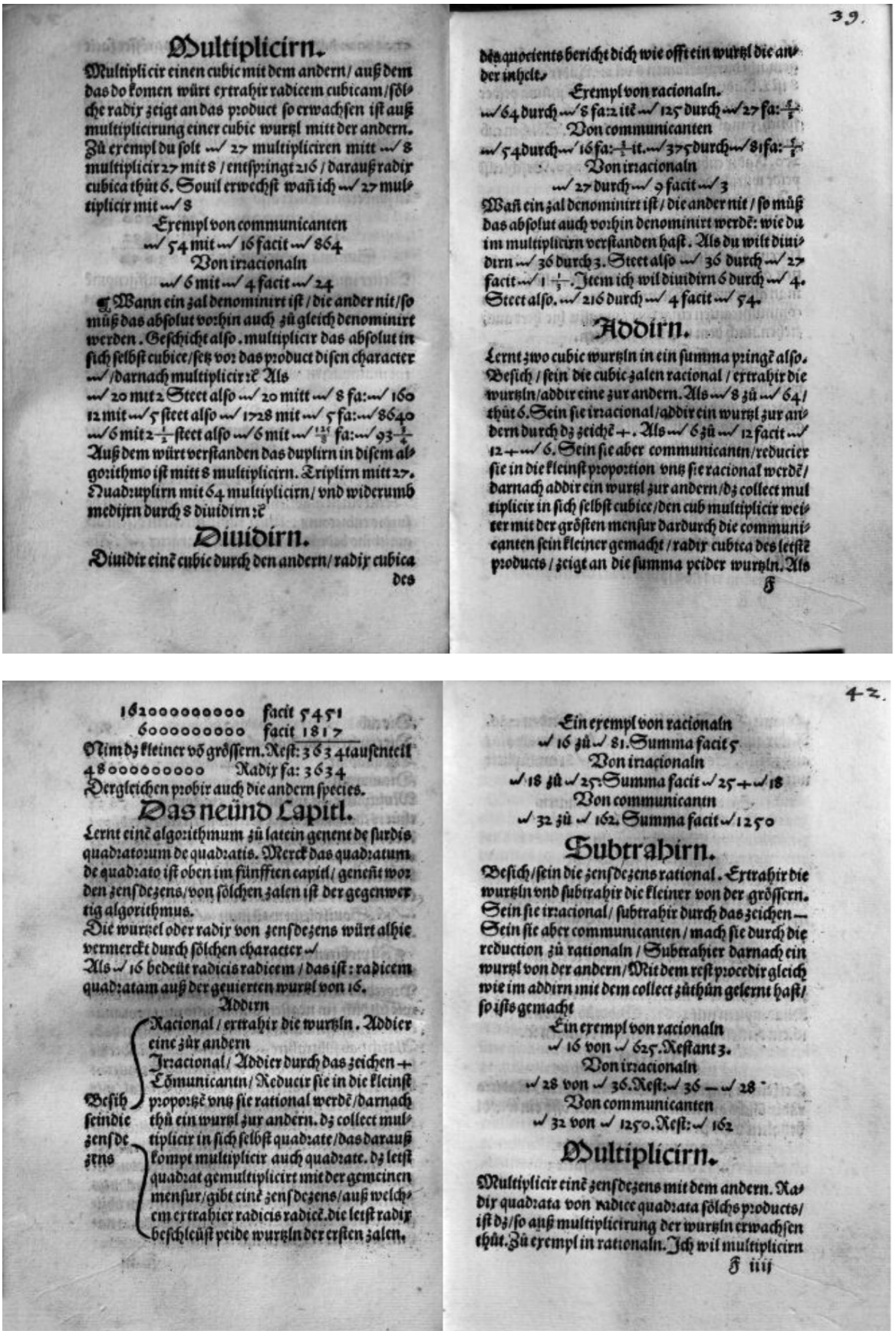


From Wallis, Operum mathematicorum, 1657 (after Cajori); shows the use of ſ LOWERCASE LONG S WITH TOP LOOP for “sursolidum”.
The ȝ (LOWERCASE KURRENT Z) has been given a sort of ‘Latinization treatment’ here, based rather on the Greek/Roman zeta shape. We regard this as a glyph variant with no distinctive meaning.





Ms. LH 4 I 4b 1v., Leibniz 1676, shows a frequent use of cossic signs: ç LOWERCASE C WITH SMALL SLASH for *cubus*, ꝛ LOWERCASE R ROTUNDA WITH LOOP for *radix* and ȝ (LOWERCASE KURRENT Z) for *zensus*.
The use of the simplier ç instead of cꝛ for *cubus* is believed to originate from writings of Descartes, from who Leibniz (and other authors) made text copies.



Multiplicirn.

Multiplicir einen cubic mit dem andern/ auß dem das do komen würt extrahir radicem cubicam/ sölche radix zeigt an das product so erwachsen ist auß multiplicirung einer cubic wurtzl mitt der andern. Zů exempl du solt ⩗ 27 multipliciren mitt ⩗ 8 multiplicir 27 mit 8 / entspringt 216 / darauß radix cubica thůt 6. Souil erwechst waß ich ⩗ 27 multiplicir mit ⩗ 8

Exempl von communicanten

⩗ 54 mit ⩗ 16 facit ⩗ 864

Von irracionaln

⩗ 6 mit ⩗ 4 facit ⩗ 24

¶ Wann ein zal denominirt ist / die ander nit/ so můß das absolut vorhin auch zů gleich denominirt werden. Geschicht also. multiplicir das absolut in sich selbst cubice/ setz vor das product disen character ⩗/ darnach multiplicir ꝛc Als

⩗ 20 mit 2 Steet also ⩗ 20 mitt ⩗ 8 fa: ⩗ 160

12 mit ⩗ 5 steet also ⩗ 1728 mit ⩗ 5 fa: ⩗ 8640

⩗ 6 mit $2\frac{1}{2}$ steet also ⩗ 6 mit ⩗ $\frac{125}{8}$ fa: ⩗ $93\frac{3}{4}$

Auß dem würt verstanden das duplirn in disem algorithmo ist mitt 8 multiplicirn. Triplirn mitt 27. Quadruplirn mit 64 multiplicirn/ vnd widerumb medijrn durch 8 dividirn ꝛc

Dividirn.

Dividir einē cubic durch den andern/ radix cubica des

des quocients bericht dich wie offt ein wurtzl die ander inhelt.

Exempl von racionaln.

⩗ 64 durch ⩗ 8 fa: 2 itē ⩗ 125 durch ⩗ 27 fa: $\frac{4}{5}$

Von communicanten

⩗ 54 durch ⩗ 16 fa: $\frac{3}{4}$ it. ⩗ 375 durch ⩗ 81 fa: $\frac{5}{6}$

Von irracionaln

⩗ 27 durch ⩗ 9 facit ⩗ 3

Waß ein zal denominirt ist / die ander nit / so můß das absolut auch vorhin denominirt werdē: wie du im multiplicirn verstanden hast. Als du wilt dividirn ⩗ 36 durch 3. Steet also ⩗ 36 durch ⩗ 27 facit ⩗ $1\frac{1}{3}$. Item ich wil dividirn 6 durch ⩗ 4. Steet also. ⩗ 216 durch ⩗ 4 facit ⩗ 54.

Addirn.

Lernt zwo cubic wurtzln in ein summa pringē also. Besich / sein die cubic zalen racional / extrahir die wurtzln/ addir eine zur andern. Als ⩗ 8 zů ⩗ 64 / thůt 6. Sein sie irracional/ addir ein wurtzl zur andern durch dz zeichē +. Als ⩗ 6 zů ⩗ 12 facit ⩗ 12 + ⩗ 6. Sein sie aber communicantn/ reducier sie in die kleinst proportion vntz sie racional werdē/ darnach addir ein wurtzl zur andern/ dz collect multiplicir in sich selbst cubice/ den cub multiplicir weiter mit der grösten mensur dardurch die communicanten sein kleiner gemacht / radix cubica des letstē products / zeigt an die summa peider wurtzln. Als

F

1820000000000 facit 5451

6000000000 facit 1817

Nim dz kleiner vō grössern. Rest: 3634 tausentteil

4800000000 Radix fa: 3634

Dergleichen probir auch die andern species.

Das neünd Capitl.

Lernt einē algorithmum zů latein genent de surdis quadratorum de quadratis. Merck das quadratum de quadrato ist oben im fünfften capitl/ genest worden zensdezens/ von sölchen zalen ist der gegenwertig algorithmus.

Die wurtzel oder radix von zensdezens würt alhie vermerckt durch sölchen character ⩗

Als ⩗ 16 bedeůt radicis radicem / das ist: radicem quadratam auß der genierten wurtzl von 16.

Addirn

Besih seindie zensdezens

Racional / extrahir die wurtzln. Addier eine zůr andern

Irracional/ Addier durch das zeichen + Cōmunicantn/ Reducir sie in die kleinst proportē vntz sie rational werdē/ darnach thů ein wurtzl zur andern. dz collect multiplicir in sich selbst quadrate/ das darauß kompt multiplicir auch quadrate. dz letst quadrat gemultiplicirt mit der gemeinen mensur/ gibt einē zensdezens/ auß welchem extrahier radicis radicē. die letst radix beschleüst peide wurtzln der ersten zalen.

Ein exempl von racionaln

⩗ 16 zů ⩗ 81. Summa facit 5

Von irracionaln

⩗ 18 zů ⩗ 25: Summa facit ⩗ 25 + ⩗ 18

Von communicantn

⩗ 32 zů ⩗ 162. Summa facit ⩗ 1250

Subtrahirn.

Besich/ sein die zensdezens rational. Extrahir die wurtzln vnd subtrahir die kleiner von der grössern. Sein sie irracional/ subtrahir durch das zeichen — Sein sie aber communicanten / mach sie durch die reduction zů rationaln / Subtrahier darnach ein wurtzl von der andern/ Mit dem rest procedir gleich wie im addirn mit dem collect zůthůn gelernt hast/ so ists gemacht

Ein exempl von racionaln

⩗ 16 von ⩗ 625. Restant 3.

Von irracionaln

⩗ 28 von ⩗ 36. Rest: ⩗ 36 — ⩗ 28

Von communicanten

⩗ 32 von ⩗ 1250. Rest: ⩗ 162

Multiplicirn.

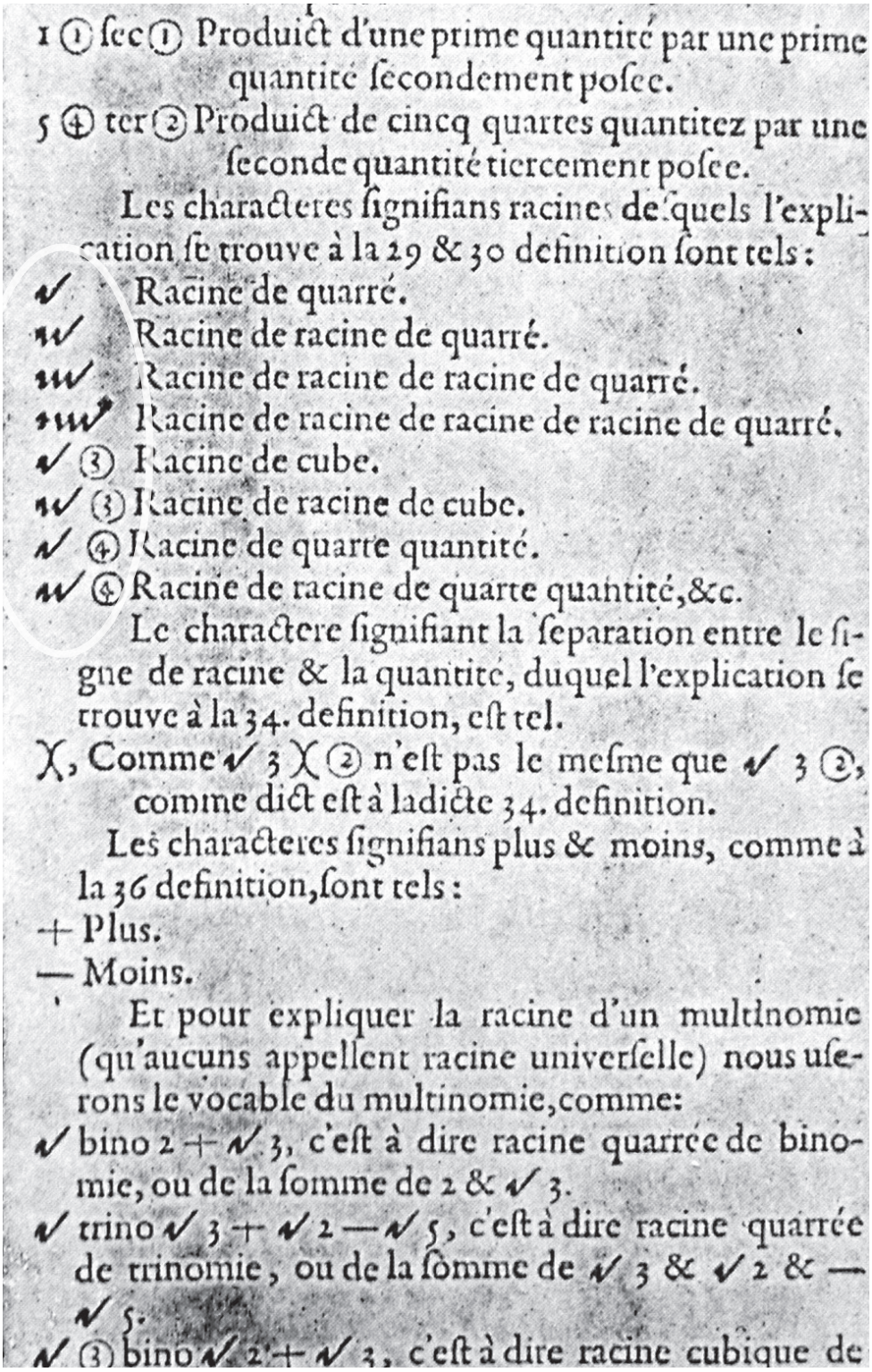
Multiplicir einē zensdezens mit dem andern. Radix quadrata von radice quadrata sölchs products/ ist dz/ so auß multiplicirung der wurtzln erwachsen thůt. Zů exempl in rationaln. Ich wil multiplicirn

F iiij

⩗ FOURTH ROOT, ⩗⩗ EIGHTH ROOT

Christoff Rudolff: *Behend und hübsch Rechnung durch die kunstreichen regeln Algebre, so gemeinicklich die Coss genennt werden*. Straßburg 1525. fol. 38v-39r., 41v-42r

see also next page



1 ① sec ① Produict d'une prime quantité par une prime quantité secondement posee.

5 ④ ter ② Produict de cincq quartes quantitez par une seconde quantité tiercement posee.

Les characteres signifians racines de quels l'explication se trouve à la 29 & 30 definition sont tels:

√ Racine de quarré.

√√ Racine de racine de quarré.

√√√ Racine de racine de racine de quarré.

√√√√ Racine de racine de racine de racine de quarré.

√ ③ Racine de cube.

√√ ③ Racine de racine de cube.

√ ④ Racine de quarte quantité.

√√ ④ Racine de racine de quarte quantité, &c.

Le charactere signifiant la separation entre le signe de racine & la quantité, duquel l'explication se trouve à la 34. definition, est tel.

χ, Comme √ 3 χ ② n'est pas le mesme que √ 3 ②, comme dict est à ladicte 34. definition.

Les characteres signifians plus & moins, comme à la 36 definition, sont tels:

+ Plus.

— Moins.

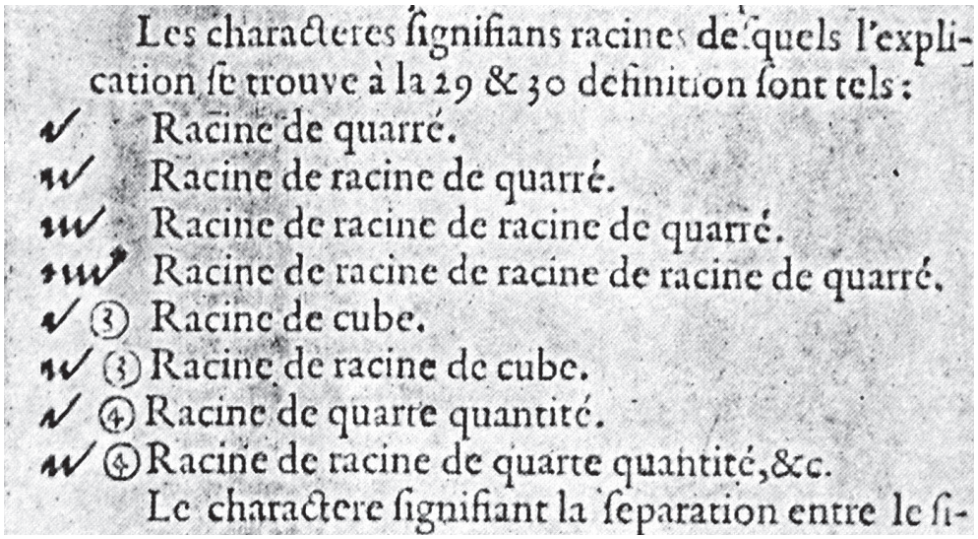
Et pour expliquer la racine d'un multinomie (qu'aucuns appellent racine universelle) nous userons le vocable du multinomie, comme:

√ bino 2 + √ 3, c'est à dire racine quarrée de binomie, ou de la somme de 2 & √ 3.

√ trino √ 3 + √ 2 — √ 5, c'est à dire racine quarrée de trinomie, ou de la somme de √ 3 & √ 2 & — √ 5.

√ ③ bino √ 2 + √ 3, c'est à dire racine cubique de

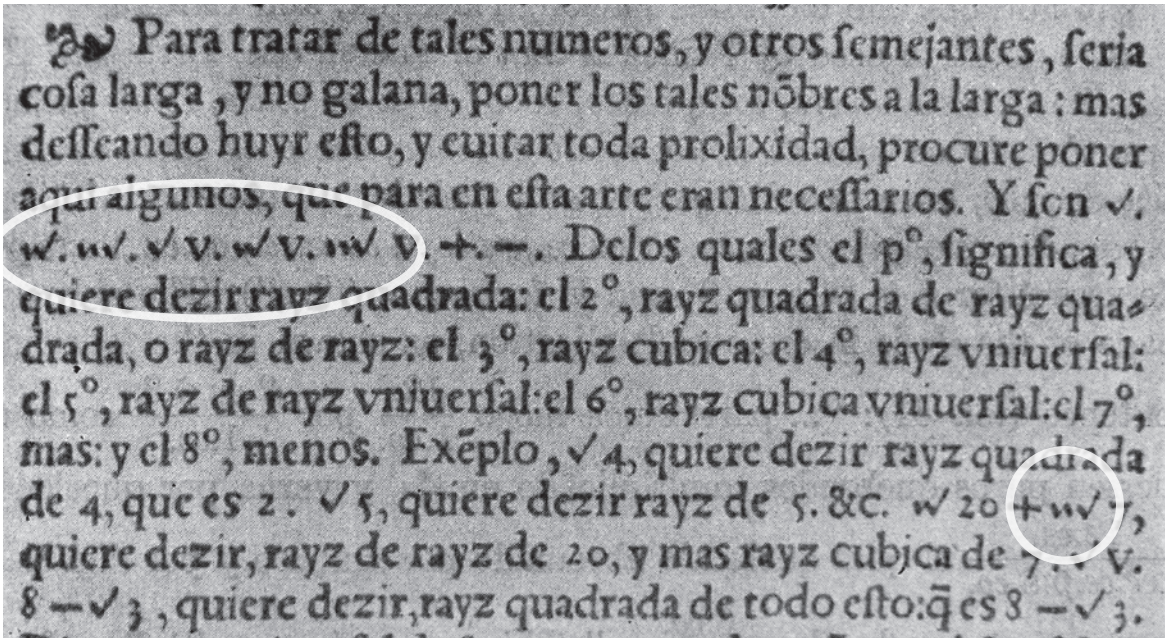
√ FOURTH ROOT, √√ EIGHTH ROOT, √√√ SIXTEENTH ROOT
These characters can be seen related to the established radix symbol √ (221A).
Simon Stevin, L'arithmétique in Œuvres mathématiques, 1634 (after Cajori)


Les characteres ſignifians racines de quels l'explication ſe trouve à la 29 & 30 definition ſont tels:
√ Racine de quarré.
√√ Racine de racine de quarré.
√√√ Racine de racine de racine de quarré.
√√√ Racine de racine de racine de racine de quarré.
√ ③ Racine de cube.
√√ ③ Racine de racine de cube.
√ ④ Racine de quarte quantité.
√√ ④ Racine de racine de quarte quantité, &c.
Le charactere ſignifiant la ſeparation entre le ſi-

Simon Stevin, L'arithmétique in Œuvres mathématiques, 1634 (after Cajori)

The number of ascending lines indicates how often an operation of root determination is performed on an expression. In the Stevin example the combination with an encircled number indicates, which type of root is meant. If there is no such number, the square root is to be considered. For example, the combinations denote the following:

√√ square root of square root, which corresponds to the forth root;
√√√ square root of square root of square root, which corresponds to the eighth root;
√√√√ square root of square root of square root of square root, which corresponds to the sixteenth root;
√√ ③ cubic root of cubic root, which corresponds to the ninth root;
√√ ④ forth root of forth root, which corresponds to the sixteenth root.


Para tratar de tales numeros, y otros ſemejantes, ſeria coſa larga, y no galana, poner los tales nõbres a la larga: mas deſſeando huyr eſto, y cuitar toda prolixidad, procure poner aqui algunos, que para en eſta arte eran neceſſarios. Y ſon √. √√. √√√. √ v. √√ v. √√√ v. +. −. Delos quales el p°, ſignifica, y quiere dezir rayz quadrada: el 2°, rayz quadrada de rayz quadrada, o rayz de rayz: el 3°, rayz cubica: el 4°, rayz vniuerſal: el 5°, rayz de rayz vniuerſal: el 6°, rayz cubica vniuerſal: el 7°, mas: y el 8°, menos. Exẽplo, √ 4, quiere dezir rayz quadrada de 4, que es 2. √ 5, quiere dezir rayz de 5. &c. √ 20 + √√ 7 v. 8 − √ 3, quiere dezir, rayz de rayz de 20, y mas rayz cubica de 7 v. 8 − √ 3, quiere dezir, rayz quadrada de todo eſto: q̃ es 8 − √ 3.

√√ FOURTH ROOT, √√√ EIGHTH ROOT, √√√√ SIXTEENTH ROOT
Marco Aurel, Arithmetica algebratica, 1552 (after Cajori)

9. Synopsis (Group 1)

	Glyph								
	Character	LOWERCASE D ROTUNDA WITH CROSSING LOOP	LOWERCASE R ROTUNDA WITH LOOP	LOWERCASE KURRENT Z SIGN	LOWERCASE C WITH DESCENDER	LOWERCASE C WITH SMALL SLASH	LOWERCASE C WITH RIGHT LOOP	LATIN SMALL LIGATURE LONG S WITH DESCENDER S	LOWERCASE LONG S WITH TOP LOOP
	Meaning	dragma	radix	zensus	census	cubus	cubus	solidus sursolidum semis	sursolidum
1	Rudolf 1525								
2	Stifel 1544								
3	Aurel 1552								
4	Peletier 1554								
5	Recorde 1557								
6	Dee 1570								
7	Peletier 1620								
8	Clavius 1608/12								
9	Beeckmann 1628								
10	Wallis 1657								
11	Leibniz MS 1676								
12	MS Leiden 17. c.								
13	MS Hamburg 17. c.								

Comparative survey of Coss characters in various sources, 1525 to 1676.

10. Unicode Character Properties

```
1D4CF FE00; kurrent style; # MATHEMATICAL SCRIPT SMALL Z

xi01;LOWERCASE C WITH SMALL SLASH;Sm;0;ON;;;;;N;;;;;
xi02;LOWERCASE C WITH DESCENDER;Sm;0;ON;;;;;N;;;;;
xi03;LOWERCASE C WITH RIGHT LOOP;Sm;0;ON;;;;;N;;;;;
xi04;LOWERCASE D ROTUNDA WITH CROSSING LOOP;Sm;0;ON;;;;;N;;;;;
xi05;LOWERCASE R ROTUNDA WITH LOOP;Sm;0;ON;;;;;N;;;;;
xi06;LATIN SMALL LIGATURE LONG S WITH DESCENDER S;Ll;0;L,017F 0073;;;;N;;;;;
xi07;MATHEMATICAL ITALIC LIGATURE LONG S WITH DESCENDER S;Sm;0;ON;;;;;N;;;;;
xi08;LOWERCASE LONG S WITH TOP LOOP;Sm;0;ON;;;;;N;;;;;
xi09;FOURTH ROOT;Sm;0;ON;;;;;N;;;;;
xi10;EIGHTH ROOT;Sm;0;ON;;;;;N;;;;;
xi11;SIXTEENTH ROOT;Sm;0;ON;;;;;N;;;;;
```

"x" stands for *unspecified codespace*. "i" refers to our internal characters classification, see N5277.

11. Bibliography

LAA – refers to: Leibniz, Gottfried Wilhelm: Sämtliche Schriften und Briefe. ('Leibniz-Akademie-Ausgabe', many volumes)
LBr – refers to: Leibniz's original correspondence papers, GWLB Hanover
LH – refers to: Leibniz's original manuscripts, GWLB Hanover

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— : Operum mathematicorum, Oxford 1657
— : Treatise of Algebra. London 1685

ISO/IEC JTC 1/SC 2/WG 2
PROPOSAL SUMMARY FORM TO ACCOMPANY SUBMISSIONS
FOR ADDITIONS TO THE REPERTOIRE OF ISO/IEC 10646[1]
Please fill all the sections A, B and C below.
Please read Principles and Procedures Document (P & P) from http://std.dkuug.dk/JTC1/SC2/WG2/docs/principles.html **for guidelines and details before filling this form.**
Please ensure you are using the latest Form from http://std.dkuug.dk/JTC1/SC2/WG2/docs/summaryform.html**.**
See also http://std.dkuug.dk/JTC1/SC2/WG2/docs/roadmaps.html **for latest *Roadmaps*.**

A. Administrative

1. **Title:** **Proposal to add 12 cossic characters**
2. Requester's name: Uwe Mayer, Siegmund Probst, David Rabouin, Elisabeth Rinner, Andreas Stötzner, Achim Trunk, Charlotte Wahl
3. Requester type (Member body/Liaison/Individual contribution): Individual (work group)
4. Submission date: 2025-05.13.
5. Requester's reference (if applicable): LUCP L-2518
6. Choose one of the following:
   This is a complete proposal: Yes
   (or) More information will be provided later:

B. Technical – General

1. Choose one of the following:
   a. This proposal is for a new script (set of characters): No
      Proposed name of script:
   b. The proposal is for addition of character(s) to an existing block:
      Name of the existing block: *not yet specified*
2. Number of characters in proposal: 12
3. Proposed category (select one from below - see section 2.2 of P&P document):
   A-Contemporary ( ) B.1-Specialized (small collection) (Yes) B.2-Specialized (large collection) ( )
   C-Major extinct ( ) D-Attested extinct ( ) E-Minor extinct ( )
   F-Archaic Hieroglyphic or Ideographic ( ) G-Obscure or questionable usage symbols ( )
4. Is a repertoire including character names provided? Yes
   a. If YES, are the names in accordance with the "character naming guidelines" in Annex L of P&P document? Yes
   b. Are the character shapes attached in a legible form suitable for review? Yes
5. Fonts related:
   a. Who will provide the appropriate computerized font to the Project Editor of 10646 for publishing the standard?
      Andreas Stötzner
   b. Identify the party granting a license for use of the font by the editors (include address, e-mail, ftp-site, etc.):
      Andreas Stötzner Gestaltung, Klauflügelweg 21, 88400 Biberach/R., Germany, as@signographie.de
6. References:
   a. Are references (to other character sets, dictionaries, descriptive texts etc.) provided? Yes
   b. Are published examples of use (such as samples from newspapers, magazines, or other sources) of proposed characters attached? Yes
7. Special encoding issues:
   Does the proposal address other aspects of character data processing (if applicable) such as input, presentation, sorting, searching, indexing, transliteration etc. (if yes please enclose information)? No

8. Additional Information:
Submitters are invited to provide any additional information about Properties of the proposed Character(s) or Script that will assist in correct understanding of and correct linguistic processing of the proposed character(s) or script. Examples of such properties are: Casing information, Numeric information, Currency information, Display behaviour information such as line breaks, widths etc., Combining behaviour, Spacing behaviour, Directional behaviour, Default Collation behaviour, relevance in Mark Up contexts, Compatibility equivalence and other Unicode normalization related information. See the Unicode standard at http://www.unicode.org for such information on other scripts. Also see Unicode Character Database (http://www.unicode.org/reports/tr44/) and associated Unicode Technical Reports for information needed for consideration by the Unicode Technical Committee for inclusion in the Unicode Standard.

[1] Form number: N4502-F (Original 1994-10-14; Revised 1995-01, 1995-04, 1996-04, 1996-08, 1999-03, 2001-05, 2001-09, 2003-11, 2005-01, 2005-09, 2005-10, 2007-03, 2008-05, 2009-11, 2011-03, 2012-01)

C. Technical - Justification

1. Has this proposal for addition of character(s) been submitted before? Yes
   If YES explain *see L2/25-123 (L-2509)*
2. Has contact been made to members of the user community (for example: National Body, user groups of the script or characters, other experts, etc.)? Yes
   If YES, with whom? Leibniz-Archiv, Forschungsstelle der Leibniz-Edition, Niedersächsische Landesbibliothek (GWLB), Hanover, Göttingen Academy of Science and Humanities in Lower Saxony (DE), Philiumm research group of CNRS (UMR 7219, laboratoire SPHERE) / Université de Paris VII; general: scholars, researchers, authors and editors working in the field of science history and upon editions of historic text corpora (e.g. of G. W. Leibniz, but also many others)
   If YES, available relevant documents: L-2409, L-2410
3. Information on the user community for the proposed characters (for example: size, demographics, information technology use, or publishing use) is included? Yes
   Reference:
4. The context of use for the proposed characters (type of use; common or rare) Common
   Reference: mainly specialist usage, scholarly, worldwide
5. Are the proposed characters in current use by the user community? Yes
   If YES, where? Reference: mainly Europe, Americas; other countries
6. After giving due considerations to the principles in the P&P document must the proposed characters be entirely in the BMP? No
   If YES, is a rationale provided?
   If YES, reference:
7. Should the proposed characters be kept together in a contiguous range (rather than being scattered)? No
8. Can any of the proposed characters be considered a presentation form of an existing character or character sequence? No
   If YES, is a rationale for its inclusion provided?
   If YES, reference:
9. Can any of the proposed characters be encoded using a composed character sequence of either existing characters or other proposed characters? Yes
   If YES, is a rationale for its inclusion provided? *Yes*
   If YES, reference: *see p. 4*
10. Can any of the proposed character(s) be considered to be similar (in appearance or function) to, or could be confused with, an existing character? No
    If YES, is a rationale for its inclusion provided?
    If YES, reference:
11. Does the proposal include use of combining characters and/or use of composite sequences? No
    If YES, is a rationale for such use provided?
    If YES, reference:
    Is a list of composite sequences and their corresponding glyph images (graphic symbols) provided? No
    If YES, reference:
12. Does the proposal contain characters with any special properties such as control function or similar semantics? No
    If YES, describe in detail (include attachment if necessary)
13. Does the proposal contain any Ideographic compatibility characters? No
    If YES, are the equivalent corresponding unified ideographic characters identified?
    If YES, reference:

Appendix

following pages:

On the *sursolidum* character

Source: Andreas Stötzner, Asmus Freytag
Number: L-2516n
Version: 2nd extended vs.
Related: L-2509
Date: 25-05.12.
Status: FYI, for discussion

About the definition and name of the proposed *sursolidum* character

A. Stötzner <as@signographie.de> 30.4.2025 10:44

RE: Doc listing Script Encoding WG comments for tomorrow's discussion

An Peter Constable <pgcon6@msn.com> • dwanders@sonic.net Kopie
Robin Leroy <eggrobin@unicode.org> • Asmus Freytag <asmusf@ix.netcom.com> • kucera@unicode.org • kirk miller <kirk.miller@gmail.com>

> And my notes regarding L2/25-123 (cossic characters):
>
> - General consensus that sharp s with descender needs to be encoded as two characters:
>   - LATIN LETTER SMALL SHARP S WITH DESCENDER
>   - MATHEMATICAL ITALIC SMALL SHARP S WITH DESCENDER
>   - in publications, this is shown in math zones as an italic math symbol, but otherwise (e.g. footnotes) as an upright character

I still have a problem with the naming “SHARP S”, because it is incorrect and misleading. To the uninformed eye the char. gives the *impression* being ‘some sort of ß’, but that is only accidental, hence of no meaning.
On the one hand, ß (sharp s) is of **German** origin, its *essence* is a long ſ with some extension on the right side, which over time took on various shapes. To the day the form of that right part is not strictly defined, there are various legitimate options.
On the other hand, our new letter is *definitely* derived from **long ſ and s** and from nothing else. Its origin is **Latin**, its function is either an abbreviation (**ſ**ur**s**olidum, **ſ**emi**s**) or sometimes it can serve as a typographic ligature (illustri**ſs**imus).
Therefore it is not appropriate to name the Latin ſ_s character with the German term “sharp s”. We would give a false information to generations to come.

We should define things properly and accurate. A naming decision should not be guided by accidental optical similarities which are (more or less) deceiving.
Would these names be acceptable:

- **LATIN LETTER SMALL SS WITH DESCENDER**
- **MATHEMATICAL ITALIC SMALL SS WITH DESCENDER**

that would be consistent with the scheme in names like LATIN SMALL LETTER NJ (01CC) or LATIN SMALL LETTER AE WITH MACRON (01E3).

with regards,
Andreas Stötzner.

Asmus Freytag <asmusf@ix.netcom.com> 1.5.2025 01:24

Sursolidum: On character naming

An A. Stötzner <as@signographie.de> • Peter Constable <pgcon6@msn.com> • dwanders@sonic.net
Kopie Robin Leroy <eggrobin@unicode.org> • kucera@unicode.org • kirk miller <kirk.miller@gmail.com>

On 4/30/2025 1:44 AM, A. Stötzner wrote:

> > And my notes regarding L2/25-123 (cossic characters):
> >
> > - General consensus that sharp s with descender needs to be encoded as two characters:
> >   - LATIN LETTER SMALL SHARP S WITH DESCENDER
> >   - MATHEMATICAL ITALIC SMALL SHARP S WITH DESCENDER
> >   - in publications, this is shown in math zones as an italic math symbol, but otherwise (e.g. footnotes) as an upright character
>
> I still have a problem with the naming “SHARP S”, because it is incorrect and misleading. To the uninformed eye the char. gives the *impression* being ‘some sort of ß’, but that is only accidental, hence of no meaning.

Character names are tricky.

They serve two purposes. One is as a ***human-readable identifier.*** For that purpose, the name must be unique, and should be mnemonic. Beyond identifying a character, it should also help with selecting among similar characters.

For letters, we do this by basing a name on some more or less traditional name for that item in the alphabet. Which works well for that purpose, because letters, for the most part, are encoded by their identities as members of an alphabet, which sometimes allows a wider variety of shapes to be encoded by a single character: the selection of the actual shape is then not a matter of plain text, but also doesn't or shouldn't affect the meaning of the text as a whole.

However, once we go beyond base letters, composites or derivatives are named by modifying the name of the base letter plus a modifier or modifying phrase. Rotated, reversed, inverted, or "with ..." are common. This is done, even where some language, using these in their alphabet, may have a traditional name for that modified character.

For symbols, we often name them by function, particularly if that association is near universal, such as for radix (root) or integral. But often, we name the symbol by a description of its shape; that more easily accommodates multiple, unrelated uses of the symbol. But it also means that related symbols are named so that they end up with related descriptions (as much as possible). That detail of descriptive names helps in selecting the correct character for the intended symbol, independent of the font's glyph choice.

Symbols that aren't universally related to a single concept also don't necessarily have an agreed-upon range of permissible glyph shapes, unlike letters. Like with modified letters, descriptive names of derived symbols help focus on the distinguishing feature between the base shape and the derivative. They thus help reign in the range of acceptable glyph representations.

At the same times, names are neither exhaustive or perfect. The fact that we never change them, even if they are incorrect, means that we prioritize their uniqueness and stability over the other aspects and sometimes accept that names are primarily identifiers and do not always give an exhaustive or detailed description. The way we address this, most often, is by providing an annotation in the nameslist, either to provide an alternate informative name, or to indicate that an expanded or restricted range of glyphic variation is intended.

In this instance, there's a case to be made for noting the descender as a feature. But if we do that, then the other part of the name needs to be the name of a base form. That gets complicated if the base form is not itself a named character. However, as you notice, the existing letter can have representative glyphs that could be analyzed they way you indicate, but crucially also allows a number of different variations that matter to font designers but not to actual readers.

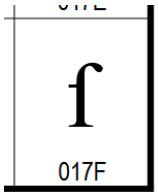
In this context, let's note that some of the submitted material substituted the sharp s for the *sursolidum.* This indicates that it would be useful to establish the relation between a form with descender and one of the

equivalent forms without a descender that is a valid subrange of the full glyph range for sharp s.

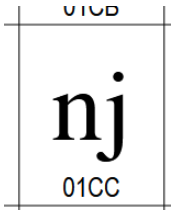
A fully descriptive name of the symbol used for *sursolidum* would be something like

- **LATIN SMALL LIGATURE LONG S WITH DESCENDER S**

The long s we encode in Unicode is clearly one without a descender.



and I would argue, if analyzed as a ligature, the descender is on the long s and not on the ligature. However, the shape of the character is most assuredly not a digraph of "ss". Compare the example you gave of

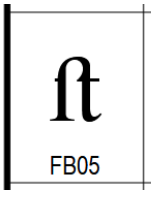


which clearly looks like "nj" and is not a ligature of "n" with "j". (Letter ae is an outlier, because it's a ligature treated as a named letter in an alphabet for some languages and there was a political compromise made to prioritize that over typographical naming which would have been more correct for other languages.)

To get back to the character at hand, a true digraph of long s and s would look like:

with the forms not connected, quite unlike the *sursolidum*. If we want to treat this as a ligature, the closest analogy we have today is:

- **LATIN SMALL LIGATURE LONG S T**



For the SHARP S we have a range of glyphs, such as

where the left most ones are from somewhat traditional type faces and the rightmost one is from a modern

mathematical font. For U+00DF, we explicitly note the two alternate forms as co-occurring with an annotation in the nameslist.

Based on all of the foregoing, I would be comfortable with continuing to use the ***identifier*** (name) of

- **MATHEMATICAL ITALIC SMALL SHARP S WITH DESCENDER**

together with an annotation, such as:

- if used for *sursolidum*, the form should be a ligature of a long s with descender and s.

Alternatively, we could suggest that the form with descender has a restricted glyph range:

- unlike U+00DF the form with descender is always a ligature with s

The same for the non-italic version. Plus cross references between and to U+00DF.

To me, this solution has several advantages:

1. We establish that, in the context of a mathematical font, this contrasts with U+00DF by addition of a descender. So that users know that when the need a form with descender to not use U+00DF.
2. We are also covered, in case there's a later request for a true mathematical italic sharp s with descender (true meaning that the letter sharp s is intended, not just the ligature shape).
3. If we get a request of a mathematical sharp s (without a descender) we are also covered, because we've set up the contrast correctly.
4. We head off encoding the distinction between an ss and sz version of these ligated forms by making clear that they are always unified in encoding and any glyph preference in a certain context needs to be specified outside plain text.

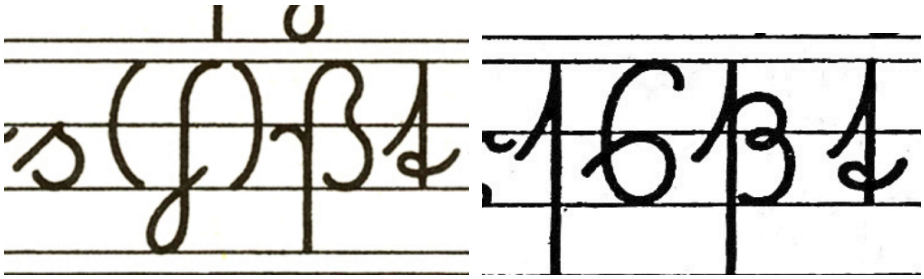
Summary

We have a number of constraints on character names that we can't easily satisfy all at once in this case, partially because we are dealing with what essentially is a ***letterlike symbol*** or something that combines features of both letters and symbols. If this forces a compromise in naming, that's not the first time.

We also have overlapping glyph ranges and need to be careful whether we want to establish in the encoding that we are disunifying some glyph ranges in the encoding (even if only in the context of a derived shape), or whether we want to simply indicate that either the derived form or its use for a certain context have a restricted glyph range.

A./

PS: a more crucial question is whether we aren't making a mistaken identification here. The reason is that it can be argued that the "descender" on the ß is a ***feature of handwriting***, not something intrinsic to the letter. Here are some excerpts from early 20th century handwriting styles intended for elementary school instruction.



Both clearly show a descender on the ß, even though they are using a different ligature. An interesting detail is that both were designed by the same person and the one on the right is of course *Sütterlin.*

The Wikipedia gives this shape for the ß in its example of Kurrentschrift:

which is arguably an "s" shape with a connecting look, despite a claim to the contrary on the German Wikipedia, and not a "z". Note for comparison that "tz" looks like this in the same sample:

Given that as a background, we can ask ourselves if we should not simply encode a

- MATHEMATICAL SCRIPT SHARP S
- the form based on a ligature of long as and s is preferred

This would mean that the usage of some upright shape with descender in the footnotes would be erroneous. Any upright form should not have a descender (but for the purpose of designating *sursolidum* would have to use a font that is based on the long s s ligature).

- uuHkbKaf7bXY31zD.png (3 KB)
- J6ZbTb5Aq0vHF6wV.png (5 KB)
- uRpR82MwDbEmPCcd.png (3 KB)
- 3lUYdx5HbOd1k6JT.png (3 KB)
- GFLCsZ07HEPim93L.png (14 KB)
- moWZdvA8jWb2fQDy.png (196 KB)
- jG0554yJa4igkAIb.png (82 KB)
- vSXc8Dk6SJ22hF8k.png (5 KB)
- NA340vcaWp5I0U3V.png (6 KB)

A. Stötzner <as@signographie.de> 7.5.2025 19:59

Re: Sursolidum: On character naming

An Asmus Freytag <asmusf@ix.netcom.com> • Peter Constable <pgcon6@msn.com> • dwanders@sonic.net Kopie Robin Leroy <eggrobin@unicode.org> • kucera@unicode.org • kirk miller <kirk.miller@gmail.com> • Siegmund Probst <siegmund.probst@gwlb.de>

Thanks to Asmus Freytag for the impressive contribution to this topic, which seems to me being a typical “three-whiskies-problem”…
Here some further thoughts upon it from my perspective.

> Asmus Freytag <asmusf@ix.netcom.com> hat am 01.05.2025 01:23 CEST geschrieben:

> Character names are tricky.

Given they are (in general), that would not free us from the task of making them as clear and fitting as possible.

I don’t think character names are tricky. They need to be “made to measure” and be based on precise thinking.

> They serve two purposes. One is as a ***human-readable identifier.*** For that purpose, the name must be unique, and should be mnemonic. Beyond identifying a character, it should also help with selecting among similar characters.

Good point. The task of selecting is not served by a misleading name part. Moreover, even **because** there is a close visual similarity, it becomes **the more important** to mark the relevant difference(s), in order to make people understand those differences and enable them to make suitable decisions.

> For letters, we do this by basing a name on some more or less traditional name for that item in the alphabet. Which works well for that purpose, because letters, for the most part, are encoded by their identities as members of an alphabet, which sometimes allows a wider variety of shapes to be encoded by a single character:

> the selection of the actual shape is then not a matter of plain text, but also doesn't or shouldn't affect the meaning of the text as a whole.

Another good point. The *actual shape* is not what matters in the first place. What matters is **what is meant** by a shape. How a shape is to be understood. What is meant by the shape of the *fursolidum* character is: a ligation of ſ and s which (in most cases) serves as a stand-in or abbreviation for words like *fursolidum* or *femis.* The rationale of choosing “…sharp s…” here can only be justified by appearance of shape, a visual similarity with ß. But that resemblance is merely accidental and therefore potentially misleading.
The modern ß varieties which visually resemble long ſ and **s** are, at least in upright typography, a product of a misconception in the early 20th century. Whereas the *fursolidum* character is a child of the Renaissance period, hence ~500 years older. So it should be obvious that the *fursolidum* character just can’t be a derivative of sharp s.

> However, once we go beyond base letters, composites or derivatives are named by modifying the name of the base letter

The *fursolidum* character is a composite or ligature, made of a long **ſ** and a round **s**, which are the base letters in this case. But that implies: the German ß (sharp s) is **not** the base letter of it. The base letters which the char. derives from are ſ and s. Therefore it seems not appropriate to declare ‘sharp s’ in the characters name as the base letter. Unlike the very old German ß which has several handed down, well-known names (of which *‘scharfes s’* is one of the most usual and, seen linguistically, the most suitable), the *fursolidum* character has no

inherited, popular name we could fall back to.

> plus a modifier or modifying phrase. Rotated, reversed, inverted, or "with ..." are common. This is done, even where some language, using these in their alphabet, may have a traditional name for that modified character.

> For symbols, we often name them by function, particularly if that association is near universal, such as for radix (root) or integral. But often, we name the symbol by a description of its shape; that more easily accommodates multiple, unrelated uses of the symbol.

> But it also means that related symbols are named so that they end up with related descriptions (as much as possible).

The boundary between letters and symbols can not be drawn categorically. For instance, 211E and 211F (℞, ℟) are basically **letters** with some graphic attachment, but they are widely **used** as **symbols** or (more precisely), as ideograms (graphic expression for a certain item or concept). It is a very similar situation with the *fursolidum* character.

> That detail of descriptive names helps in selecting the correct character for the intended symbol, independent of the font's glyph choice.

– only if one can be sure that the descriptive name is 100% correct. Can we be sure? Example: 211E is named PRESCRIPTION TAKE. That is roughly OK as long as we are in Pharmacy matters (RECIPE would be less anglosaxon-centristic but more appropriate because it is originally a Latin abbreviation for an R… word), but when we are in Mathematics it can mean “Radix“ (rare), when we are in Numismatics it happens to stand for “Reverse” (side of a coin). So this char. would have been better off as LATIN CAPITAL R WITH CROSSBAR or similar. Because **that** description is **always** right.

> Symbols that aren't universally related to a single concept also don't necessarily have an agreed-upon range of permissible glyph shapes, unlike letters.

The variability of acceptable glyph shapes depends on time and cultural environment, local customs, certain writing traditions and more. This is valid for letters and symbols alike and also for letters or letter derivates which function as abbreviations or symbols. The accepted variability usually doesn’t overrun certain limitations, there’s always a (more or less) limited *field* of possibilities. Again, I would not draw a sharp line between letters and symbols, generally.

> Like with modified letters, descriptive names of derived symbols help focus on the distinguishing feature between the base shape and the derivative. They thus help reign in the range of acceptable glyph representations.

It could be said that in the case of the *fursolidum* character the descender of the base character ſ is a distinguishing feature, although it is a weak one because the ordinary long ſ can also feature a descender (– field of possibilities). However, the Latin *fursolidum* character is **definitely not** a derivative of the German ß. Although the sharp s is much older than the *fursolidum* abbreviation it is obvious that Latin authors (or French or Italian) had not a German ß in mind when they abbreviated *fursolidum* to ſ_s. The base character of the *fursolidum* abbr. is long ſ.

> At the same times, names are neither exhaustive or perfect.

Agreed. They don’t have to. But they ought to be appropriate.

> The fact that we never change them, even if they are incorrect, means that we prioritize their uniqueness and stability over the other aspects and sometimes accept that names are primarily identifiers and do not always give an exhaustive or detailed description. The way we address this, most often, is by providing an annotation in the nameslist, either to provide an alternate informative name, or to indicate that an expanded or restricted range of glyphic variation is intended.

A possible annotation to the character would be: • glyph should always resemble long ſ and s.

> In this instance, there's a case to be made for noting the descender as a feature. But if we do that, then the other part of the name needs to be the name of a base form. That gets complicated if the base form is not itself a named character. However, as you notice, the existing letter can have representative glyphs that could be analyzed they way you indicate, but crucially also allows a number of different variations that matter to font designers but not to actual readers.
> In this context, let's note that some of the submitted material substituted the sharp s for the *sursolidum.*

That is right, but, as we have explained, this phenomenon is to be seen just as a **makeshift** because of shortage of the proper letters in the composer's workshop;. It occurs **only** in some German print works. In prints from other countries (e.g. Aurel/Valencia-1552, Peletier/Lyon-1554) the ſ_s char. is used, it is also used in Rudolff/Straßburg-1525/first edition, and in Stifel/Nuremberg-1544. Therefore the usage of ſ_s is the **relevant** usage, and the occasional use of ß is not actually relevant for the character's definition.

> This indicates that it would be useful to establish the relation between a form with descender and one of the equivalent forms without a descender that is a valid subrange of the full glyph range for sharp s.

The permissible glyph shape range of ß is not relevant here.

> A fully descriptive name of the symbol used for *sursolidum* would be something like
>
> - **LATIN SMALL LIGATURE LONG S WITH DESCENDER S**

– that would be an agreeable definition.

> The long s we encode in Unicode is clearly one without a descender.

The same counts for ß. But this is just because it is customary for modern Roman-style typefaces to have ſ and ß **without** descender. In fonts of other styles (e.g. script, chancery, Italic, Kurrent, blackletter, Fraktur…) ſ and ß frequently feature a descender because it is customary in those styles.
For that reason – ſ and ß **may have** a descender (as well) –, the feature of the descender is not what **in any case** makes the difference.

> and I would argue, if analyzed as a ligature, the descender is on the long s and not on the ligature. However, the shape of the character is most assuredly not a digraph of "ss". Compare the example you gave of

– agreed, the part "ss" is not sufficiently unambiguous.

> which clearly looks like "nj" and is not a ligature of "n" with "j". (Letter ae is an outlier, because it's a ligature treated as a named letter in an alphabet for some languages and there was a political compromise made to prioritize that over typographical naming which would have been more correct for other languages.)
>
> To get back to the character at hand, a true digraph of long s and s would look like:
>
> with the forms not connected, quite unlike the *sursolidum*. If we want to treat this as a ligature, the closest analogy we have today is:
>
> - **LATIN SMALL LIGATURE LONG S T**
>
> For the SHARP S we have a range of glyphs, such as
>
> where the left most ones are from somewhat traditional type faces and the rightmost one is from a modern mathematical font. For U+00DF, we explicitly note the two alternate forms as co-occurring with an annotation in the nameslist.
>
> Based on all of the foregoing, I would be comfortable with continuing to use the ***identifier*** (name) of
>
> - **MATHEMATICAL ITALIC SMALL SHARP S WITH DESCENDER**

> together with an annotation, such as:
>
> - if used for *sursolidum*, the form should be a ligature of a long s with descender and s.
>
> Alternatively, we could suggest that the form with descender has a restricted glyph range:
>
> - unlike U+00DF the form with descender is always a ligature with s
>
> The same for the non-italic version. Plus cross references between and to U+00DF.
>
> To me, this solution has several advantages:
>
> 1. We establish that, in the context of a mathematical font, this contrasts with U+00DF by addition of a descender. So that users know that when the need a form with descender to not use U+00DF.
> 2. We are also covered, in case there's a later request for a true mathematical italic sharp s with descender (true meaning that the letter sharp s is intended, not just the ligature shape).
> 3. If we get a request of a mathematical sharp s (without a descender) we are also covered, because we've set up the contrast correctly.
> 4. We head off encoding the distinction between an ss and sz version of these ligated forms by making clear that they are always unified in encoding and any glyph preference in a certain context needs to be specified outside plain text.
>
> **Summary**
>
> We have a number of constraints on character names that we can't easily satisfy all at once in this case, partially because we are dealing with what essentially is a ***letterlike symbol*** or something that combines features of both letters and symbols. If this forces a compromise in naming, that's not the first time.
>
> We also have overlapping glyph ranges and need to be careful whether we want to establish in the encoding that we are disunifying some glyph ranges in the encoding (even if only in the context of a derived shape), or whether we want to simply indicate that either the derived form or its use for a certain context have a restricted glyph range.
>
> A./

> [… …]

> Given that as a background, we can ask ourselves if we should not simply encode a
>
> - MATHEMATICAL SCRIPT SHARP S

– rather not.

> - the form based on a ligature of long as and s is preferred
>
> This would mean that the usage of some upright shape with descender in the footnotes would be erroneous. Any upright form should not have a descender (but for the purpose of designating *sursolidum* would have to use a font that is based on the long s s ligature).

I propose the following solution:

(1)
LATIN SMALL LIGATURE LONG S WITH DESCENDER S
= sursolidum
• glyph always resembles long s and s

(2)
MATHEMATICAL ITALIC LIGATURE LONG S WITH DESCENDER S
= sursolidum

- glyph always resembles long s and s
- corresponding text character is (1) *[optional]*

With this we achieve:

- compliance with practised naming conventions
- a structural and historical correct explanation
- a clearly understandable definition of the characters nature
- to avoid confusion of the Latin abbreviation character with the German ß
- allow (1) to be implemented in both a typeface's Regular and Italic fonts with appropriate shapes
- maintain a safe distinction between a plain-text character and a specific math character
- allow a different treatment of (1) and (2) in an Italic font, if there should be any need for that
- leave the door open for a possible later request for a *mathematical sharp s* (without a descender)

—

Andreas Stötzner

ps: further reading: on the origin of ß (in German)