ISO/IEC JTC1/SC2/WG2 N5334
LUCP L-2519

Universal Multiple-Octet Coded Character Set
International Organization for Standardization
Internationale Standardisierungs-Organisation
Organisation Internationale de Normalisation
Διεθνής Οργανισμός Τυποποίησης
Международная организация по стандартизации

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1. Background

This proposal is part of the research program upon historical mathematical sources, conducted by the CNRS Philiumm project (headed by Prof. David Rabouin, University of Paris) and supported by researchers from the Landesbibliothek Hanover (Germany). The aim of this project work is to achieve a standardized encoding for special mathematical characters in historic texts, which is required for accurate facsilime editions of those sources.
For more background information about the Philiumm project and the related research work, please visit the Philiumm website or see doc. no. N5277.

2. Mathematical relation symbols in historic sources

The topic of this proposal is 31 symbols for relations, like *equal, congruence, greater-than* or *commensurability*. They are testified in works of G. W. Leibniz and many other authors, mainly of the 17th century. Some of the proposed characters basically represent the same meaning as e.g. 003D = EQUAL SIGN or 003E > GREATER-THAN SIGN. However, for the purpose of historiographically exact transcriptions and editions it is neccessary to encode the difference between such modern symbols and historic ones, since either of them may occur in the very same edition.
The UCS already contains cases of closely related symbols which represent different writing customs for relations. For instance:

22DC ⋜ EQUAL TO OR LESS-THAN
2A95 ⪕ SLANTED EQUAL TO OR LESS-THAN

Following the logic of such instances, one of our proposed characters is:

xxxx ⩹ HORIZONTAL EQUAL TO OR LESS-THAN

In character names we left out the component 'SIGN' as we see this in line with most of comparable names of symbols already encoded. In some cases we propose personal identifiers as name parts ('LEIBNIZIAN', 'CARTESIAN') because we regard this as a suitable means of clarification. However, other naming options for the proposed character names could be discussed as well.

3. Characters

If this proposal gets accepted, the following characters will exist:

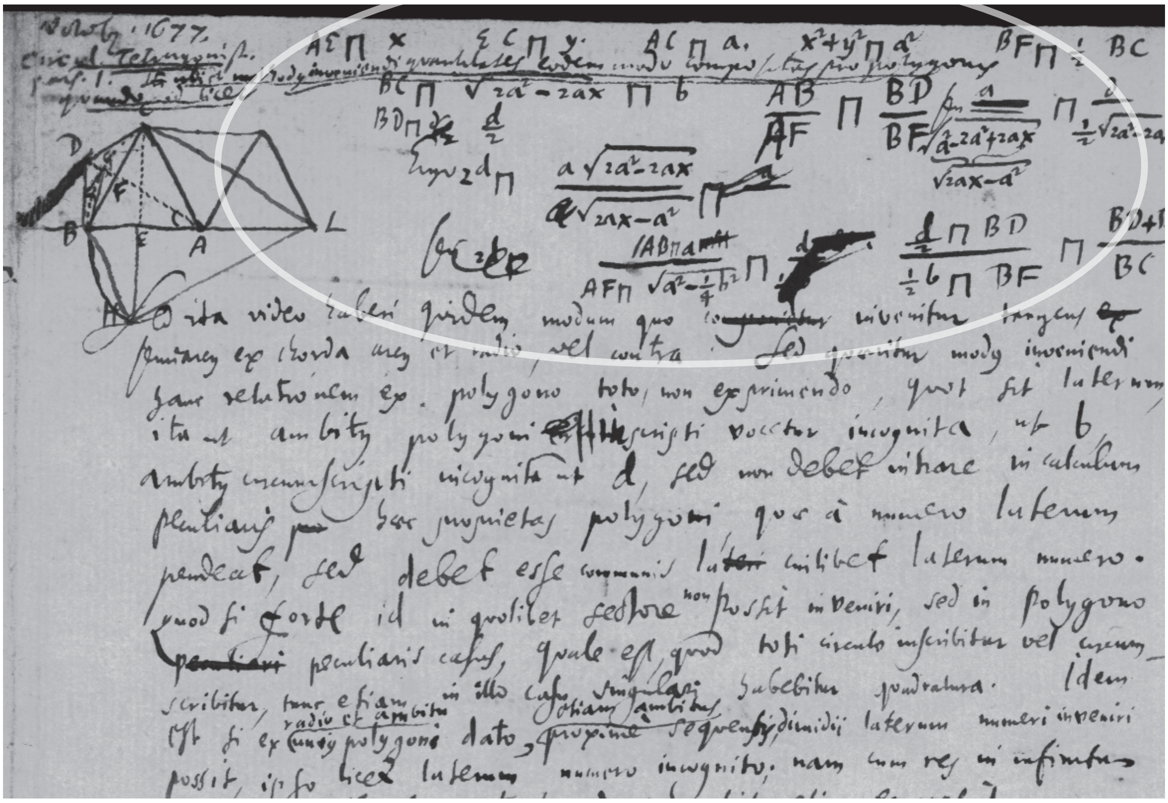
LEIBNIZIAN EQUAL
LEIBNIZIAN EQUAL WITH DOUBLE VERTICALS
LEIBNIZIAN EQUAL WITH SMALL S
LEIBNIZIAN GREATER
LEIBNIZIAN LESS
LEIBNIZIAN GREATER WITH SMALL P
LEIBNIZIAN LESS WITH SMALL P
LEIBNIZIAN GREATER-LESS
RECTANGULAR GREATER OPEN RIGHT
RECTANGULAR GREATER OPEN LEFT
RECTANGULAR LESS OPEN RIGHT
RECTANGULAR LESS OPEN LEFT
TWO-LINE GREATER
TWO-LINE LESS
COMMENSURABILITY
INCOMMENSURABILITY
COMMENSURABILITY IN SQUARE
INCOMMENSURABILITY IN SQUARE
HORIZONTAL EQUAL TO OR GREATER-THAN
HORIZONTAL EQUAL TO OR LESS-THAN
CARTESIAN EQUAL
LEIBNIZIAN CONGRUENCE-1
LEIBNIZIAN CONGRUENCE-2
LEIBNIZIAN CONGRUENCE-3
LEIBNIZIAN CONGRUENCE-4
LEIBNIZIAN CONGRUENCE-4 WITH COINCIDENCE
LEIBNIZIAN CONGRUENCE-4 WITHOUT COINCIDENCE
LEIBNIZIAN SIMILARITY-1
LEIBNIZIAN SIMILARITY-2
FACIT SYMBOL

For one character we propose a variation sequence:

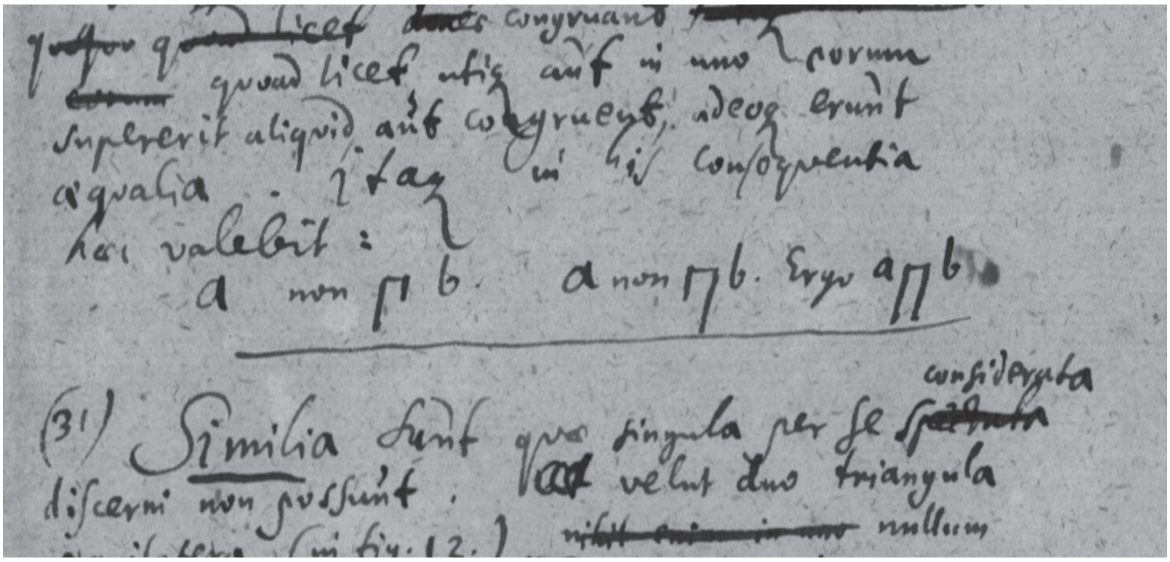
CARTESIAN EQUAL – *variation sequence to CARTESIAN EQUAL*

4. Figures and explanations

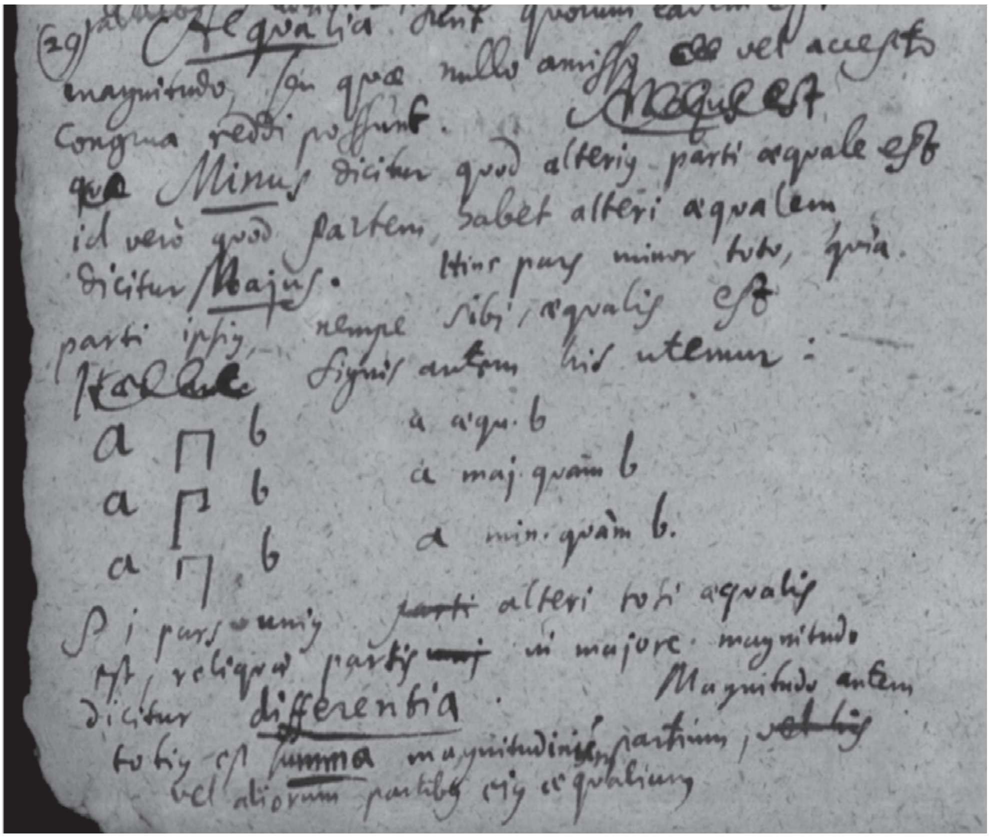
Leibniz made use of a fine differentiation of notions of equality and inequality in his mathematical writings. The character ⊓ LEIBNIZIAN EQUAL signifies in many of his mathematical works *equality* in the common meaning as it denotes the equality of two things with regard to some property.



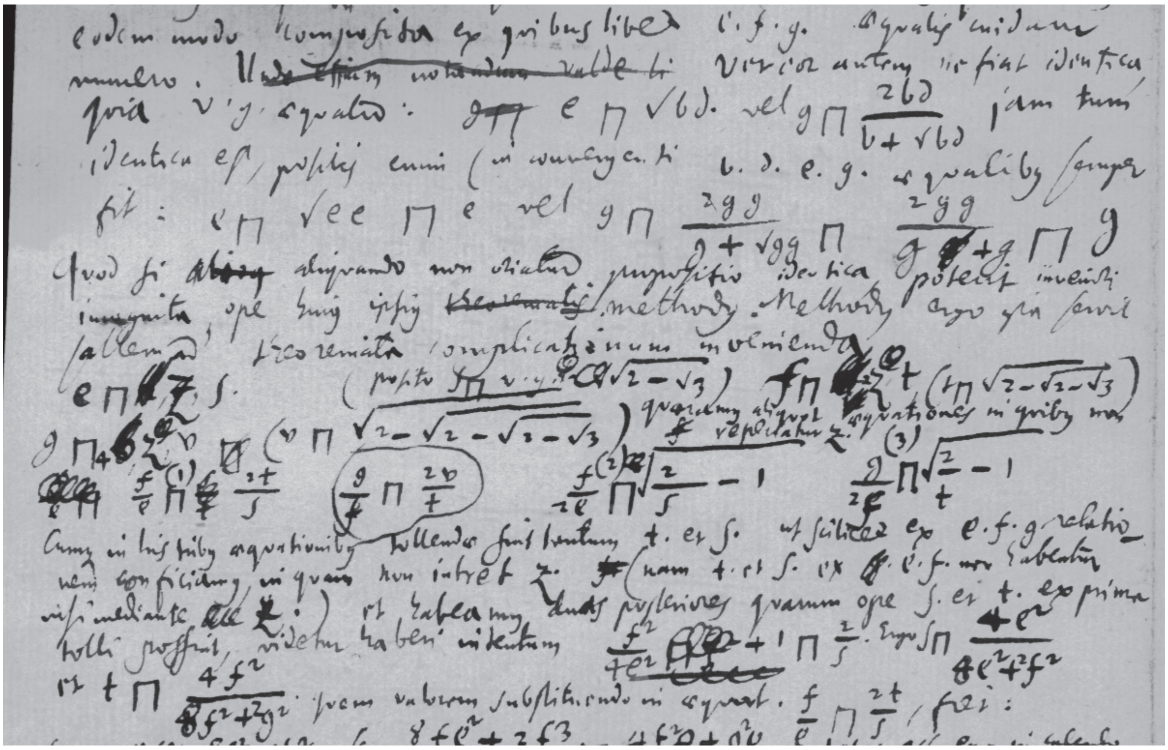
⊓ LEIBNIZIAN EQUAL
LH 35 XIII 3, fol. 72r



⊓ LEIBNIZIAN EQUAL, ⊓ LEIBNIZIAN GREATER, ⊓ LEIBNIZIAN LESS
LH 35 I 11, fol. 8r

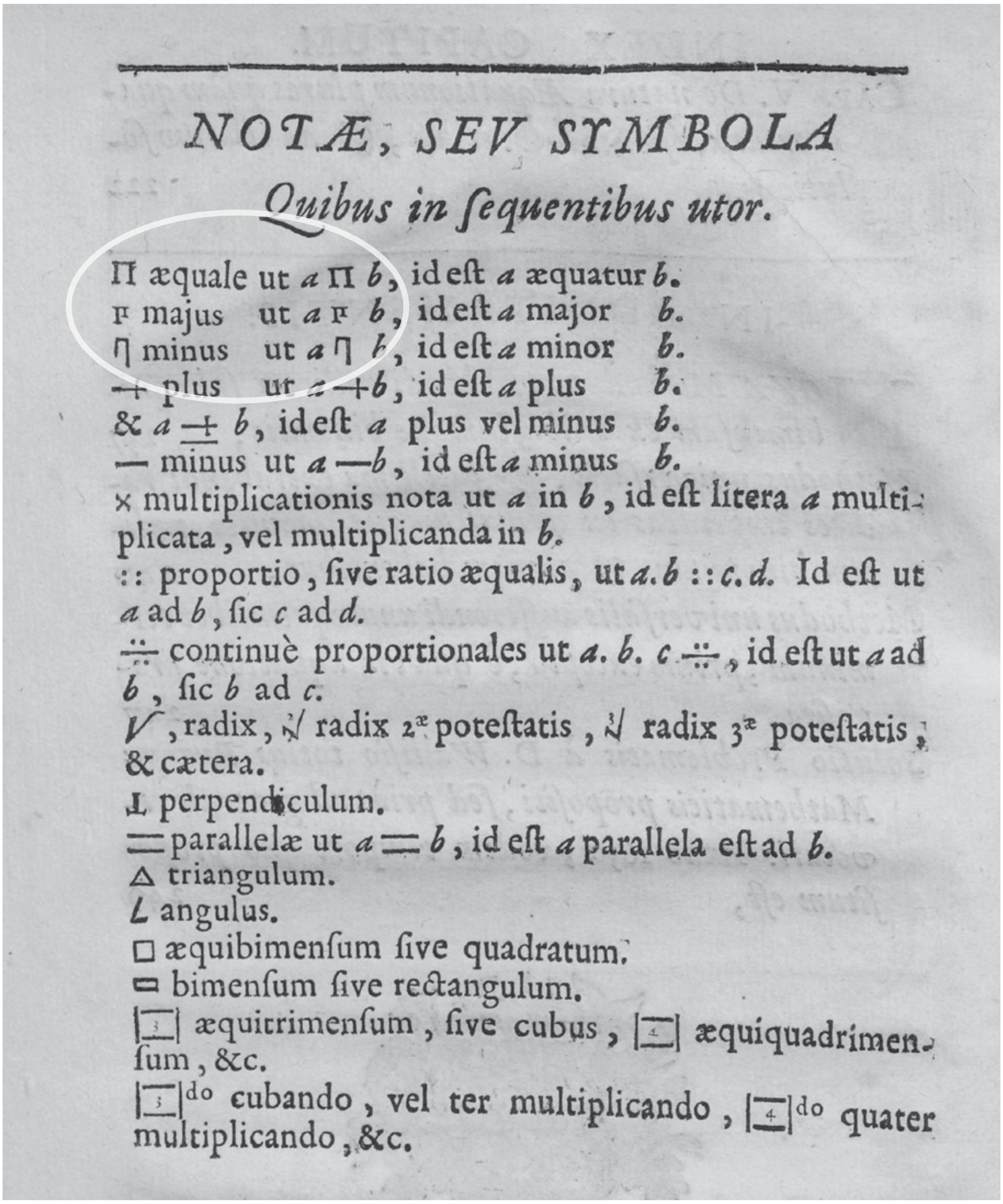


⊓ LEIBNIZIAN EQUAL, ⊓ LEIBNIZIAN GREATER, ⊓ LEIBNIZIAN LESS
LH 35 I 11, fol. 7v



⊓ LEIBNIZIAN EQUAL
LH 35 XIII 3, fol. 73v

Leibniz adopted the symbol (as well as the related symbols for “greater than” and “less than”) probably in 1674, after reading François Dulaurens: Specimina Mathematica Duobus Libris Comprehensa, Paris, 1667.

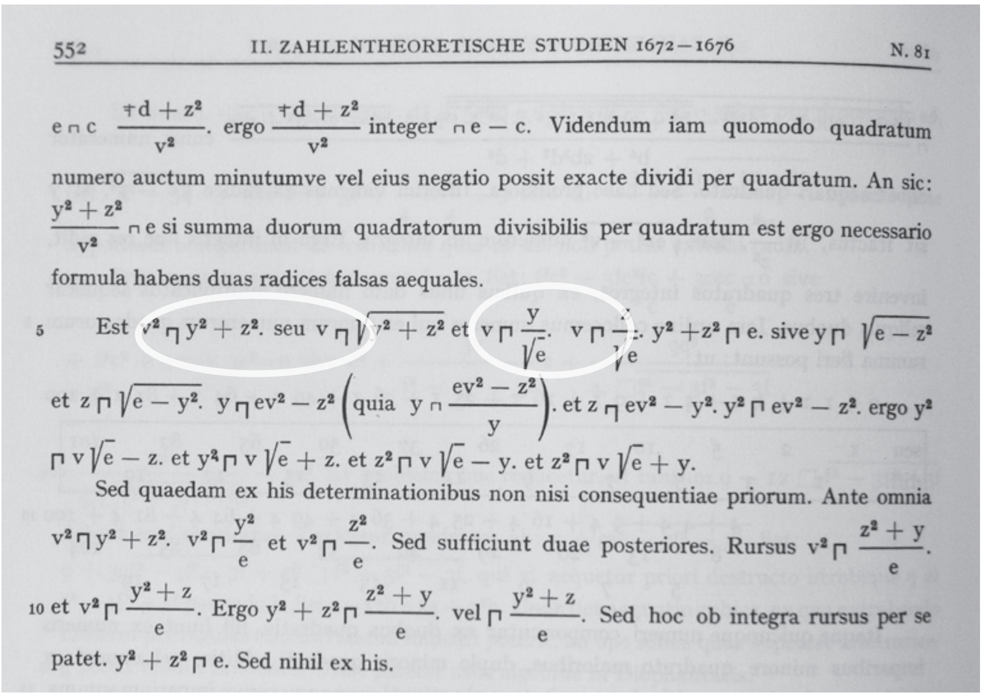


NOTÆ, SEV SYMBOLA

Quibus in ſequentibus utor.

Π æquale ut *a* Π *b*, id eſt *a* æquatur *b*.
ꟼ majus ut *a* ꟼ *b*, id eſt *a* major *b*.
⊓ minus ut *a* ⊓ *b*, id eſt *a* minor *b*.
+ plus ut *a* + *b*, id eſt *a* plus *b*.
& *a* ± *b*, id eſt *a* plus vel minus *b*.
— minus ut *a* — *b*, id eſt *a* minus *b*.
× multiplicationis nota ut *a* in *b*, id eſt litera *a* multiplicata, vel multiplicanda in *b*.
:: proportio, ſive ratio æqualis, ut *a*. *b* :: *c*. *d*. Id eſt ut *a* ad *b*, ſic *c* ad *d*.
÷ continuè proportionales ut *a*. *b*. *c* ÷, id eſt ut *a* ad *b*, ſic *b* ad *c*.
√, radix, √2 radix 2æ poteſtatis, √3 radix 3æ poteſtatis, & cætera.
⊥ perpendiculum.
= parallelæ ut *a* = *b*, id eſt *a* parallela eſt ad *b*.
△ triangulum.
∠ angulus.
□ æquibimenſum ſive quadratum.
▭ bimenſum ſive rectangulum.
[3] æquitrimenſum, ſive cubus, [4] æquiquadrimenſum, &c.
[3]do cubando, vel ter multiplicando, [4]do quater multiplicando, &c.

⊓ LEIBNIZIAN EQUAL, ⊓ LEIBNIZIAN GREATER, ⊓ LEIBNIZIAN LESS
Dulaurens, Specimina Mathematica, 1667. Note the typesetter’s makeshift solution, he borrowed two different greek Π-characters for *æquale* and *majus*.

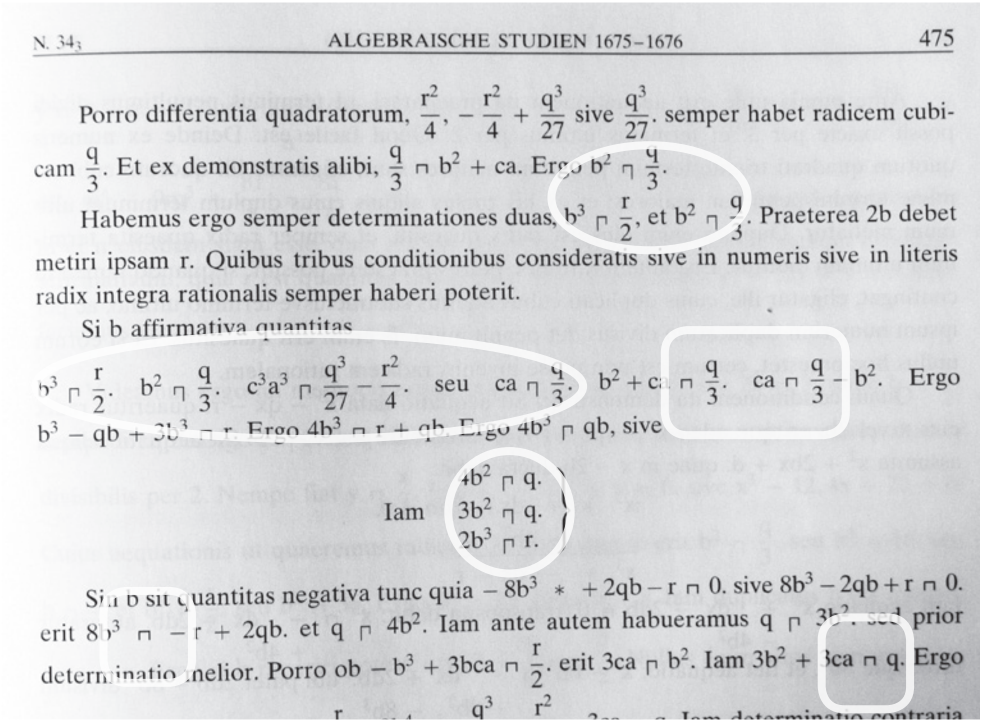

552 II. ZAHLENTHEORETISCHE STUDIEN 1672–1676 N. 81

$e \sqcap c\ \frac{\mp d + z^2}{v^2}$. ergo $\frac{\mp d + z^2}{v^2}$ integer $\sqcap e - c$. Videndum iam quomodo quadratum numero auctum minutumve vel eius negatio possit exacte dividi per quadratum. An sic: $\frac{y^2+z^2}{v^2} \sqcap e$ si summa duorum quadratorum divisibilis per quadratum est ergo necessario formula habens duas radices falsas aequales.

Est $v^2 \sqcap y^2 + z^2$. seu $v \sqcap \sqrt{y^2+z^2}$ et $v \sqcap \frac{y}{\sqrt{e}}$. $v \sqcap \frac{z}{\sqrt{e}}$. $y^2 + z^2 \sqcap e$. sive $y \sqcap \sqrt{e - z^2}$ et $z \sqcap \sqrt{e - y^2}$. $y \sqcap ev^2 - z^2$ $\left(\text{quia } y \sqcap \frac{ev^2 - z^2}{y}\right)$. et $z \sqcap ev^2 - y^2$. $y^2 \sqcap ev^2 - z^2$. ergo $y^2 \sqcap v\sqrt{e} - z$. et $y^2 \sqcap v\sqrt{e} + z$. et $z^2 \sqcap v\sqrt{e} - y$. et $z^2 \sqcap v\sqrt{e} + y$.

Sed quaedam ex his determinationibus non nisi consequentiae priorum. Ante omnia $v^2 \sqcap y^2 + z^2$. $v^2 \sqcap \frac{y^2}{e}$ et $v^2 \sqcap \frac{z^2}{e}$. Sed sufficiunt duae posteriores. Rursus $v^2 \sqcap \frac{z^2+y}{e}$. et $v^2 \sqcap \frac{y^2+z}{e}$. Ergo $y^2 + z^2 \sqcap \frac{z^2+y}{e}$. vel $\sqcap \frac{y^2+z}{e}$. Sed hoc ob integra rursus per se patet. $y^2 + z^2 \sqcap e$. Sed nihil ex his.

⊓ LEIBNIZIAN GREATER, ⊓ LEIBNIZIAN LESS
LAA VII-1 p. 552


N. 34₃ ALGEBRAISCHE STUDIEN 1675–1676 475

Porro differentia quadratorum, $\frac{r^2}{4}$, $-\frac{r^2}{4} + \frac{q^3}{27}$ sive $\frac{q^3}{27}$. semper habet radicem cubicam $\frac{q}{3}$. Et ex demonstratis alibi, $\frac{q}{3} \sqcap b^2 + ca$. Ergo $b^2 \sqcap \frac{q}{3}$.

Habemus ergo semper determinationes duas, $b^3 \sqcap \frac{r}{2}$, et $b^2 \sqcap \frac{q}{3}$. Praeterea 2b debet metiri ipsam r. Quibus tribus conditionibus consideratis sive in numeris sive in literis radix integra rationalis semper haberi poterit.

Si b affirmativa quantitas

$b^3 \sqcap \frac{r}{2}$. $b^2 \sqcap \frac{q}{3}$. $c^3a^3 \sqcap \frac{q^3}{27} - \frac{r^2}{4}$. seu $ca \sqcap \frac{q}{3}$. $b^2 + ca \sqcap \frac{q}{3}$. $ca \sqcap \frac{q}{3} - b^2$. Ergo $b^3 - qb + 3b^3 \sqcap r$. Ergo $4b^3 \sqcap r + qb$. Ergo $4b^3 \sqcap qb$, sive

Iam $4b^2 \sqcap q$. $3b^2 \sqcap q$. $2b^3 \sqcap r$.

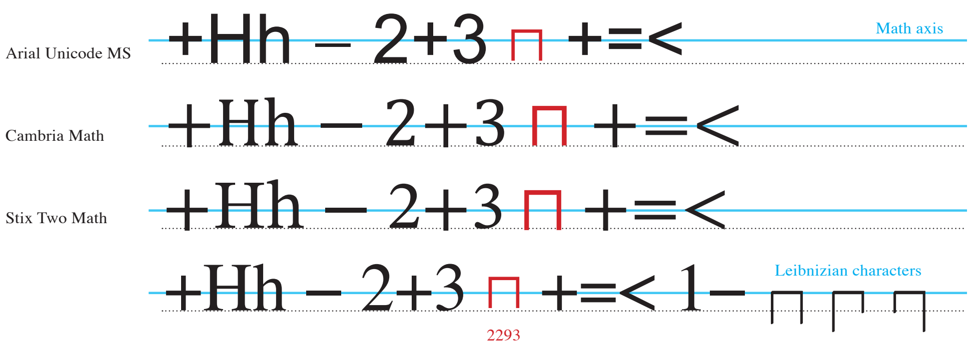
Si b sit quantitas negativa tunc quia $-8b^3 * +2qb - r \sqcap 0$. sive $8b^3 - 2qb + r \sqcap 0$. erit $8b^3 \sqcap -r + 2qb$. et $q \sqcap 4b^2$. Iam ante autem habueramus $q \sqcap 3b^2$. sed prior determinatio melior. Porro ob $-b^3 + 3bca \sqcap \frac{r}{2}$. erit $3ca \sqcap b^2$. Iam $3b^2 + 3ca \sqcap q$. Ergo

... Iam determinatio contraria

⊓ LEIBNIZIAN EQUAL, ⊓ LEIBNIZIAN GREATER, ⊓ LEIBNIZIAN LESS
LAA VII-2 p. 475

Ideally these character's glyphs are adjusted with their horizontal parts to the *math axis*, like e.g. + and –

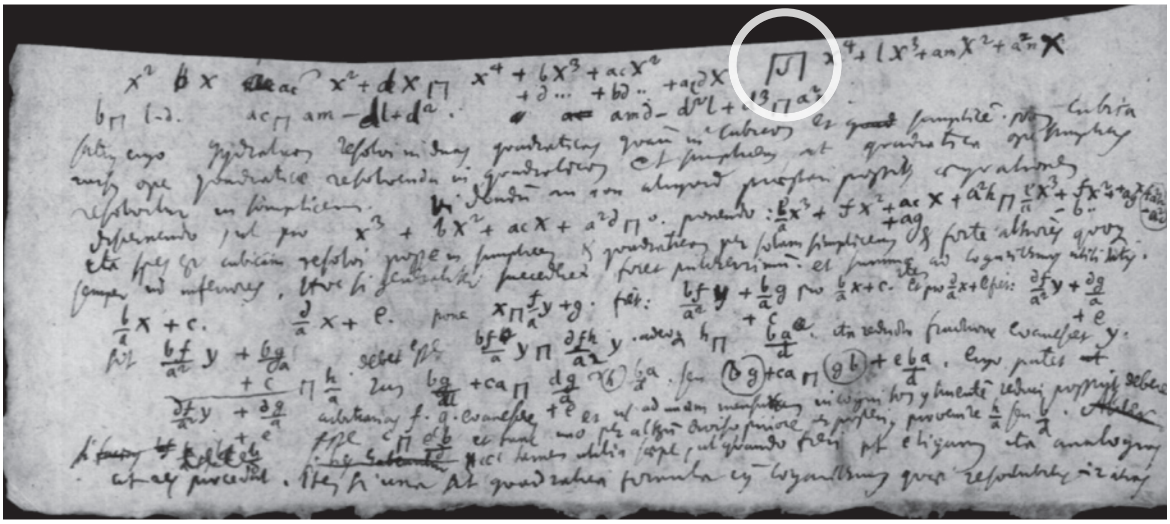
Whereas the printer of Dulaurens’ book (mis-)used capital Greek Pi types as stand-ins for *equality* and *greater,* thus getting the representations of *greater* and *less* inconsistent; in Leibniz’s manuscripts we encounter a well-considered coordination of these signs: The *equals* sign represents, as it were, a balance beam with two equal weights symbolized by the vertical strokes. For *greater* and *less,* respectively, vertical strokes of unequal length are used. These symbols have to be aligned vertically with their horizontal parts to the *math axis* which is usually represented by the vertical centres of + and – (*plus, minus*). This graphosystemic requirement together with different semantics exclude ⊓ LEIBNIZIAN EQUAL from being united with the (visually similar) character 2293 ⊓ SQUARE CAP.



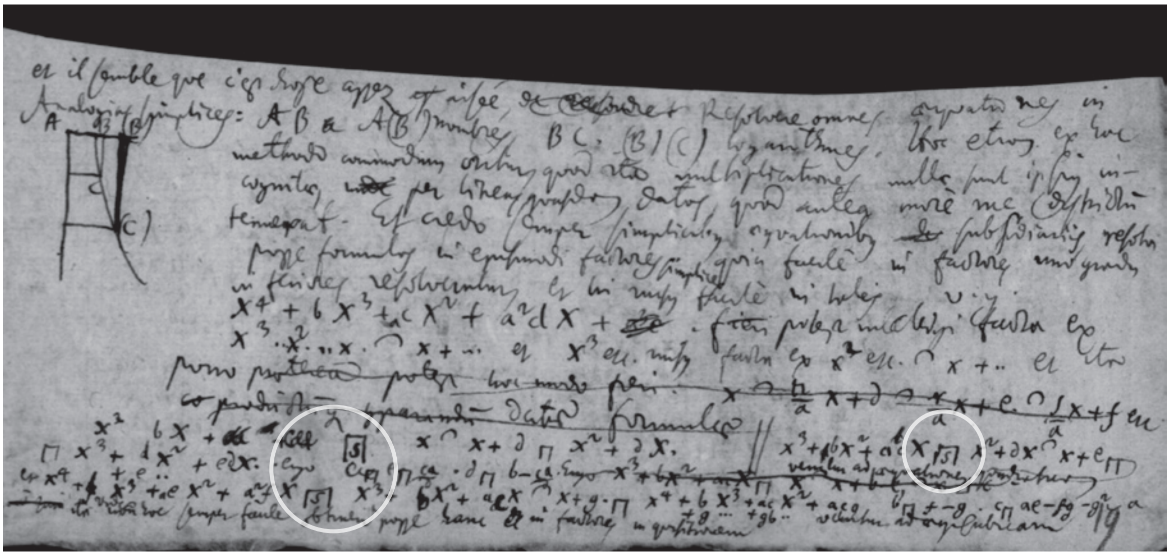
Due to their semantical connections, the 2293 ⊓ SQUARE CAP, 2229 ∩ INTERSECTION, 222A ∪ UNION and 2294 ⊔ SQUARE CUP characters need a strong consistency in their visual representation. The same is needed for ⊓ LEIBNIZIAN EQUAL, ⊓ LEIBNIZIAN EQUAL WITH DOUBLE VERTICALS, ⊓ LEIBNIZIAN EQUALITY WITH S, ┌ LEIBNIZIAN GREATER, ┐ LEIBNIZIAN LESS, ┌ LEIBNIZIAN GREATER WITH SMALL P, ┐ LEIBNIZIAN LESS WITH SMALL P and ┌┐ LEIBNIZIAN GREATER-LESS.

This is how the glyphs of the new characters may be integrated into a Roman-style typeface:

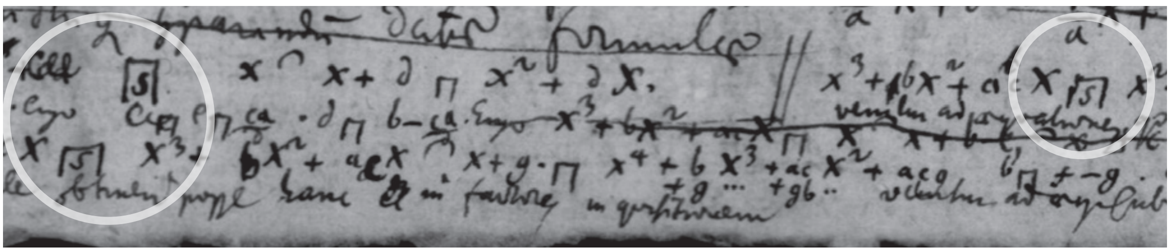
Leibniz derived the configurations of several other symbols from ⊓ LEIBNIZIAN EQUAL:
[S] LEIBNIZIAN EQUALITY WITH S denotes a kind of equality by definition that originates from equating two expressions with each other as in the phrase “let *a* be equal to *b*”. Unlike the definition sign in modern mathematics, there is no specific direction in Leibniz’s sign. The “s” in the sign is an abbreviation of the Latin word “sit”.
Combining both ⊓ and ⊓ into ⊓⊓ LEIBNIZIAN GREATER-LESS leads to an ambiguous inequality sign that denotes “greater than in the first case and less than in the second case”.

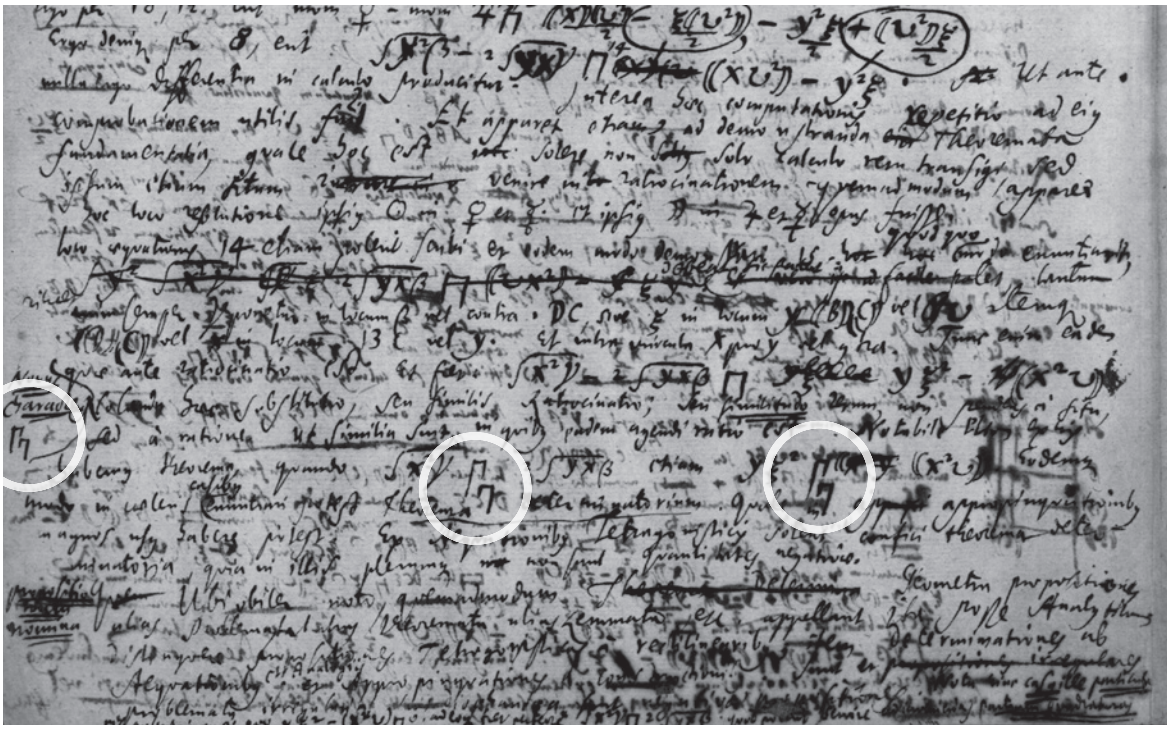


[S] LEIBNIZIAN EQUALITY WITH S
LH 35 V 14, fol. 18r. *The edition of this manuscript is currently in progress.*



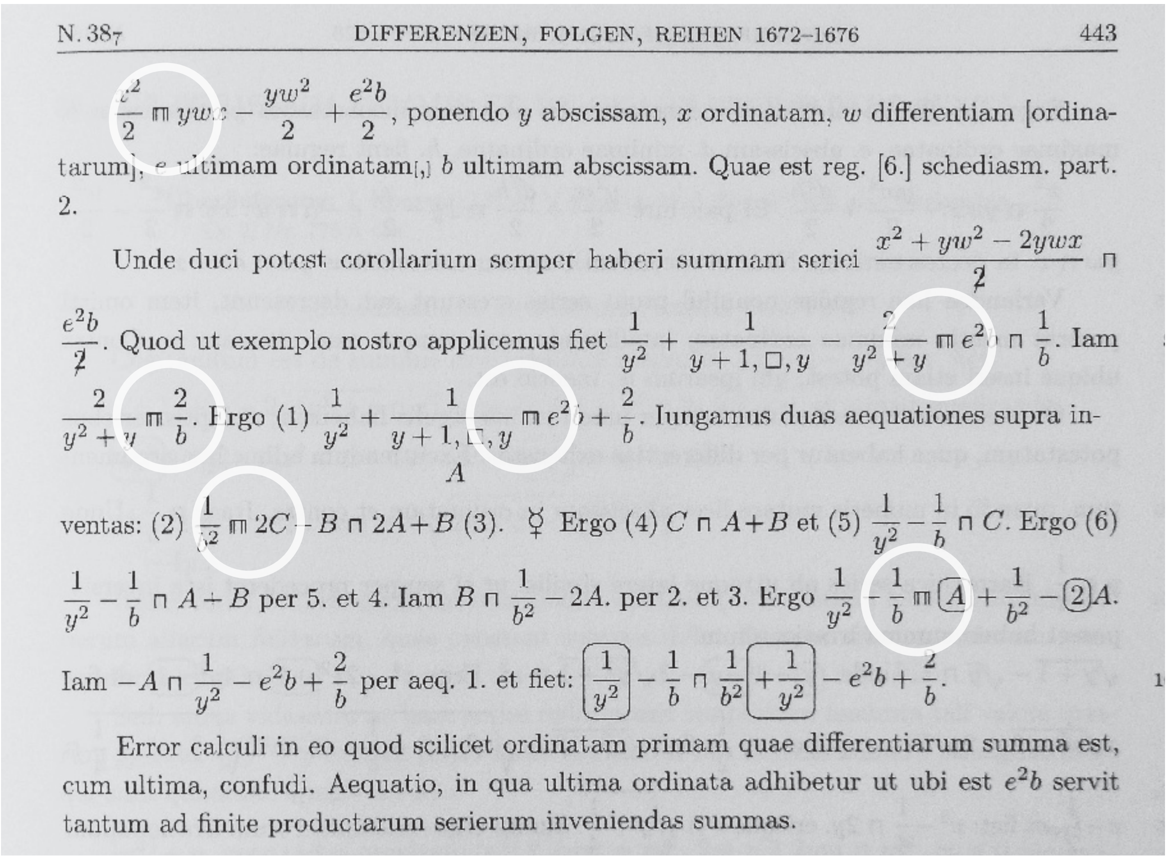
[S] LEIBNIZIAN EQUALITY WITH S
LH 35 V 14, fol. 19r. *The edition of this manuscript is currently in progress.*





⊓| LEIBNIZIAN GREATER-LESS

LH 35 XIII 3, fol. 150v. *The edition of this manuscript is currently in progress.*



N. 38₇ DIFFERENZEN, FOLGEN, REIHEN 1672–1676 443

$\frac{e^2}{2}$ ℿ $yw\ldots - \frac{yw^2}{2} + \frac{e^2b}{2}$, ponendo y abscissam, x ordinatam, w differentiam [ordinatarum], e ultimam ordinatam[,] b ultimam abscissam. Quae est reg. [6.] schediasm. part. 2.

Unde duci potest corollarium semper haberi summam seriei $\frac{x^2 + yw^2 - 2ywx}{2}$ ⊓ $\frac{e^2b}{2}$. Quod ut exemplo nostro applicemus fiet $\frac{1}{y^2} + \frac{1}{y+1, \square, y} - \frac{2}{y^2+y}$ ℿ e^2b ⊓ $\frac{1}{b}$. Iam $\frac{2}{y^2+y}$ ℿ $\frac{2}{b}$. Ergo (1) $\frac{1}{y^2} + \frac{1}{y+1, \square, y}$ ℿ $e^2b + \frac{2}{b}$. Iungamus duas aequationes supra inventas: (2) $\frac{1}{y^2}$ ℿ $2C - B$ ⊓ $2A + B$ (3). ☿ Ergo (4) C ⊓ $A + B$ et (5) $\frac{1}{y^2} - \frac{1}{b}$ ⊓ C. Ergo (6) $\frac{1}{y^2} - \frac{1}{b}$ ⊓ $A + B$ per 5. et 4. Iam B ⊓ $\frac{1}{b^2} - 2A$. per 2. et 3. Ergo $\frac{1}{y^2} - \frac{1}{b}$ ℿ $A + \frac{1}{b^2} - 2A$.

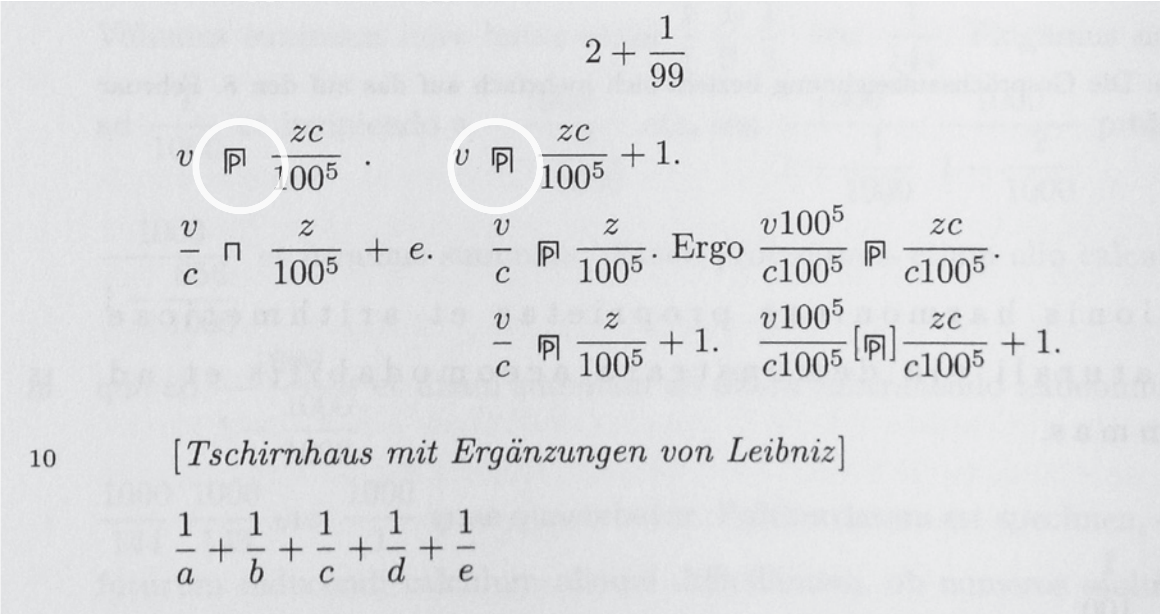
Iam $-A$ ⊓ $\frac{1}{y^2} - e^2b + \frac{2}{b}$ per aeq. 1. et fiet: $\frac{1}{y^2} - \frac{1}{b}$ ⊓ $\frac{1}{b^2} + \frac{1}{y^2} - e^2b + \frac{2}{b}$.

Error calculi in eo quod scilicet ordinatam primam quae differentiarum summa est, cum ultima, confudi. Aequatio, in qua ultima ordinata adhibetur ut ubi est e^2b servit tantum ad finite productarum serierum inveniendas summas.

ℿ LEIBNIZIAN EQUAL WITH DOUBLE VERTICALS

Leibniz uses this symbol for "equality of sums".

LAA VII-3 p. 443


$$2+\frac{1}{99}$$

$$v\ \overline{|p}\ \frac{zc}{100^5}\,. \qquad v\ \overline{p|}\ \frac{zc}{100^5}+1.$$

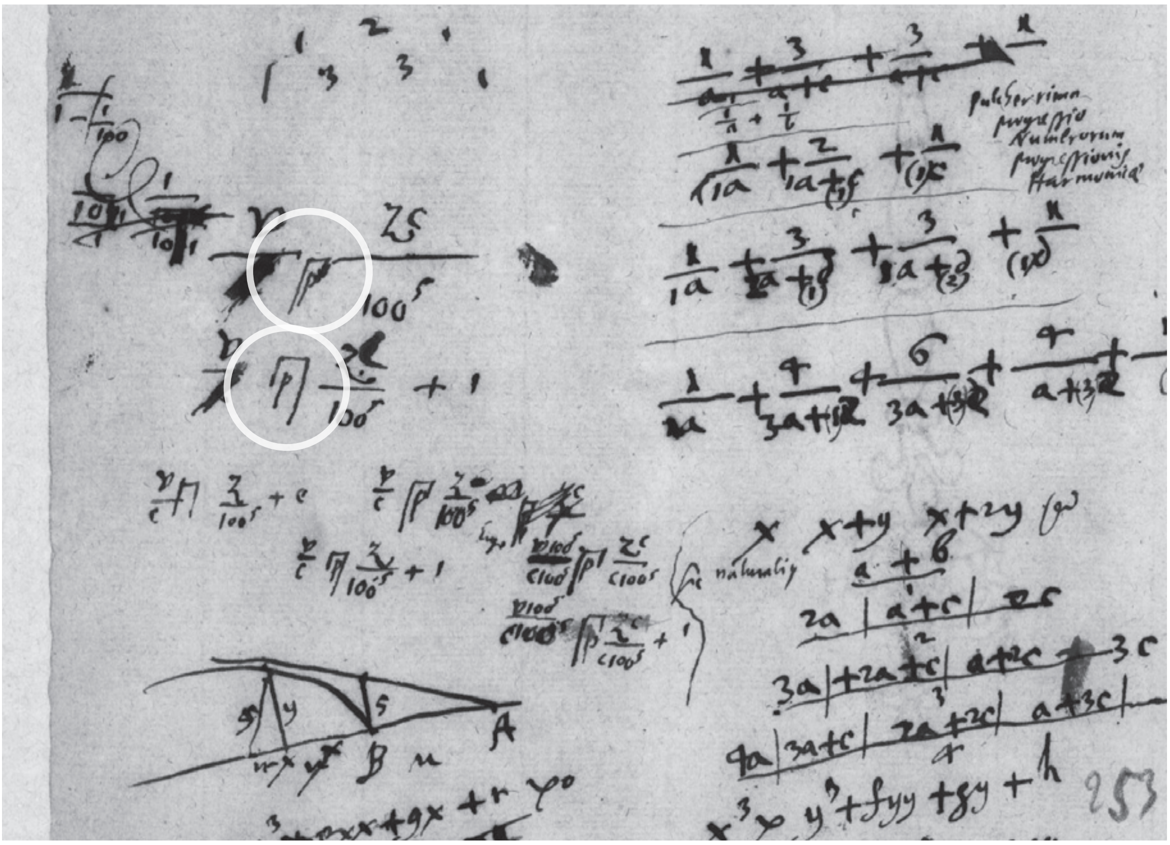
$$\frac{v}{c}\ \sqcap\ \frac{z}{100^5}+e. \qquad \frac{v}{c}\ \overline{|p}\ \frac{z}{100^5}.\ \text{Ergo}\ \frac{v100^5}{c100^5}\ \overline{|p}\ \frac{zc}{c100^5}.$$

$$\frac{v}{c}\ \overline{p|}\ \frac{z}{100^5}+1. \qquad \frac{v100^5}{c100^5}\,[\overline{p|}]\,\frac{zc}{c100^5}+1.$$

[*Tschirnhaus mit Ergänzungen von Leibniz*]

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}+\frac{1}{e}$$

$\overline{|p}$ LEIBNIZIAN GREATER WITH SMALL P, $\overline{p|}$ LEIBNIZIAN LESS WITH SMALL P
These symbols denote “a little bit greater” and “a little bit less”, the letter “p” abbreviating the Latin word “paulo” (little). – *Corresponding Ms.: see below.*
LAA VII-3 p. 732



$\overline{|p}$ LEIBNIZIAN GREATER WITH SMALL P, $\overline{p|}$ LEIBNIZIAN LESS WITH SMALL P
The handwriting shows that a lowercase p was intended by the author, so the representation of these symbols in the printed edition is not accurate in this respect.
LH 35 XII 1, fol. 253r

(7) Ungleichungen:

Zusätzlich zu den üblichen Symbolen ⊓ für „größer“ und ⊓ für „kleiner“ (N. 66) führt Leibniz noch Zeichen für „ein wenig größer“ (⊓) bzw. „ein wenig kleiner“ (⊓) ein (N. 54).

⊓ LEIBNIZIAN GREATER WITH SMALL P, ⊓ LEIBNIZIAN LESS WITH SMALL P
LAA VII-3 p. XXXI

D e m o n s t r. Productae sint particulae curvae in tangentes $DBCL$, ECG, eritque $LBI = DBA = BAC + BCA = BMC$ $(2BFC) + BCA = 2ECD + BCA = ECD + ECA = LCG + GCH = LCH$. Ergo BI parallela CH. Quod si a sit intra circulum, erit $DBa \sqsubset DBA = LBI$, quare divaricabitur a CH. Sin α sit extra circulum, erit $DB\alpha \sqsupset DBA = LBI$, quare coibit cum CH. Q. E. D.

C o r o l l. Hinc possunt inveniri puncta Causticae: Nam quia $BF = 2BM$; et

⊏ RECTANGULAR GREATER OPEN RIGHT, ⊐ RECTANGULAR LESS OPEN LEFT
LAA III-6 p. 688; corresponding manuscript part (below)

Demonstr. Productae sint particulae curvae in tangentes $DBCL$, ECG, eritque $LBI = DBA = BAC + BCA = BMC\ (2BFC) + BCA = 2ECD + BCA = ECD + ECA = LCG + GCH = LCH$. Ergo BI parallela CH. Quod si autem sit intra circulum, erit $DBa \sqsubset DBA = LBI$, quare divaricabitur à CH. sin α sit extra Circulum, erit $DB\alpha \sqsupset DBA = LBI$. quare coibit cum CH. Q. E. D.

... Caustica: Nam quia $BF = 2BM$.

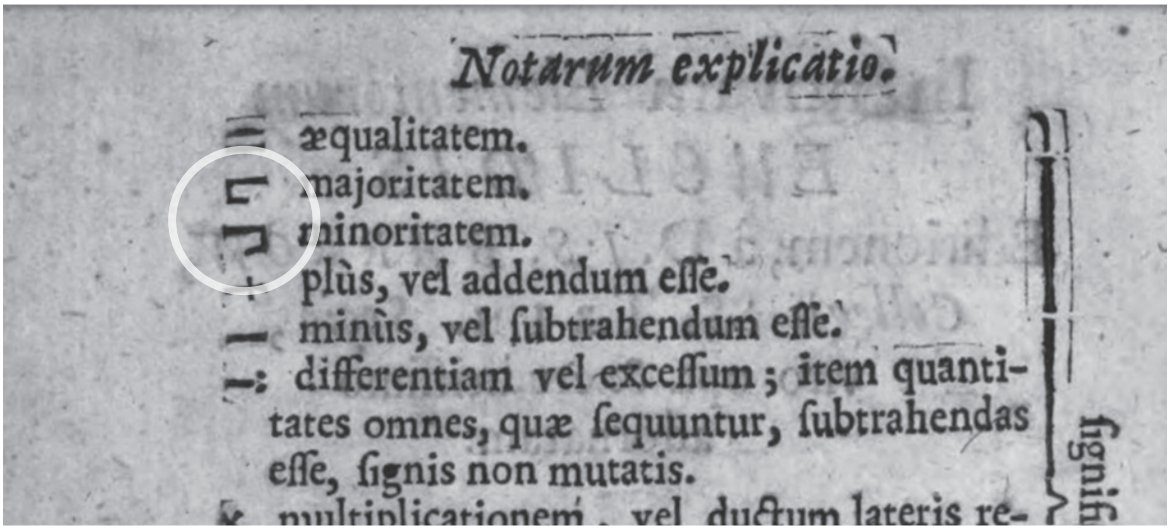
Distinct from the above signs are these two greater / less signs, which lack the vertical part:

Et pro $\dfrac{\sqrt{aa + b\sqrt{cc + dd}}}{e + \sqrt{f\sqrt{gg + hh} + kk}}$

scribi poterit $\sqrt{(aa + b\sqrt{(cc + dd)})}:, e + \sqrt{(f\sqrt{(gg + hh)} + kk)}$

Hactenus notas exposuimus, quibus termini, id est numeri vel quantitates formantur, tanquam subjecta aut prædicata in veritatibus. Sequuntur notæ quæ explicant modum prædicationis, seu quomodo quantitates quæ Terminos constituunt in propositiones conjungantur, potissimum autem de iis enuntiatur, *Aequales* esse, vel *majores*, aut *minores* aliis, itaque $a = b$ significat, a, esse æquale ipsi b, & a ⩦ b significat a esse majus quàm b, & a ⩦ b significat a esse minus quàm b. Sed

⩦ TWO-LINE GREATER, ⩦ TWO-LINE LESS
Monitum de Characteribus Algebraica, Miscellanea Berolinensia, 1710, p. 158



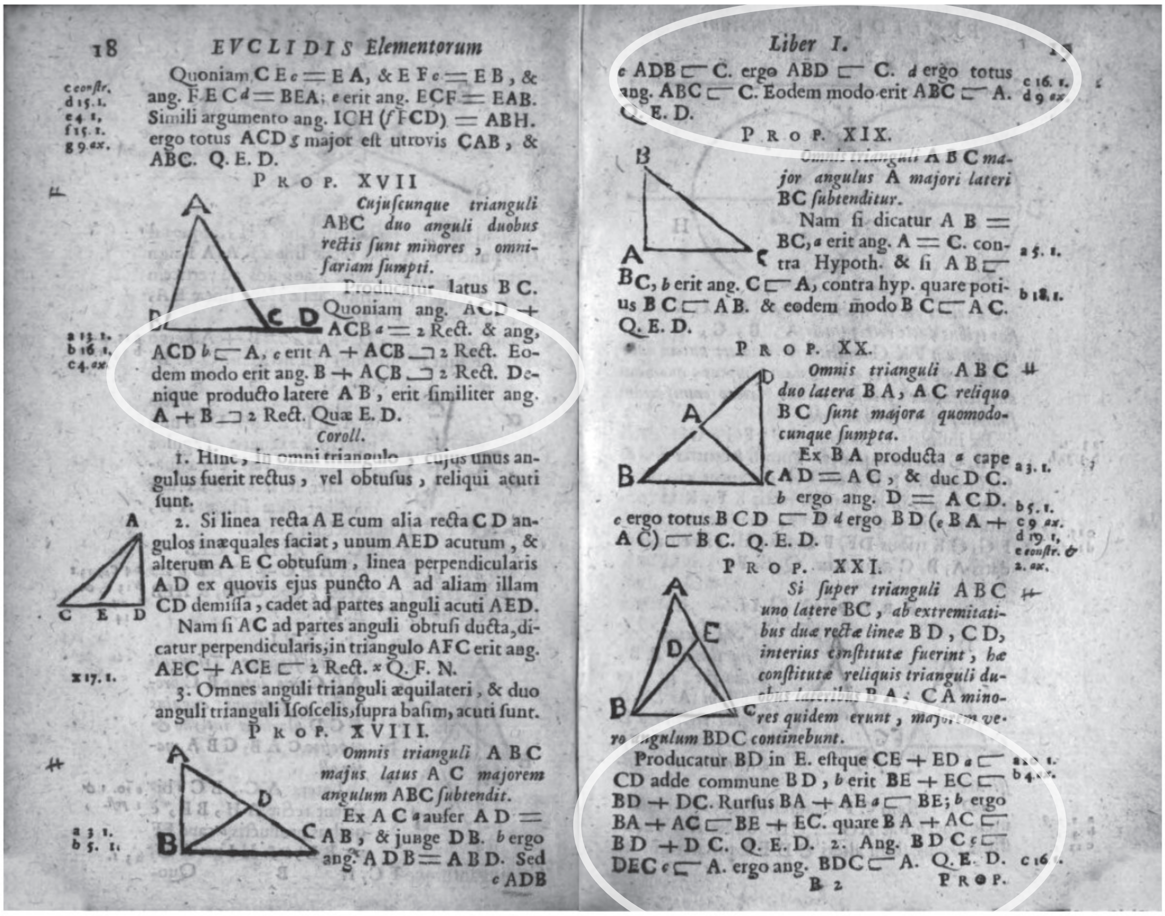
Notarum explicatio.

= æqualitatem.
⊏ majoritatem.
⊐ minoritatem.
+ plùs, vel addendum esse.
− minùs, vel subtrahendum esse.
−: differentiam vel excessum; item quantitates omnes, quæ sequuntur, subtrahendas esse, signis non mutatis.
× multiplicationem, vel ductum lateris re-

signif

⊏ RECTANGULAR GREATER OPEN RIGHT, ⊐ RECTANGULAR GREATER OPEN LEFT, ⊐ RECTANGULAR LESS OPEN LEFT

Barrow 1655 (top), Barrow 1659 (below)



18 EVCLIDIS Elementorum

Quoniam C E c = E A, & E F c = E B, & ang. F E C d = BEA; e erit ang. ECF = EAB. Simili argumento ang. ICH (f FCD) = ABH. ergo totus ACD g major est utrovis CAB, & ABC. Q. E. D.

PROP. XVII

Cujuscunque trianguli ABC duo anguli duobus rectis sunt minores, omnifariam sumpti.

Producatur latus B C. Quoniam ang. ACD + ACB a = 2 Rect. & ang. ACD b ⊏ A, c erit A + ACB ⊐ 2 Rect. Eodem modo erit ang. B + ACB ⊐ 2 Rect. Denique producto latere A B, erit similiter ang. A + B ⊐ 2 Rect. Quæ E. D.

Coroll.

1. Hinc, in omni triangulo, cujus unus angulus fuerit rectus, vel obtusus, reliqui acuti sunt.

2. Si linea recta A E cum alia recta C D angulos inæquales faciat, unum AED acutum, & alterum A E C obtusum, linea perpendicularis A D ex quovis ejus puncto A ad aliam illam CD demissa, cadet ad partes anguli acuti AED. Nam si AC ad partes anguli obtusi ducta, dicatur perpendicularis, in triangulo AFC erit ang. AEC + ACE ⊏ 2 Rect. x Q. F. N.

3. Omnes anguli trianguli æquilateri, & duo anguli trianguli Isoscelis, supra basim, acuti sunt.

PROP. XVIII.

Omnis trianguli A B C majus latus A C majorem angulum ABC subtendit.

Ex A C a aufer A D = A B, & junge D B. b ergo ang. A D B = A B D. Sed c ADB

Liber I.

c ADB ⊏ C. ergo ABD ⊏ C. d ergo totus ang. ABC ⊏ C. Eodem modo erit ABC ⊏ A. Q. E. D.

PROP. XIX.

Omnis trianguli A B C major angulus A majori lateri BC subtenditur.

Nam si dicatur A B = BC, a erit ang. A = C. contra Hypoth. & si A B ⊏ BC, b erit ang. C ⊏ A, contra hyp. quare potius B C ⊏ A B. & eodem modo B C ⊏ A C. Q. E. D.

PROP. XX.

Omnis trianguli A B C duo latera B A, A C reliquo B C sunt majora quomodocunque sumpta.

Ex B A producta a cape A D = A C, & duc D C. b ergo ang. D = A C D. c ergo totus B C D ⊏ D d ergo B D (e B A + A C) ⊏ B C. Q. E. D.

PROP. XXI.

Si super trianguli A B C uno latere BC, ab extremitatibus duæ rectæ lineæ B D, C D, interius constitutæ fuerint, hæ constitutæ reliquis trianguli duobus lateribus B A, C A minores quidem erunt, majorem vero angulum BDC continebunt.

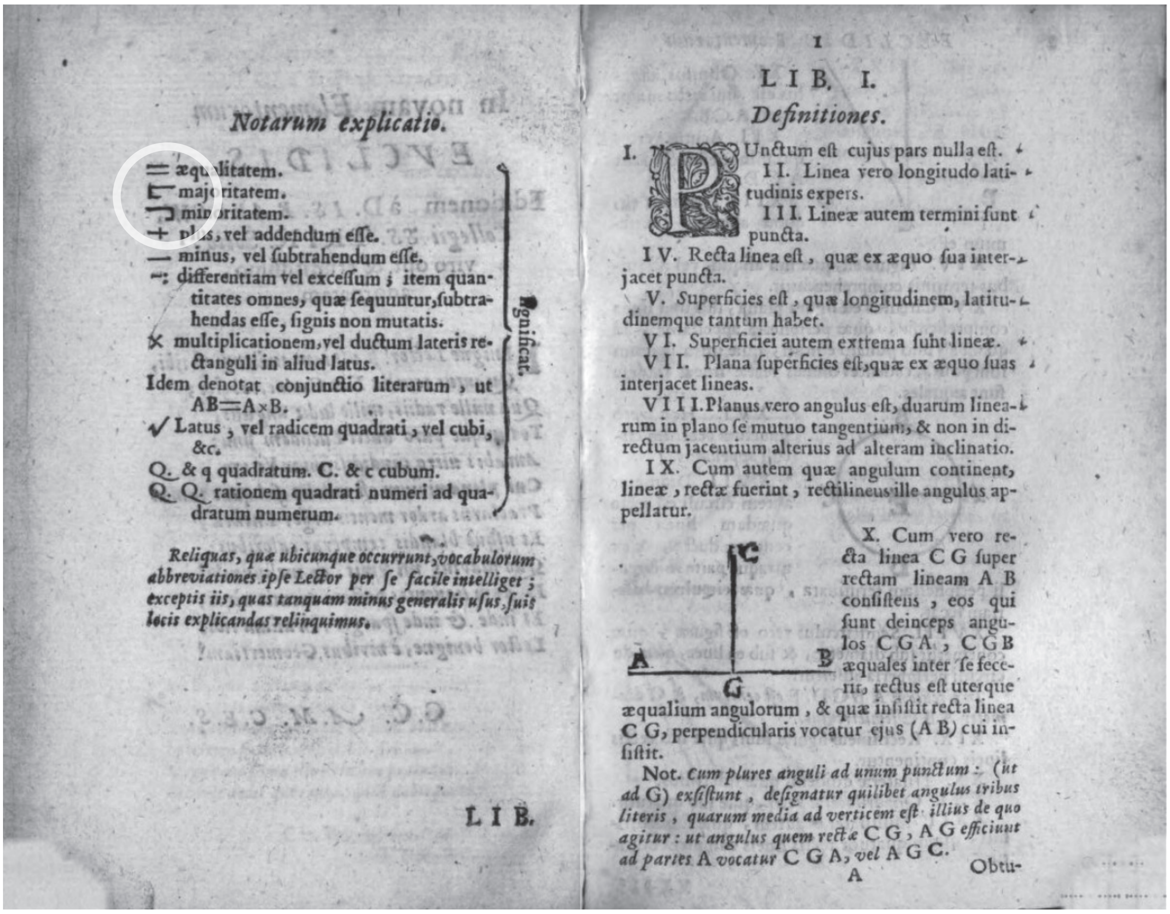
Producatur BD in E. estque CE + ED a ⊏ CD adde commune B D, b erit BE + EC ⊏ BD + DC. Rursus BA + AE a ⊏ BE; b ergo BA + AC ⊏ BE + EC. quare B A + AC ⊏ B D + D C. Q. E. D. 2. Ang. B D C c ⊏ DEC c ⊏ A. ergo ang. BDC ⊏ A. Q. E. D.

B 2 PROP.

⊏ RECTANGULAR GREATER OPEN RIGHT, ⊐ RECTANGULAR GREATER OPEN LEFT, ⊐ RECTANGULAR LESS OPEN LEFT

Barrow 1659

(next page)



Notarum explicatio.

═ æqualitatem.
⊏ majoritatem.
⊐ minoritatem.
+ plus, vel addendum eſſe.
— minus, vel ſubtrahendum eſſe.
=: differentiam vel exceſſum ; item quantitates omnes, quæ ſequuntur, ſubtrahendas eſſe, ſignis non mutatis.
× multiplicationem, vel ductum lateris rectanguli in aliud latus.
Idem denotat conjunctio literarum, ut AB ═ A×B.
√ Latus, vel radicem quadrati, vel cubi, &c.
Q. & q quadratum. C. & c cubum.
Q. Q. rationem quadrati numeri ad quadratum numerum.

(significat.)

Reliquas, quæ ubicunque occurrunt, vocabulorum abbreviationes ipſe Lector per ſe facile intelliget; exceptis iis, quas tanquam minus generalis uſus ſuis locis explicandas relinquimus.

LIB.

1

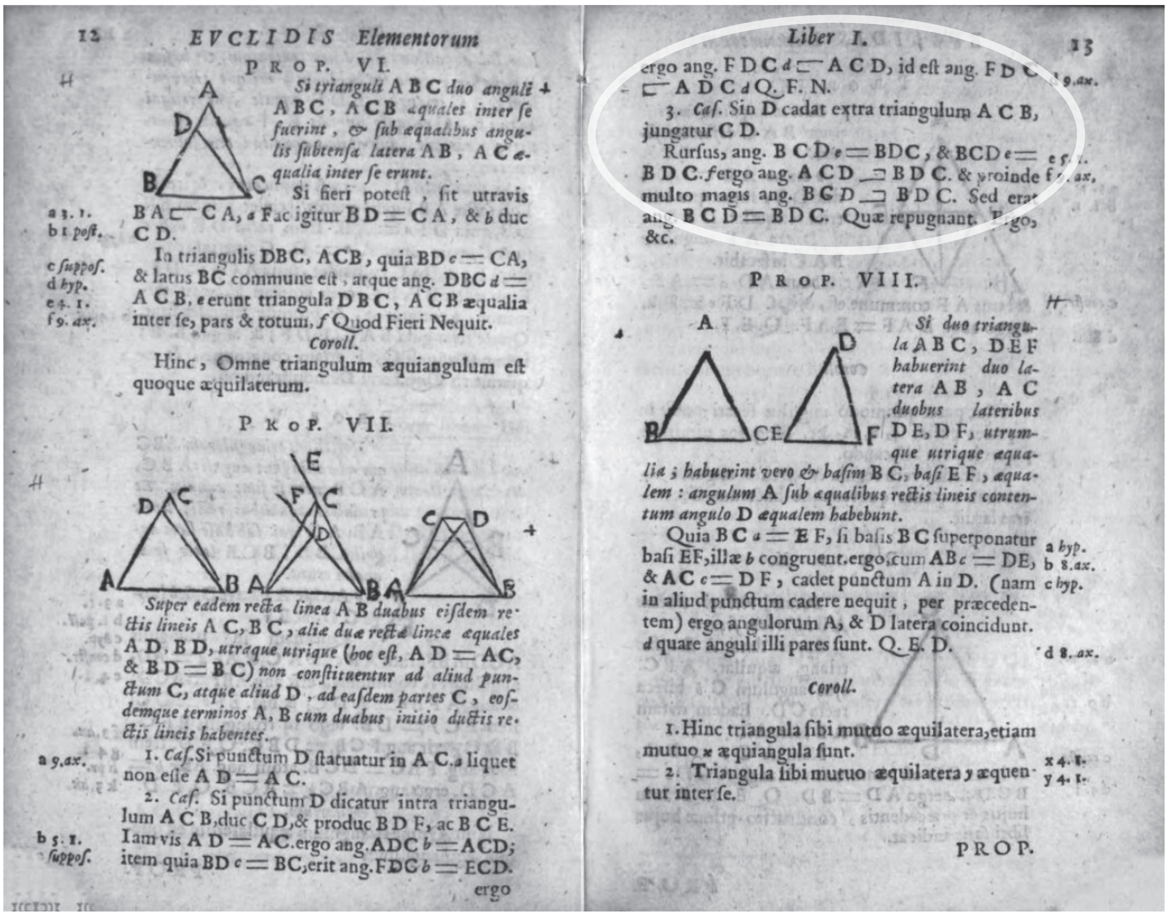
LIB. I.

Definitiones.

I. Punctum eſt cujus pars nulla eſt.
II. Linea vero longitudo latitudinis expers.
III. Lineæ autem termini ſunt puncta.
IV. Recta linea eſt, quæ ex æquo ſua interjacet puncta.
V. Superficies eſt, quæ longitudinem, latitudinemque tantum habet.
VI. Superficiei autem extrema ſunt lineæ.
VII. Plana ſuperficies eſt, quæ ex æquo ſuas interjacet lineas.
VIII. Planus vero angulus eſt, duarum linearum in plano ſe mutuo tangentium, & non in directum jacentium alterius ad alteram inclinatio.
IX. Cum autem quæ angulum continent, lineæ, rectæ fuerint, rectilineus ille angulus appellatur.
X. Cum vero recta linea C G ſuper rectam lineam A B conſiſtens, eos qui ſunt deinceps angulos C G A, C G B æquales inter ſe fecerit, rectus eſt uterque æqualium angulorum, & quæ inſiſtit recta linea C G, perpendicularis vocatur ejus (A B) cui inſiſtit.

Not. *Cum plures anguli ad unum punctum: (ut ad G) exſiſtunt, deſignatur quilibet angulus tribus literis, quarum media ad verticem eſt illius de quo agitur: ut angulus quem rectæ* C G, A G *efficiunt ad partes* A *vocatur* C G A, *vel* A G C.

Obtu-



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PROP. VI.

Si trianguli A B C *duo anguli* A B C, A C B *æquales inter ſe fuerint, & ſub æqualibus angulis ſubtenſa latera* A B, A C *æqualia inter ſe erunt.*

Si fieri poteſt, ſit utravis B A ⊏ C A, *a* Fac igitur B D ═ C A, & *b* duc C D.

In triangulis DBC, ACB, quia BD *c* ═ CA, & latus BC commune eſt, atque ang. DBC *d* ═ A C B, *e* erunt triangula D B C, A C B æqualia inter ſe, pars & totum. *f* Quod Fieri Nequit.

Coroll.

Hinc, Omne triangulum æquiangulum eſt quoque æquilaterum.

PROP. VII.

Super eadem recta linea A B *duabus iiſdem rectis lineis* A C, B C, *aliæ duæ rectæ lineæ æquales* A D, B D, *utraque utrique (hoc eſt,* A D ═ A C, & B D ═ B C) *non conſtituentur ad aliud punctum* C, *atque aliud* D, *ad eaſdem partes* C, *eoſdemque terminos* A, B *cum duabus initio ductis rectis lineis habentes.*

1. *Caſ.* Si punctum D ſtatuatur in A C, *a* liquet non eſſe A D ═ A C.
2. *Caſ.* Si punctum D dicatur intra triangulum A C B, duc C D, & produc B D F, ac B C E. Iam vis A D ═ AC. ergo ang. ADC *b* ═ ACD; item quia BD *c* ═ BC, erit ang. FDC *b* ═ ECD.

ergo

Liber I. 13

ergo ang. F D C *d* ⊏ A C D, id eſt ang. F D C ⊏ A D C *d* Q. F. N.

3. *Caſ.* Sin D cadat extra triangulum A C B, jungatur C D.

Rurſus, ang. B C D *e* ═ BDC, & BCD *e* ═ B D C. *f* ergo ang. A C D ⊐ B D C. & proinde multo magis ang. B C D ⊐ B D C. Sed erat ang. B C D ═ B D C. Quæ repugnant. Ergo, &c.

PROP. VIII.

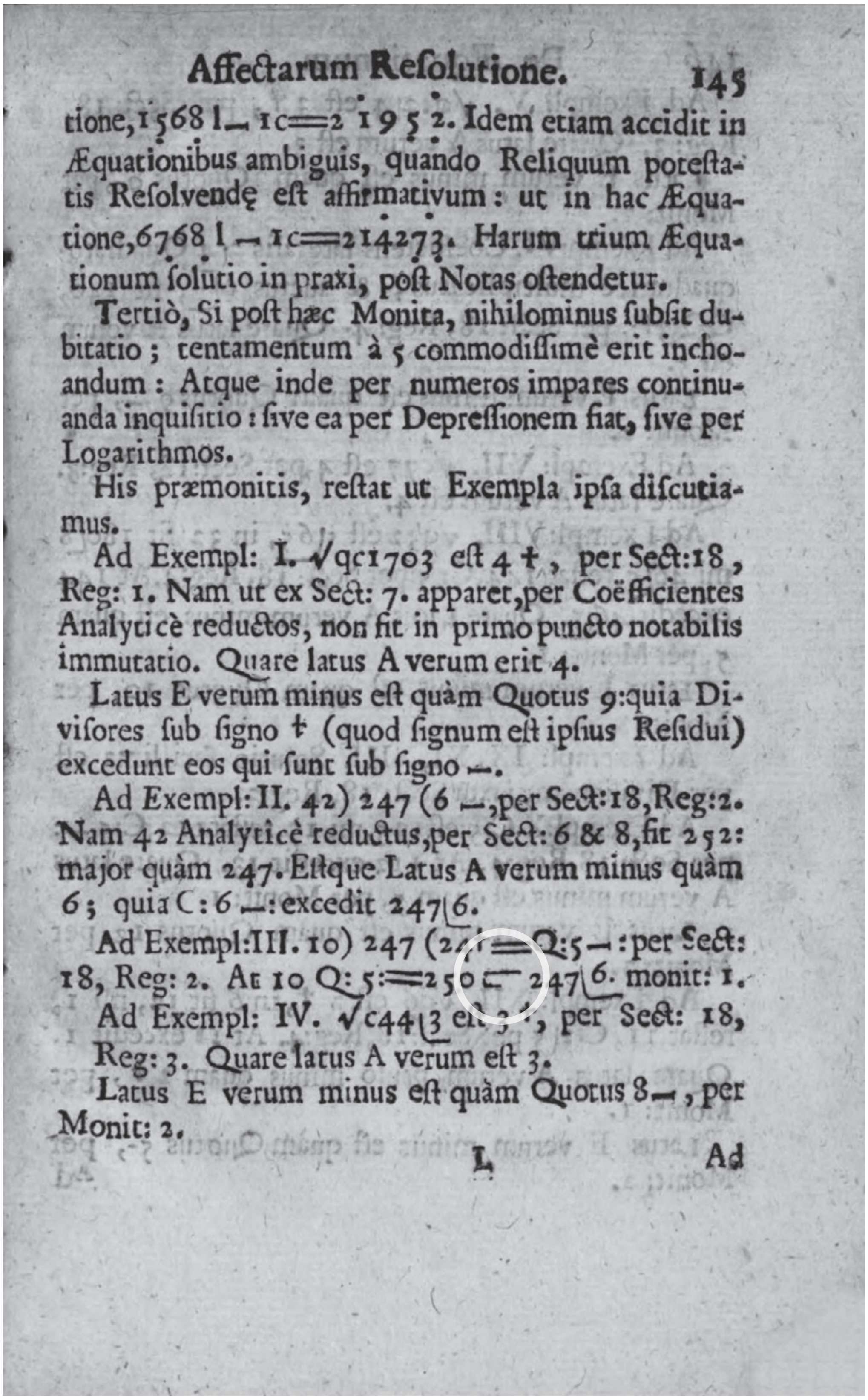
Si duo triangula A B C, D E F *habuerint duo latera* A B, A C *duobus lateribus* D E, D F, *utrumque utrique æqualia; habuerint vero & baſim* B C, *baſi* E F, *æqualem: angulum* A *ſub æqualibus rectis lineis contentum angulo* D *æqualem habebunt.*

Quia B C *a* ═ E F, ſi baſis B C ſuperponatur baſi EF, illæ *b* congruent. ergo, cum AB *c* ═ DE, & A C *c* ═ D F, cadet punctum A in D. (nam in aliud punctum cadere nequit, per præcedentem) ergo angulorum A, & D latera coincidunt. *d* quare anguli illi pares ſunt. Q. E. D.

Coroll.

1. Hinc triangula ſibi mutuo æquilatera, etiam mutuo *x* æquiangula ſunt.
2. Triangula ſibi mutuo æquilatera *y* æquentur inter ſe.

PROP.



Affectarum Reſolutione. 145

tione, 1568 l — 1c ═ 2195 2. Idem etiam accidit in Æquationibus ambiguis, quando Reliquum poteſtatis Reſolvendę eſt affirmativum: ut in hac Æquatione, 6768 l — 1c ═ 214273. Harum trium Æquationum ſolutio in praxi, poſt Notas oſtendetur.

Tertiò, Si poſt hæc Monita, nihilominus ſubſit dubitatio; tentamentum à 5 commodiſſimè erit inchoandum: Atque inde per numeros impares continuanda inquiſitio: ſive ea per Depreſſionem fiat, ſive per Logarithmos.

His præmonitis, reſtat ut Exempla ipſa diſcutiamus.

Ad Exempl: I. √qc1703 eſt 4 +, per Sect:18, Reg: 1. Nam ut ex Sect: 7. apparet, per Coëfficientes Analyticè reductos, non fit in primo puncto notabilis immutatio. Quare latus A verum erit 4.

Latus E verum minus eſt quàm Quotus 9: quia Diviſores ſub ſigno + (quod ſignum eſt ipſius Reſidui) excedunt eos qui ſunt ſub ſigno —.

Ad Exempl: II. 42) 247 (6 —, per Sect:18, Reg:2. Nam 42 Analyticè reductus, per Sect: 6 & 8, fit 252: major quàm 247. Eſtque Latus A verum minus quàm 6; quia C: 6 —: excedit 247|6.

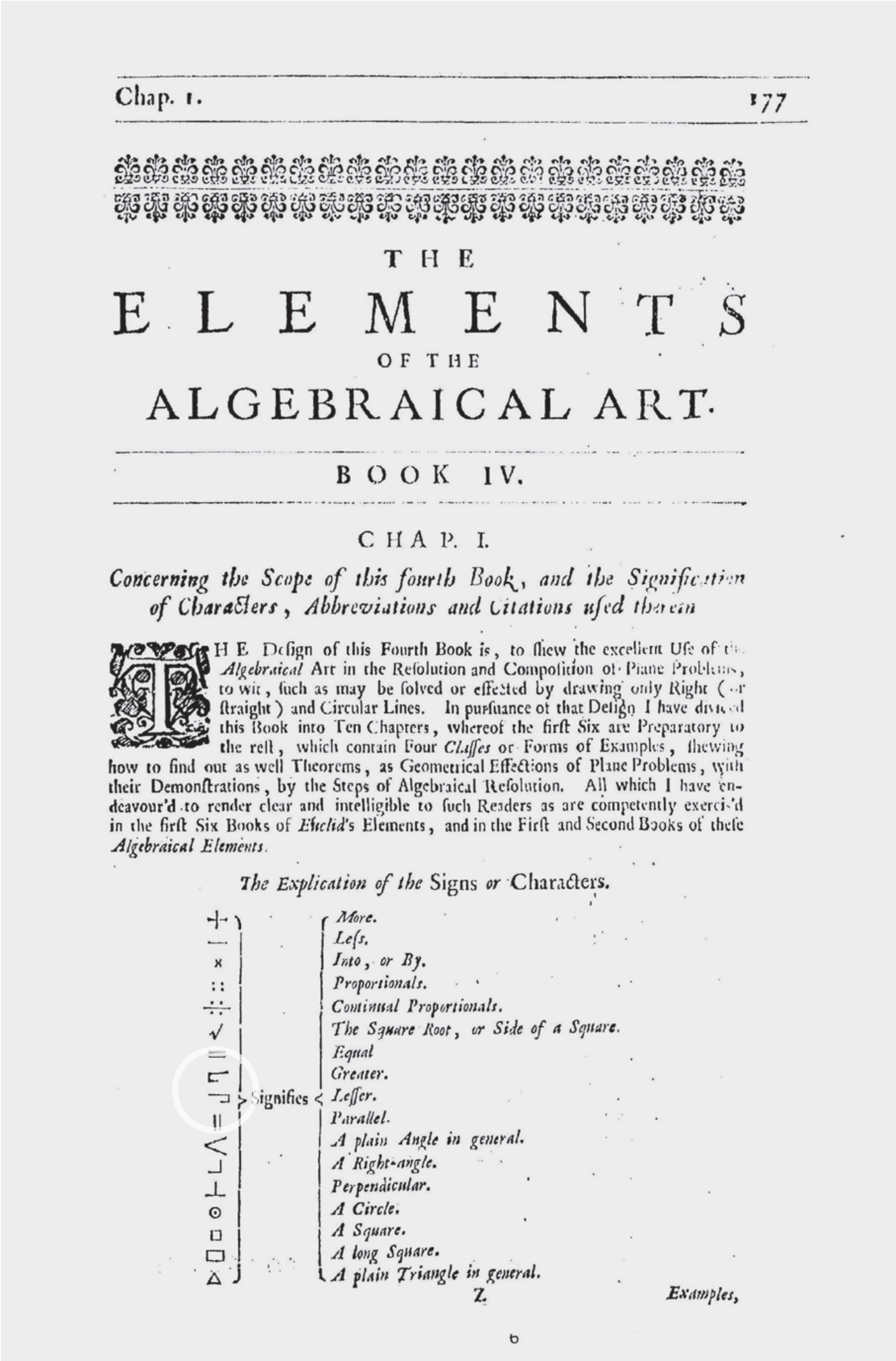
Ad Exempl: III. 10) 247 (24 ═ Q:5 —: per Sect: 18, Reg: 2. At 10 Q: 5: ═ 250 ⊏ 247|6. monit: 1.

Ad Exempl: IV. √c44|3 eſt 3 +, per Sect: 18, Reg: 3. Quare latus A verum eſt 3.

Latus E verum minus eſt quàm Quotus 8 —, per Monit: 2.

L Ad

⊏ RECTANGULAR GREATER OPEN RIGHT
William Oughtred 1667



THE

ELEMENTS

OF THE

ALGEBRAICAL ART.

BOOK IV.

CHAP. I.

Concerning the Scope of this fourth Book, and the Signification of Characters, Abbreviations and Citations used therein

THE Design of this Fourth Book is, to shew the excellent Use of the *Algebraical* Art in the Resolution and Composition of Plane Problems, to wit, such as may be solved or effected by drawing only Right (or straight) and Circular Lines. In pursuance of that Design I have divided this Book into Ten Chapters, whereof the first Six are Preparatory to the rest, which contain Four *Classes* or Forms of Examples, shewing how to find out as well Theorems, as Geometrical Effections of Plane Problems, with their Demonstrations, by the Steps of Algebraical Resolution. All which I have endeavour'd to render clear and intelligible to such Readers as are competently exercis'd in the first Six Books of *Euclid*'s Elements, and in the First and Second Books of these *Algebraical Elements*.

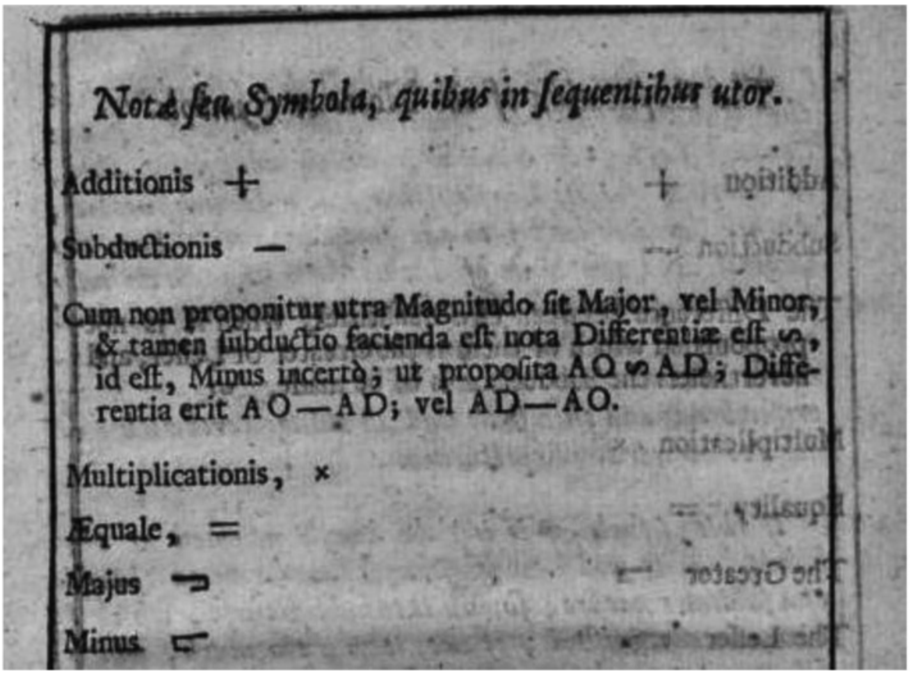
The Explication of the Signs *or* Characters.

Sign		Signification
+	Signifies	*More.*
—		*Less.*
×		*Into, or By.*
∷		*Proportionals.*
∺		*Continual Proportionals.*
√		*The Square Root, or Side of a Square.*
=		*Equal*
⊏		*Greater.*
⊐		*Lesser.*
‖		*Parallel.*
∠		*A plain Angle in general.*
⌐		*A Right-angle.*
⊥		*Perpendicular.*
⊙		*A Circle.*
□		*A Square.*
▭		*A long Square.*
△		*A plain Triangle in general.*

Z *Examples,*

⊏ RECTANGULAR GREATER OPEN RIGHT; ⊐ RECTANGULAR GREATER OPEN LEFT is used here for “lesser”, unlike in other sources. Kersey 1674

The use of the rectangular symbols for *greater* and *less* varies from one source to another. In this 1684 London edition of Thomas Baker's *Clavis Geometrica* ⊐ RECTANGULAR GREATER OPEN LEFT is used for *the greater,* but ⊏ RECTANGULAR GREATER OPEN RIGHT represents *the lesser.* Source: Google books



Notæ ſeu Symbola, quibus in ſequentibus utor.

Additionis +

Subductionis —

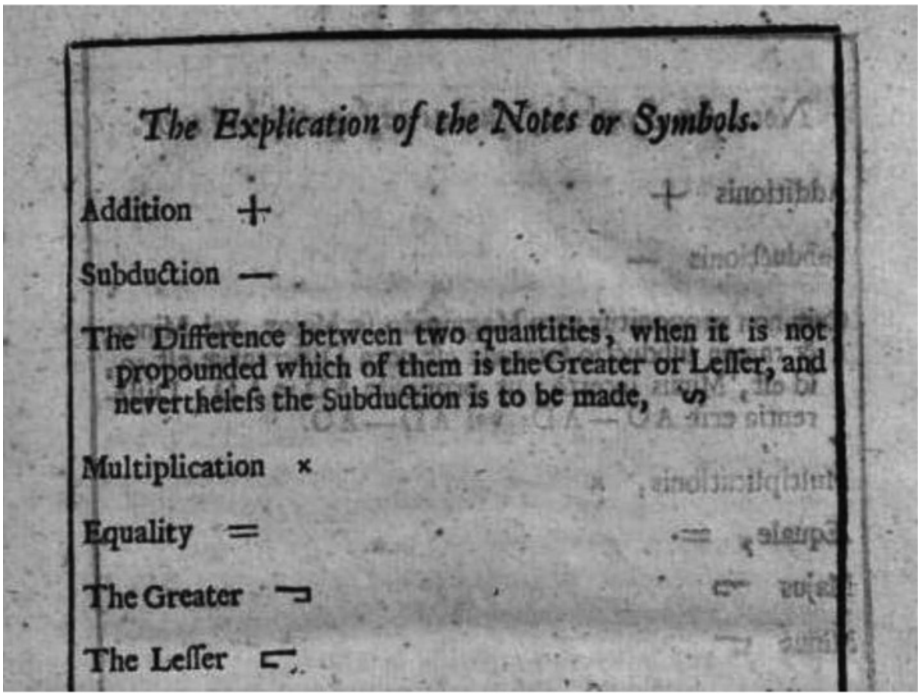
Cum non proponitur utra Magnitudo ſit Major, vel Minor, & tamen ſubductio facienda eſt nota Differentiæ eſt ∽, id eſt, Minus incertò; ut propoſita AO ∽ AD; Differentia erit AO—AD; vel AD—AO.

Multiplicationis, ×

Æquale, =

Majus ⊐

Minus ⊏



The Explication of the Notes or Symbols.

Addition +

Subduction —

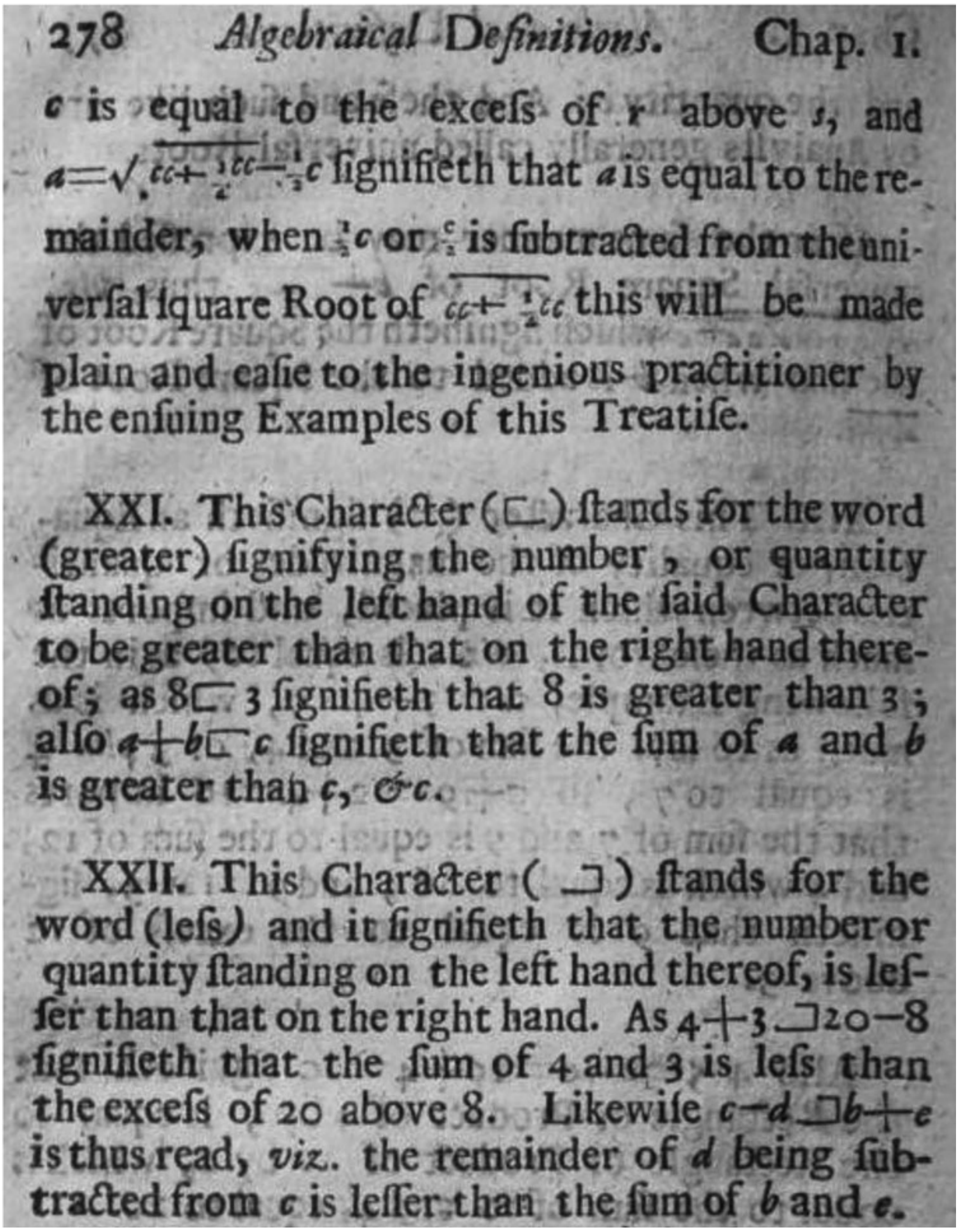
The Difference between two quantities, when it is not propounded which of them is the Greater or Leſſer, and neverthelefs the Subduction is to be made, ∽

Multiplication ×

Equality =

The Greater ⊐

The Leſſer ⊏.

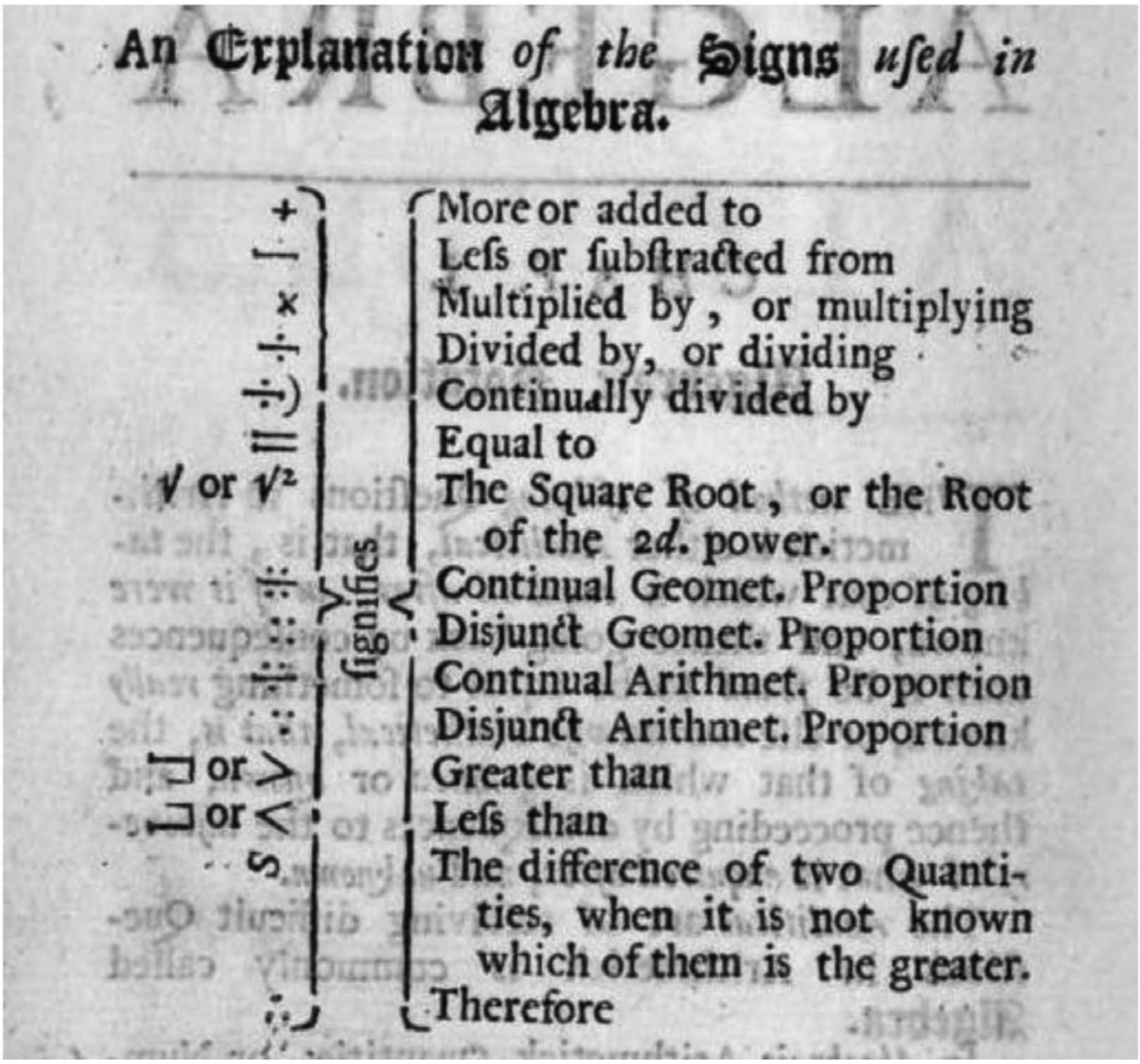


c is equal to the exceſs of *r* above *s*, and *a*=√ *cc*+¼*cc*−½*c* ſignifieth that *a* is equal to the remainder, when ½*c* or *c*/2 is ſubtracted from the univerſal ſquare Root of *cc*+¼*cc* this will be made plain and eaſie to the ingenious practitioner by the enſuing Examples of this Treatiſe.

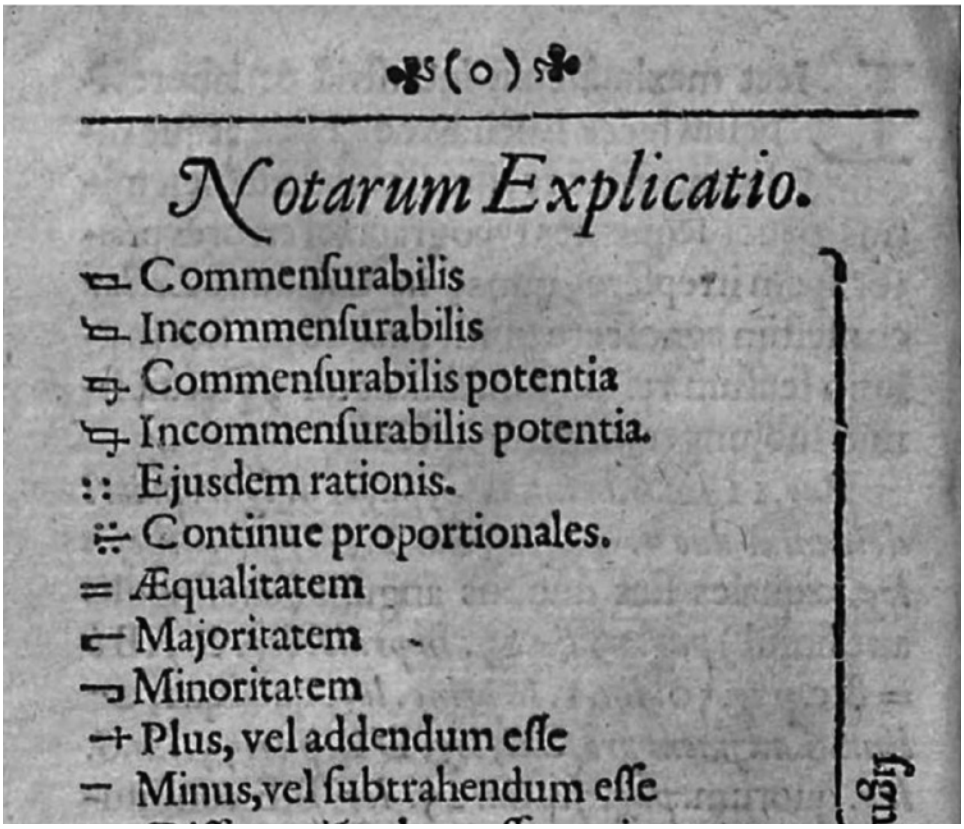
XXI. This Character (⊏) ſtands for the word (greater) ſignifying the number, or quantity ſtanding on the left hand of the ſaid Character to be greater than that on the right hand thereof; as 8⊏ 3 ſignifieth that 8 is greater than 3; alſo *a*+*b*⊏ *c* ſignifieth that the ſum of *a* and *b* is greater than *c*, &c.

XXII. This Character (⊐) ſtands for the word (leſs) and it ſignifieth that the number or quantity ſtanding on the left hand thereof, is leſſer than that on the right hand. As 4+3⊐20−8 ſignifieth that the ſum of 4 and 3 is leſs than the exceſs of 20 above 8. Likewiſe *c*−*d*⊐*b*+*e* is thus read, *viz.* the remainder of *d* being ſubtracted from *c* is leſſer than the ſum of *b* and *e*.

In this 1685 edition of Edward Cocker's *Decimal Arithmetick* both ⊏ RECTANGULAR LESS OPEN RIGHT and ⊏ RECTANGULAR GREATER OPEN RIGHT are used to denote *greater*, whereas ⊐ RECTANGULAR LESS OPEN LEFT stands for *less*. Source: Google books

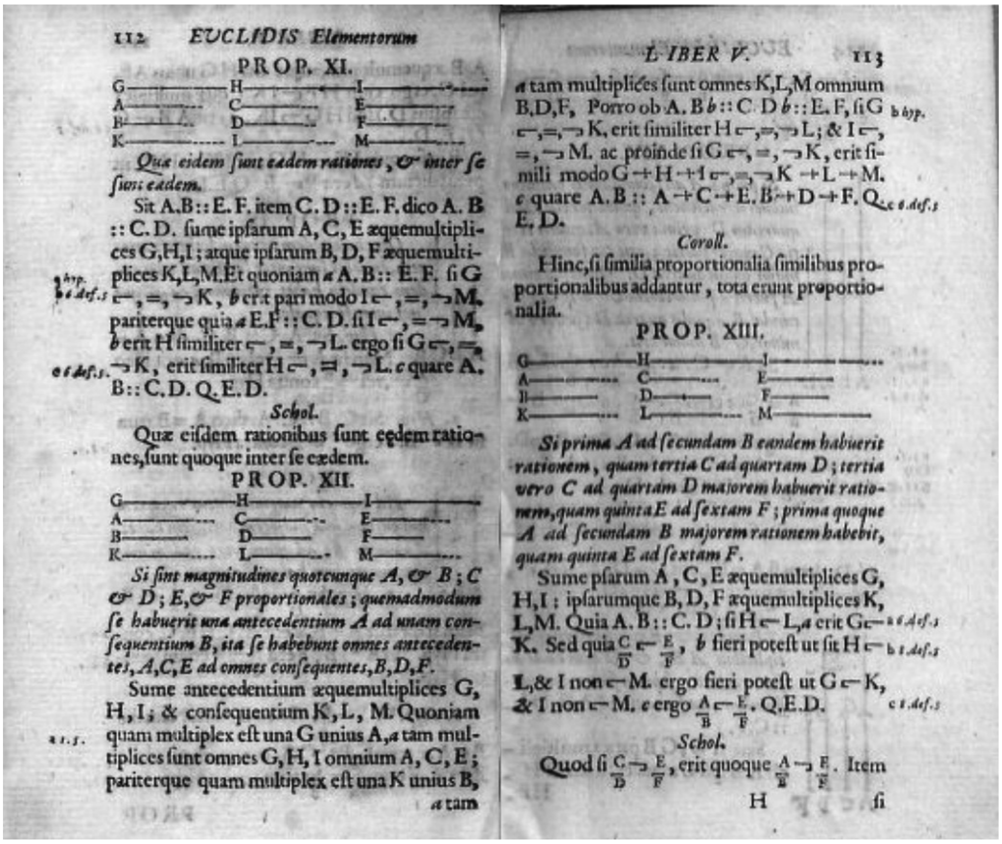


⊐ RECTANGULAR GREATER OPEN LEFT for *greater than,* ⊐ RECTANGULAR LESS OPEN LEFT for *less than*. John Parsons, Thomas Wastell: Clavis Arithmeticae, 1705. Source: Google books



⊏⊐ COMMENSURABILITY, ⊏⊐ INCOMMENSURABILITY, ⊏⊐ COMMENSURABILITY IN SQUARE, ⊏⊐ INCOMMENSURABILITY IN SQUARE; ⊏— RECTANGULAR GREATER OPEN RIGHT, —⊐ RECTANGULAR GREATER OPEN LEFT

Barrow 1676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PROP. XI.

G H I / A C E / B D F / K L M

Quæ eidem ſunt eadem rationes, & inter ſe ſunt eadem.

Sit A.B :: E. F. item C. D :: E. F. dico A. B :: C. D. ſume ipſarum A, C, E æquemultiplices G,H,I; atque ipſarum B, D, F æquemultiplices K,L,M. Et quoniam *a* A. B :: E. F. ſi G ⊏—, =, —⊐ K, *b* erit pari modo I ⊏—, =, —⊐ M. pariterque quia *a* E.F :: C. D. ſi I ⊏—, = —⊐ M, *b* erit H ſimiliter ⊏—, =, —⊐ L. ergo ſi G ⊏—, =, —⊐ K, erit ſimiliter H ⊏—, =, —⊐ L. *c* quare A. B :: C. D. Q. E. D. (a hyp. b 6 def. 5; c 6 def. 5.)

Schol.

Quæ eiſdem rationibus ſunt eædem rationes, ſunt quoque inter ſe eædem.

PROP. XII.

G H I / A C E / B D F / K L M

Si ſint magnitudines quotcunque A, & B; C & D; E, & F proportionales; quemadmodum ſe habuerit una antecedentium A ad unam conſequentium B, ita ſe habebunt omnes antecedentes, A, C, E ad omnes conſequentes, B, D, F.

Sume antecedentium æquemultiplices G, H, I; & conſequentium K, L, M. Quoniam quam multiplex eſt una G unius A, *a* tam multiplices ſunt omnes G, H, I omnium A, C, E; pariterque quam multiplex eſt una K unius B, (a 1. 5.)

a tam

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a tam multiplices ſunt omnes K,L,M omnium B,D,F, Porro ob A. B *b* :: C. D *b* :: E. F, ſi G ⊏—, =, —⊐ K, erit ſimiliter H ⊏—, =, —⊐ L; & I ⊏—, =, —⊐ M. ac proinde ſi G ⊏—, =, —⊐ K, erit ſimili modo G -+ H -+ I ⊏—, =, —⊐ K -+ L -+ M. *c* quare A. B :: A -+ C -+ E. B -+ D -+ F. Q. E. D. (b hyp. c 6. def. 5)

Coroll.

Hinc, ſi ſimilia proportionalia ſimilibus proportionalibus addantur, tota erunt proportionalia.

PROP. XIII.

G H I / A C E / B D F / K L M

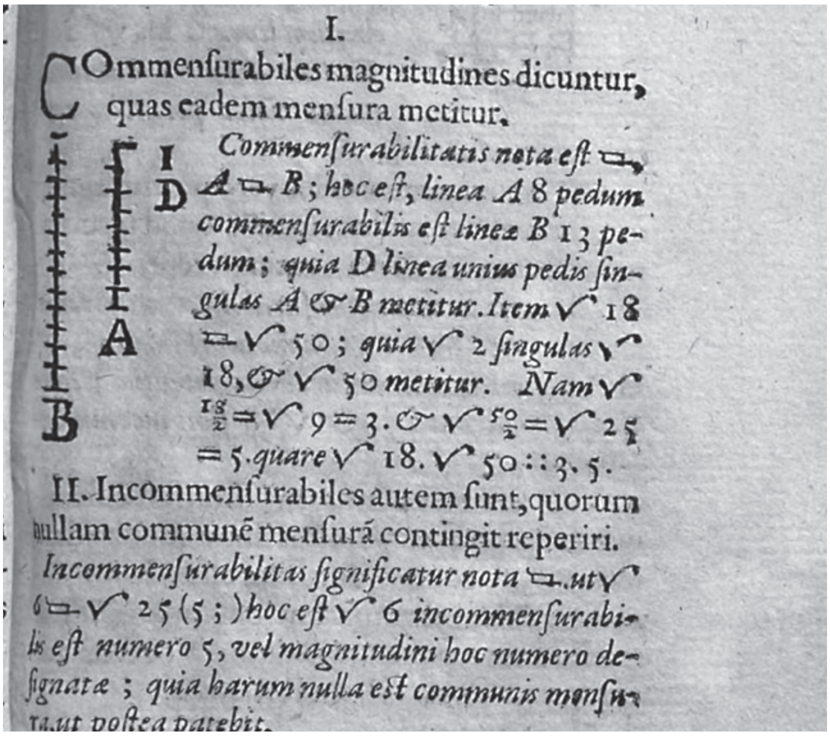
Si prima A ad ſecundam B eandem habuerit rationem, quam tertia C ad quartam D; tertia vero C ad quartam D majorem habuerit rationem, quam quinta E ad ſextam F; prima quoque A ad ſecundam B majorem rationem habebit, quam quinta E ad ſextam F.

Sume pſarum A, C, E æquemultiplices G, H, I: ipſarumque B, D, F æquemultiplices K, L, M. Quia A. B :: C. D; ſi H ⊏— L, *a* erit G ⊏— K. Sed quia $\frac{C}{D}$ ⊏— $\frac{E}{F}$, *b* fieri poteſt ut ſit H ⊏— L, & I non ⊏— M. ergo fieri poteſt ut G ⊏— K, & I non ⊏— M. *c* ergo $\frac{A}{B}$ ⊏— $\frac{E}{F}$. Q. E. D. (a 6 def. 5; b 8 def. 5; c 8 def. 5)

Schol.

Quod ſi $\frac{C}{D}$ —⊐ $\frac{E}{F}$, erit quoque $\frac{A}{B}$ —⊐ $\frac{E}{F}$. Item

H ſi


I.

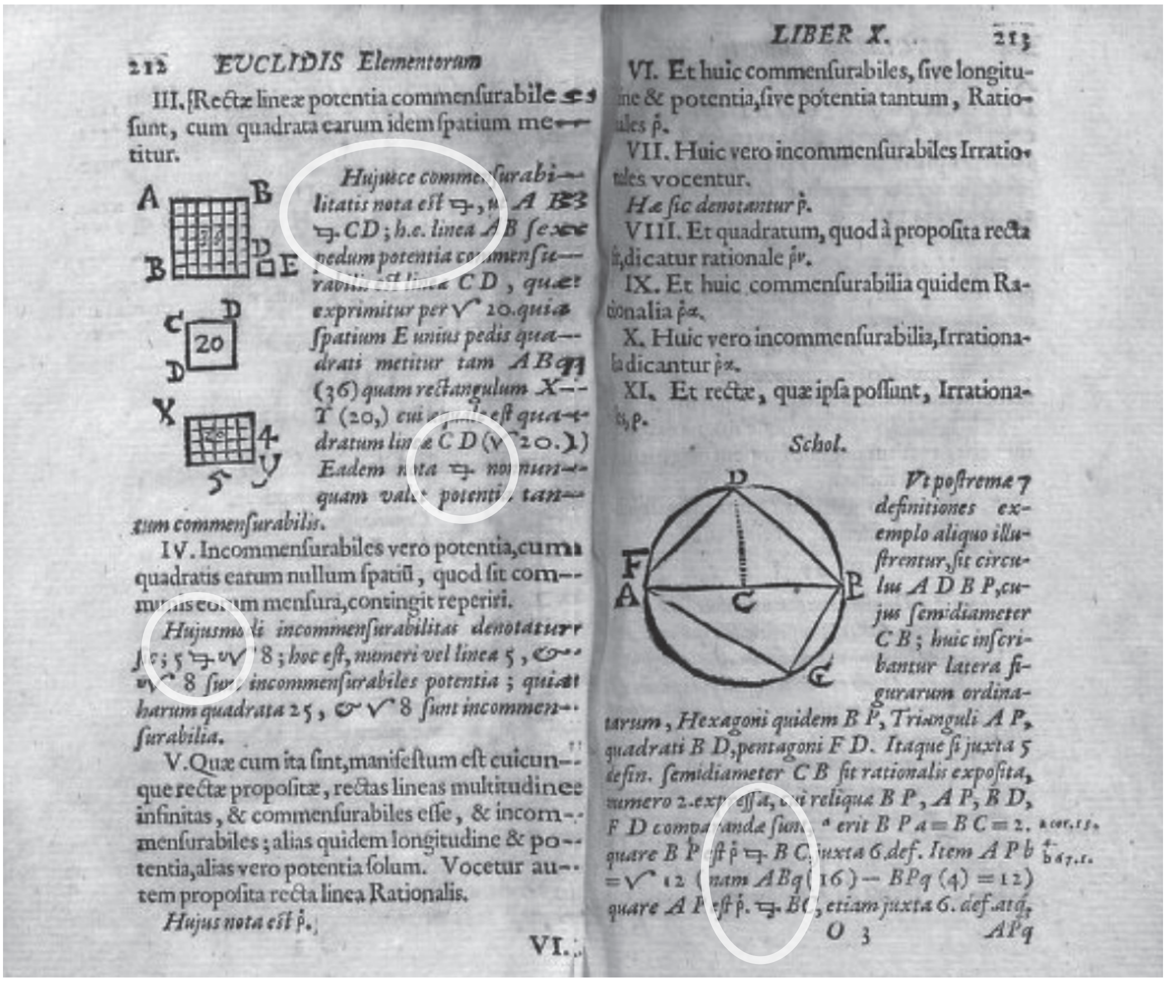
Commensurabiles magnitudines dicuntur, quas eadem mensura metitur.

Commensurabilitatis nota est ⊏, *A* ⊏ *B; hoc est, linea A 8 pedum commensurabilis est lineæ B 13 pedum; quia D linea unius pedis singulas A & B metitur. Item* $\sqrt{18}$ ⊏ $\sqrt{50}$; *quia* $\sqrt{2}$ *singulas* $\sqrt{18}$, & $\sqrt{50}$ *metitur. Nam* $\sqrt{\frac{18}{2}} = \sqrt{9} = 3$. & $\sqrt{\frac{50}{2}} = \sqrt{25} = 5$. *quare* $\sqrt{18}$. $\sqrt{50}$ :: 3. 5.

II. Incommensurabiles autem sunt, quorum nullam communē mensurā contingit reperiri.

Incommensurabilitas significatur nota ⊏. *ut* $\sqrt{6}$ ⊏ $\sqrt{25}$ (5;) *hoc est* $\sqrt{6}$ *incommensurabilis est numero 5, vel magnitudini hoc numero designatæ; quia harum nulla est communis mensura, ut postea patebit.*

COMMENSURABILITY, INCOMMENSURABILITY, COMMENSURABILITY IN SQUARE, INCOMMENSURABILITY IN SQUARE
Barrow 1676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III. Rectæ lineæ potentia commensurabiles sunt, cum quadrata earum idem spatium metitur.

Hujusce commensurabilitatis nota est ⊏̣, *ut A B* ⊏̣ *CD; h.e. linea A B sex pedum potentia commensurabilis est lineæ C D, quæ exprimitur per* $\sqrt{20}$. *quia spatium E unius pedis quadrati metitur tam A Bq (36) quam rectangulum X T (20,) cui æquale est quadratum lineæ C D (*$\sqrt{20}$.*) Eadem nota* ⊏̣ *nonnunquam valet potentia tantum commensurabilis.*

IV. Incommensurabiles vero potentia, cum quadratis earum nullum spatiū, quod sit communis eorum mensurā, contingit reperiri.

Hujusmodi incommensurabilitas denotatur [illegible] *5* ⊏̣ $\sqrt{8}$; *hoc est, numeri vel linea 5, &* $\sqrt{8}$ *sunt incommensurabiles potentia; quia harum quadrata 25, &* $\sqrt{8}$ *sunt incommensurabilia.*

V. Quæ cum ita sint, manifestum est cuicunque rectæ propositæ, rectas lineas multitudine infinitas, & commensurabiles esse, & incommensurabiles; alias quidem longitudine & potentia, alias vero potentia solum. Vocetur autem proposita recta linea Rationalis.

Hujus nota est ρ̇.

VI.

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VI. Et huic commensurabiles, sive longitudine & potentia, sive potentia tantum, Rationales ρ̇.

VII. Huic vero incommensurabiles Irrationales vocentur.

Hæ sic denotantur ρ̇.

VIII. Et quadratum, quod à proposita recta fit, dicatur rationale ρ̇ν.

IX. Et huic commensurabilia quidem Rationalia ρ̇α.

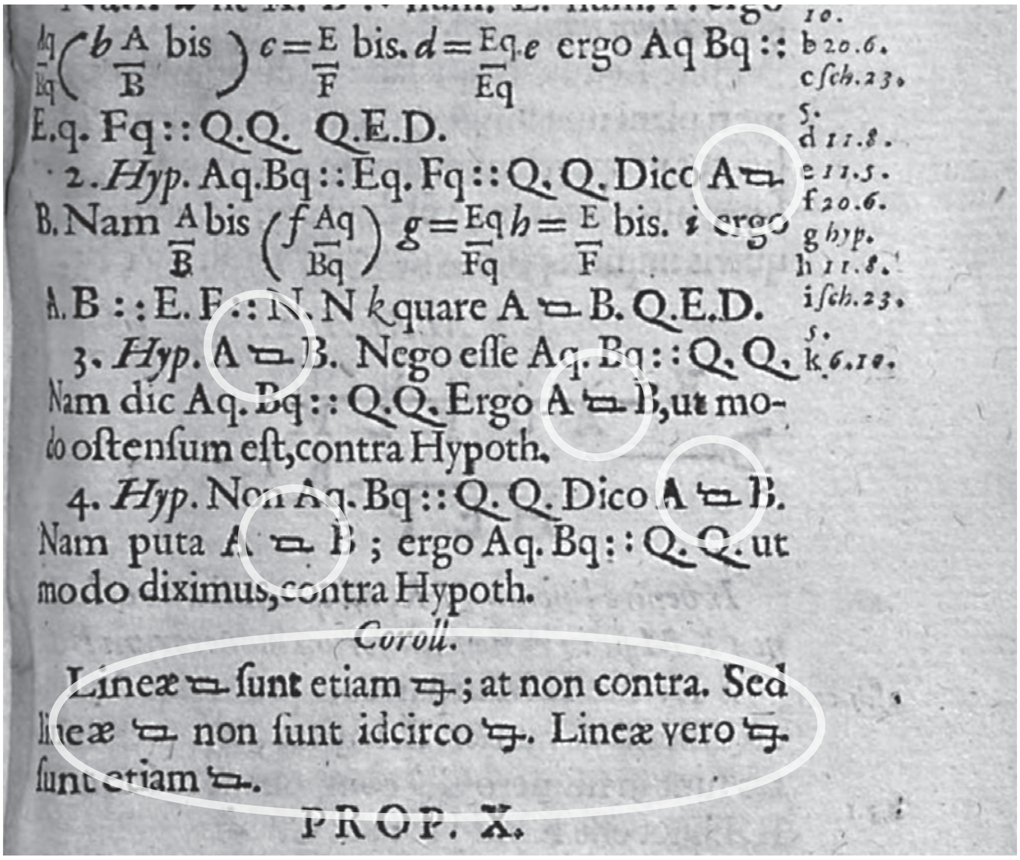
X. Huic vero incommensurabilia, Irrationalia dicantur ρ̇α.

XI. Et rectæ, quæ ipsa possunt, Irrationales, ρ̇.

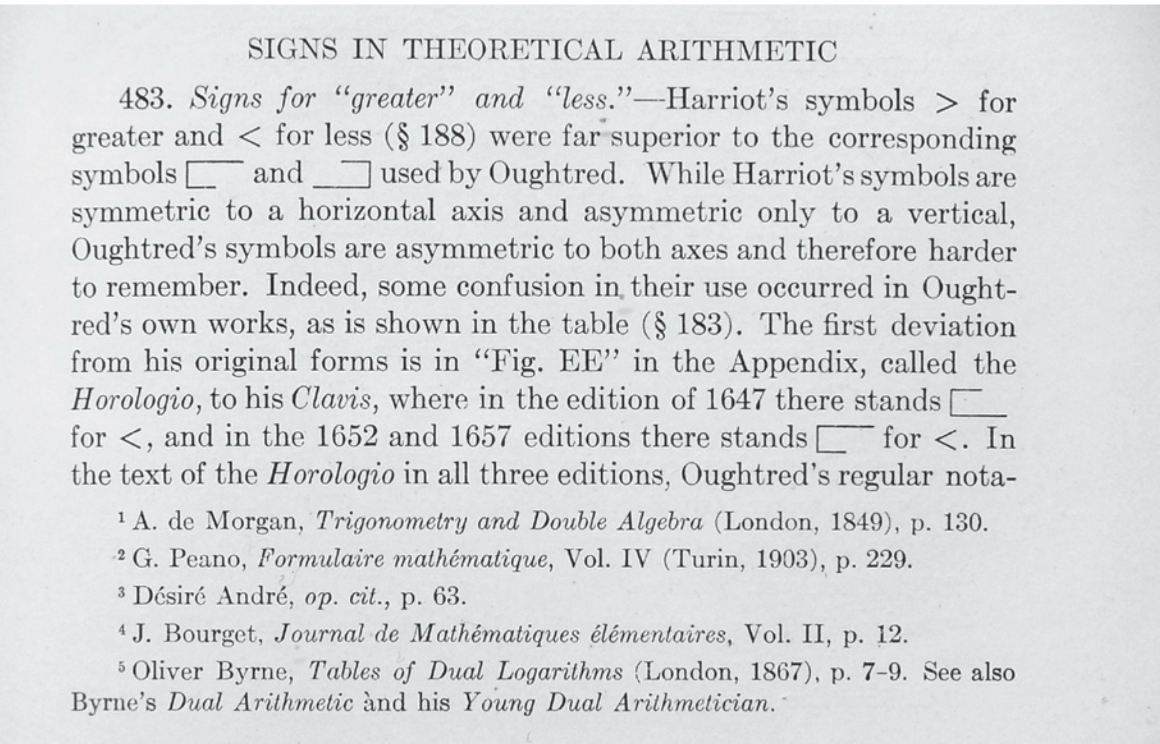
Schol.

Ut postremæ 7 definitiones exemplo aliquo illustrentur, sit circulus A D B P, cujus semidiameter C B; huic inscribantur latera figurarum ordinatarum, Hexagoni quidem B P, Trianguli A P, quadrati B D, pentagoni F D. Itaque si juxta 5 defin. semidiameter C B sit rationalis exposita, numero 2. expressa, cui reliqua B P, A P, B D, F D comparandæ sunt, erit B P a = B C = 2. quare B P est ρ̇ ⊏ *B C, juxta 6. def. Item A P b =* $\sqrt{12}$ *(nam A Bq (16) − BPq (4) = 12) quare A P est* ρ̇. ⊏̣ *B C, etiam juxta 6. def. atq;*

O 3 APq



COMMENSURABILITY, INCOMMENSURABILITY, COMMENSURABILITY IN SQUARE, INCOMMENSURABILITY IN SQUARE
Barrow 1676



SIGNS IN THEORETICAL ARITHMETIC

483. *Signs for "greater" and "less."*—Harriot's symbols > for greater and < for less (§ 188) were far superior to the corresponding symbols and used by Oughtred. While Harriot's symbols are symmetric to a horizontal axis and asymmetric only to a vertical, Oughtred's symbols are asymmetric to both axes and therefore harder to remember. Indeed, some confusion in their use occurred in Oughtred's own works, as is shown in the table (§ 183). The first deviation from his original forms is in "Fig. EE" in the Appendix, called the *Horologio*, to his *Clavis*, where in the edition of 1647 there stands for <, and in the 1652 and 1657 editions there stands for <. In the text of the *Horologio* in all three editions, Oughtred's regular nota-

[1] A. de Morgan, *Trigonometry and Double Algebra* (London, 1849), p. 130.
[2] G. Peano, *Formulaire mathématique*, Vol. IV (Turin, 1903), p. 229.
[3] Désiré André, *op. cit.*, p. 63.
[4] J. Bourget, *Journal de Mathématiques élémentaires*, Vol. II, p. 12.
[5] Oliver Byrne, *Tables of Dual Logarithms* (London, 1867), p. 7–9. See also Byrne's *Dual Arithmetic* and his *Young Dual Arithmetician*.

RECTANGULAR GREATER OPEN RIGHT, RECTANGULAR LESS OPEN RIGHT and RECTANGULAR LESS OPEN LEFT. Cajori vol. II p. 115 (1928) tion is adhered to. Isaac Barrow used ⊏ for "majus" and ⊐ for "minus" in his *Euclidis Data* (Cambridge, 1657), page 1, and also in his *Euclid's Elements* (London, 1660), Preface, as do also John Kersey,[1] Richard Sault,[2] and Roger Cotes.[3] In one place John Wallis[4] writes ⊐ for >, ⊐ for <.

Seth Ward, another pupil of Oughtred, writes in his *In Ismaelis Bullialdi astronomiae philolaicae fundamenta inquisitio brevis* (Oxoniae, 1653), page 1, ⊏ for "majus" and ⊏ for "minus." For further notices of discrepancy in the use of these symbols, see *Bibliotheca mathematica*, Volume XII[3] (1911–12), page 64. Harriot's > and < easily won out over Oughtred's notation. Wallis follows Harriot almost exclusively; so do Gibson[5] and Brancker.[6] Richard Rawlinson of Oxford used ⊐ for greater and ⊏ for less (§ 193). This notation is used also by Thomas Baker[7] in 1684, while E. Cocker[8] prefers ⊏ for ⊐. In the arithmetic of S. Jeake,[9] who gives "⊏ greater, ⊏. next greater, ⊐. lesser, ⊐. next lesser, ⊏ not greater, ⊐. not lesser, ⊏. equal or less, ⊐. equal or greater," there is close adherence to Oughtred's original symbols.

Ronayne[10] writes in his *Algebra* ⊏ for "greater than," and ⊐ for "less than." As late as 1808, S. Webber[11] says: ". . . . . we write a ⊏ b, or $a \mathbin{7} b$; . . . . a ⊐ b, or $a<b$." In Isaac Newton's *De Analysi per Aequationes*, as printed in the *Commercium Epistolicum* of 1712, page 20, there occurs x ⊐ $\frac{1}{2}$, probably for $x<\frac{1}{2}$; apparently, Newton used here the symbolism of his teacher, I. Barrow, but in Newton's *Opuscula* (Castillion's ed., 1744) and in Lefort's *Commercium Epistolicum* (1856), page 74, the symbol is interpreted as meaning $x>\frac{1}{2}$. Eneström[12]

[1] John Kersey, *Elements of Algebra* (London, 1674), Book IV, p. 177.

[2] Richard Sault, *A New Treatise of Algebra* (London, n.d.).

[3] Roger Cotes, *Harmonia mensurarum* (Cambridge, 1722), p. 115.

[4] John Wallis, *Algebra* (1685), p. 127.

[5] Thomas Gibson, *Syntaxis mathematica* (London, 1655), p. 246.

[6] Thomas Brancker, *Introduction to Algebra* (trans. of Rahn's *Algebra;* London, 1668), p. 76.

[7] Thomas Baker, *Clavis geometrica* (London, 1684), fol. *d* 2 *a*.

[8] Edward Crocker, *Artificial Arithmetick* (London, 1684), p. 278.

[9] Samuel Jeake, Sr., ΛΟΓΙΣΤΙΚΗΛΟΓΙ'Α *or Arithmetick* (London, 1696), p. 12

[10] Philip Ronayne, *Treatise of Algebra* (London, 1727), p. 3.

[11] Samuel Webber, *Mathematics*, Vol. I (Cambridge, Mass., 1808; 2d ed.), p. 233.

[12] G. Eneström, *Bibliotheca mathematica* (3d ser.), Vol. XII (1911–12), p. 74

Cajori vol. II p. 116. In this chapter Cajori discusses the differing use cases of the rectangular symbols for *greater* and *less* in the works of various authors.

argues that Newton followed his teacher Barrow in the use of ⊐ and actually took $x<\frac{1}{2}$, as is demanded by the reasoning.

In E. Stone's *New Mathematical Dictionary* (London, 1726), article "Characters," one finds ⊏ or ⊐ for "greater" and ⊐ or ⊏ for "less." In the Italian translation (1800) of the mathematical part of Diderot's *Encyclopédie*, article "Carattere," the symbols are further modified, so that ⊏ and ⌉ stand for "greater than," ⊐ for "less than"; and the remark is added, "but today they are no longer used."

Brook Taylor[1] employed ⊐ and ⊏ for "greater" and "less," respectively, while E. Hatton[2] in 1721 used ⊏ and ⊐, and also > and <. The original symbols of Oughtred are used in Colin Maclaurin's *Algebra*.[3] It is curious that as late as 1821, in an edition of Thomas Simpson's *Elements of Geometry* (London), pages 40, 42, one finds ⊏ for > and ⊐ for <.

The inferiority of Oughtred's symbols and the superiority of Harriot's symbols for "greater" and "less" are shown nowhere so strongly as in the confusion which arose in the use of the former and the lack of confusion in employing the latter. The burden cast upon the memory by Oughtred's symbols was even greater than that of double asymmetry; there was difficulty in remembering the distinction between the symbol ⊏ and the symbol ⊏. It is not strange that Oughtred's greatest admirers—John Wallis and Isaac Borrow—differed not only from Oughtred, but also from each other, in the use of these symbols. Perhaps nowhere is there another such a fine example of symbols ill chosen and symbols well chosen. Yet even in the case of Harriot's symbolism, there is on record at least one strange instance of perversion. John Frend[4] defined < as "greater than" and > as "less than."

484. *Sporadic symbols for "greater" or "less."*—A symbol constructed on a similar plan to Oughtred's was employed by Leibniz[5] in 1710, namely, "*a* ⫣ significat *a* esse majus quam *b*, et *a* ⫤ significat *a* esse minus quam *b*." Leibniz borrowed these signs from his teacher Erhard Weigel,[6] who used them in 1693. In the 1749 edition of the *Miscellanea Berolinensia* from which we now quote, these inequality

[1] Brook Taylor, *Phil. Trans.*, Vol. XXX (1717–19), p. 961.

[2] Edward Hatton, *Intire System of Arithmetic* (London, 1721), p. 287.

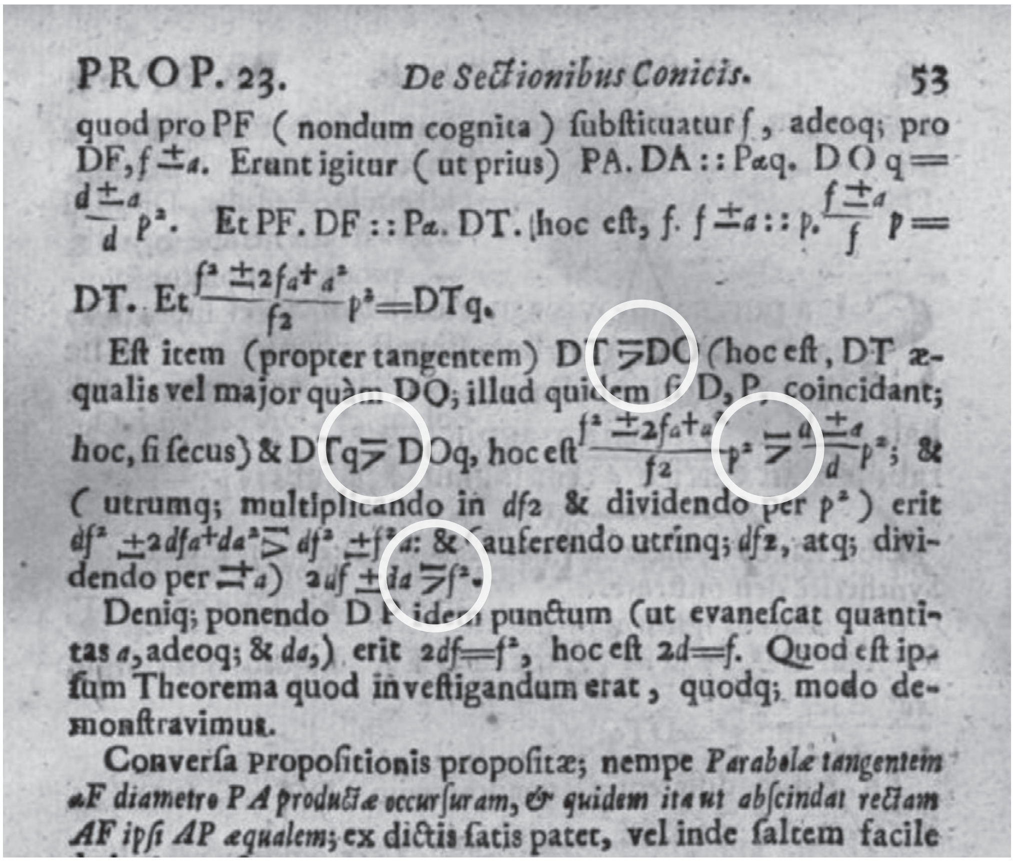
[3] Colin Maclaurin, *A Treatise of Algebra* (3d ed.; London, 1771).

[4] John Frend, *Principles of Algebra* (London, 1796), p. 3.

[5] *Miscellanea Berolinensia* (Berlin, 1710), p. 158.

[6] *Erhardi Weigelii Philosophia mathematica* (Jenae, 1693), p. 135.

Cajori vol. II p. 117. In this chapter Cajori discusses the differing use cases of the rectangular symbols for *greater* and *less* in the works of various authors.


PROP. 23. *De Sectionibus Conicis.* 53

quod pro PF (nondum cognita) ſubſtituatur ſ, adeoq; pro DF, ſ ± a. Erunt igitur (ut prius) PA. DA :: Paq. DOq = $\frac{d \pm a}{d} p^2$. Et PF. DF :: Pa. DT. (hoc eſt, ſ. ſ ± a :: p. $\frac{f \pm a}{f} p$ = DT. Et $\frac{f^2 \pm 2fa + a^2}{f2} p^2$ = DTq.

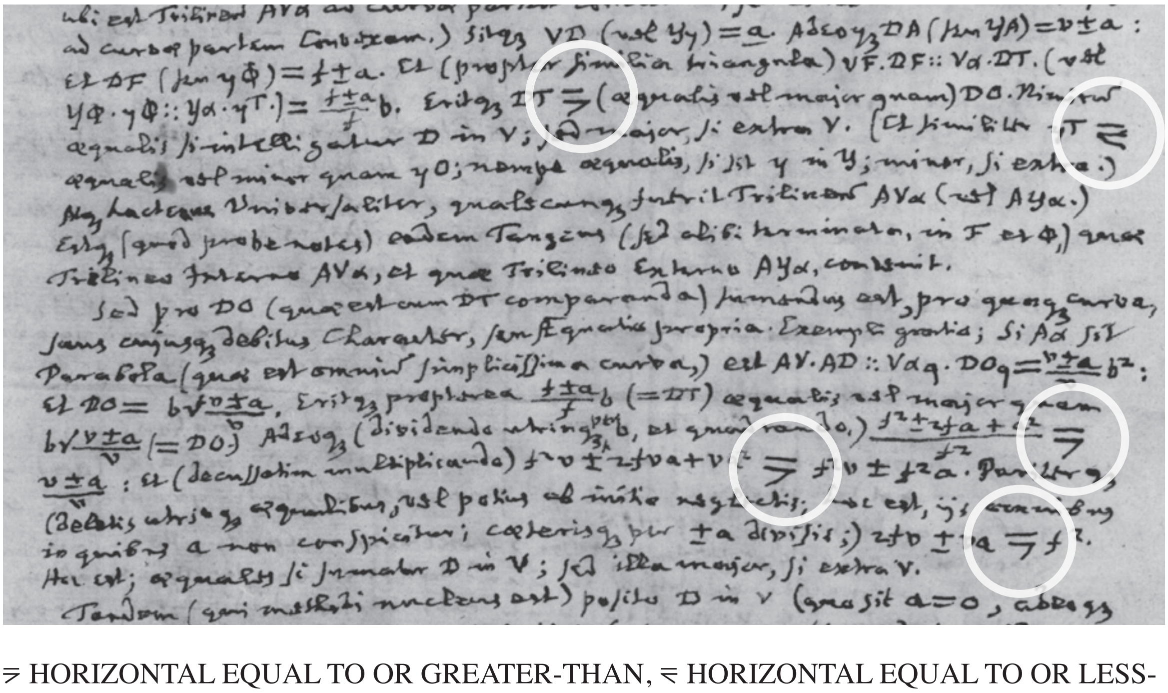
Eſt item (propter tangentem) DT ⩱ DO (hoc eſt, DT æqualis vel major quàm DO; illud quidem ſi D, P, coincidant; hoc, ſi ſecus) & DTq ⩱ DOq, hoc eſt $\frac{f^2 \pm 2fa + a^2}{f2} p^2$ ⩱ $\frac{d \pm a}{d} p^2$; & (utrumq; multiplicando in df2 & dividendo per p^2) erit $df^2 \pm 2dfa + da^2$ ⩱ $df^2 \pm f^2a$: & auferendo utrinq; df2, atq; dividendo per ± a) $2df \pm da$ ⩱ f^2.

Deniq; ponendo D P idem punctum (ut evaneſcat quantitas a, adeoq; & da,) erit $2df = f^2$, hoc eſt $2d = f$. Quod eſt ipſum Theorema quod inveſtigandum erat, quodq; modo demonſtravimus.

Converſa propoſitionis propoſitæ; nempe *Parabolæ tangentem aF diametro PA productæ occurſuram, & quidem ita ut abſcindat rectam AF ipſi AP æqualem*; ex dictis ſatis patet, vel inde ſaltem facile

⩱ HORIZONTAL EQUAL TO OR GREATER-THAN
Wallis, De sectionibus conicis nova methodo expositis tractatus, 1655; p. 53

In these historic symbols for “lessequal” and “greaterequal” the “=” strokes are on top of the glyphs, whereas in the existing characters 29A4 and 29A5 they appear on the bottom of the glyphs. We reagard this a sufficient difference to disunify the two character pairs.



⩱ HORIZONTAL EQUAL TO OR GREATER-THAN, ⩰ HORIZONTAL EQUAL TO OR LESS-THAN. Manuscript of J. Wallis, LBr 974, 28v.

LIVRE PREMIER. 303

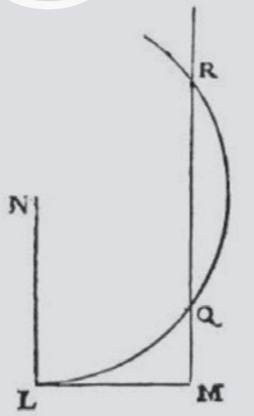
angle, iusques a O, en sorte qu'N O soit esgale a N L, la toute O M est z la ligne cherchée. Et elle s'exprime en cete sorte

z ꝏ $\frac{1}{2}a + \sqrt{\frac{1}{4}aa + bb}$.

Que si iay yy ꝏ $- ay + bb$, & qu'y soit la quantité qu'il faut trouuer, ie fais le mesme triangle rectangle N L M, & de sa baze M N i'oste N P esgale a N L, & le reste P M est y la racine cherchée. De façon que iay y ꝏ $- \frac{1}{2}a + \sqrt{\frac{1}{4}aa + bb}$. Et tout de mesme si i'auois x^4 ꝏ $- ax^2 + b^2$. P M seroit x^2. & i'aurois x ꝏ $\sqrt{-\frac{1}{2}a + \sqrt{\frac{1}{4}aa + bb}}$: & ainsi des autres.

Enfin si i'ay

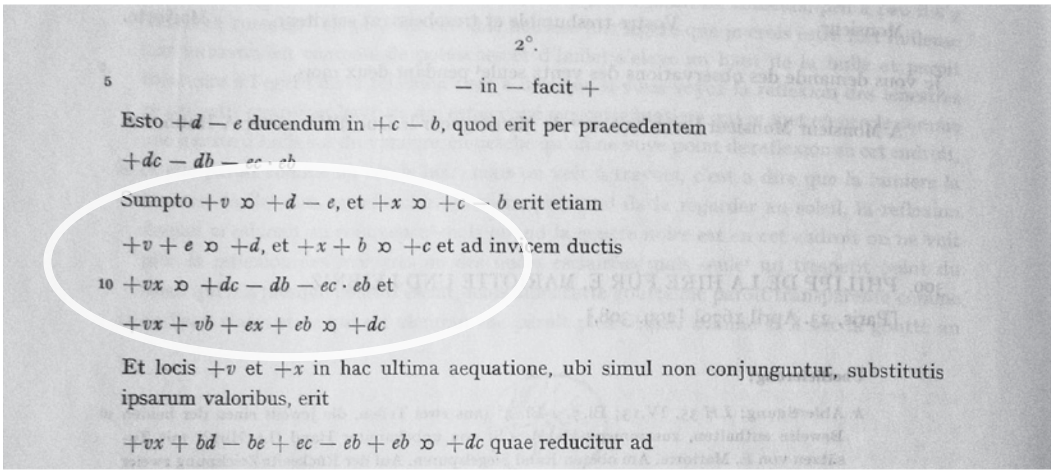
z^2 ꝏ $az - bb$:



ie fais N L esgale à $\frac{1}{2}a$, & L M esgale à b cõme deuãt, puis, au lieu de ioindre les poins M N, ie tire M Q R parallele a L N. & du centre N par L ayant descrit vn cercle qui la couppe aux poins Q & R, la ligne cherchée z est M Q, oubiẽ M R, car en ce cas elle s'exprime en deux façons, a sçauoir z ꝏ $\frac{1}{2}a + \sqrt{\frac{1}{4}aa - bb}$, & z ꝏ $\frac{1}{2}a - \sqrt{\frac{1}{4}aa - bb}$.

Et si le cercle, qui ayant son centre au point N, passe par le point L, ne couppe ny ne touche la ligne droite M Q R, il n'y a aucune racine en l'Equation, de façon qu'on peut assurer que la construction du problesme proposé est impossible.

ꝏ CARTESIAN EQUAL
Descartes, La Géométrie, 1637, p. 303
Here the type composer utilized a turned **œ** letter from which he carved off the horizontal bar of the **e**, as a makeshift for ꝏ. Rather than sticking to that desperate solution, we see ꝏ being graphically a rotated variant of 221D ∝ PROPORTIONAL TO.


2°.

— in — facit +

Esto $+d - e$ ducendum in $+c - b$, quod erit per praecedentem

$+dc - db - ec \cdot eb$

Sumpto $+v \propto +d - e$, et $+x \propto +c - b$ erit etiam

$+v + e \propto +d$, et $+x + b \propto +c$ et ad invicem ductis

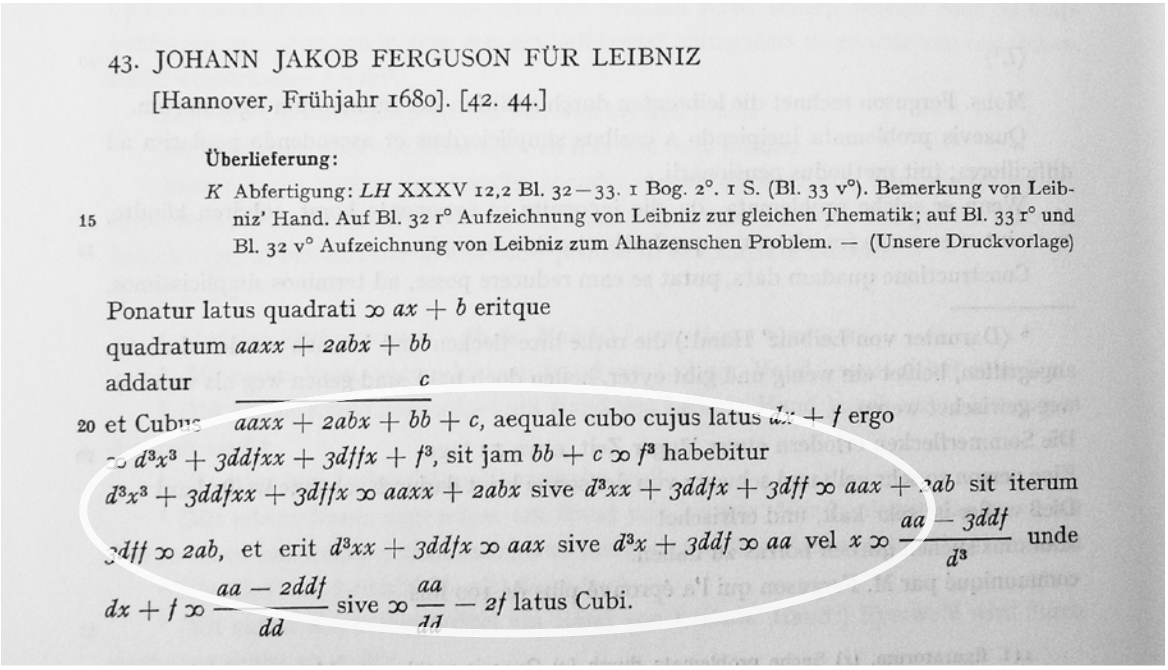
$+vx \propto +dc - db - ec \cdot eb$ et

$+vx + vb + ex + eb \propto +dc$

Et locis $+v$ et $+x$ in hac ultima aequatione, ubi simul non conjunguntur, substitutis ipsarum valoribus, erit

$+vx + bd - be + ec - eb + eb \propto +dc$ quae reducitur ad

$\propto$ CARTESIAN EQUAL
LAA III-2 p. 698. – Equal sign introduced and mainly used by René Descartes.


43. JOHANN JAKOB FERGUSON FÜR LEIBNIZ

[Hannover, Frühjahr 1680]. [42. 44.]

Überlieferung:

K Abfertigung: *LH* XXXV 12,2 Bl. 32–33. 1 Bog. 2°. 1 S. (Bl. 33 v°). Bemerkung von Leibniz' Hand. Auf Bl. 32 r° Aufzeichnung von Leibniz zur gleichen Thematik; auf Bl. 33 r° und Bl. 32 v° Aufzeichnung von Leibniz zum Alhazenschen Problem. — (Unsere Druckvorlage)

Ponatur latus quadrati $\propto ax + b$ eritque

quadratum $aaxx + 2abx + bb$

addatur c

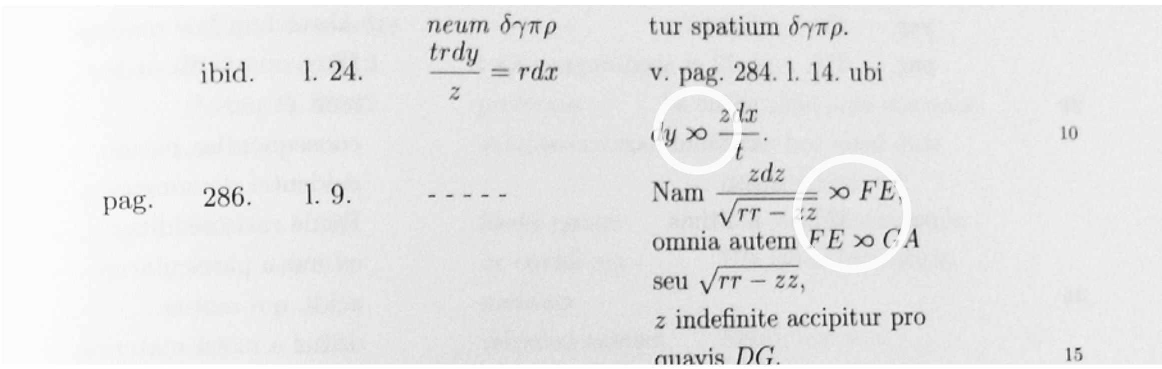
et Cubus $aaxx + 2abx + bb + c$, aequale cubo cujus latus $dx + f$ ergo

$\propto d^3x^3 + 3ddfxx + 3dffx + f^3$, sit jam $bb + c \propto f^3$ habebitur

$d^3x^3 + 3ddfxx + 3dffx \propto aaxx + 2abx$ sive $d^3xx + 3ddfx + 3dff \propto aax + 2ab$ sit iterum $3dff \propto 2ab$, et erit $d^3xx + 3ddfx \propto aax$ sive $d^3x + 3ddf \propto aa$ vel $x \propto \frac{aa - 3ddf}{d^3}$ unde

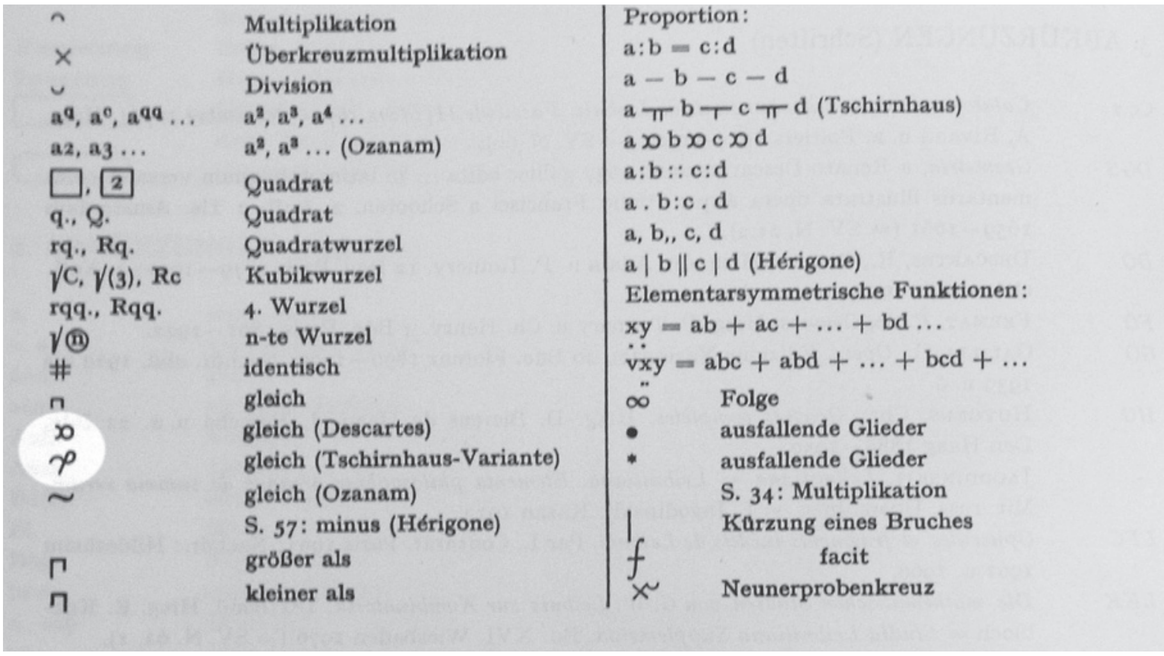
$dx + f \propto \frac{aa - 2ddf}{dd}$ sive $\propto \frac{aa}{dd} - 2f$ latus Cubi.

$\propto$ CARTESIAN EQUAL. LAA III-3 p. 102.



			neum $\delta\gamma\pi\rho$	tur spatium $\delta\gamma\pi\rho$.
ibid.		l. 24.	$\frac{trdy}{z} = rdx$	v. pag. 284. l. 14. ubi $dy \propto \frac{zdx}{t}$.
pag.	286.	l. 9.	- - - - -	Nam $\frac{zdz}{\sqrt{rr - zz}} = \propto FE$, omnia autem $FE \propto CA$ seu $\sqrt{rr - zz}$, z indefinite accipitur pro quavis DG.

$\propto$ CARTESIAN EQUAL
LAA III-7 p. 137



⌢	Multiplikation	Proportion:	
×	Überkreuzmultiplikation	a:b = c:d	
⌣	Division	a — b — c — d	
aq, ac, aqq …	a², a³, a⁴ …	a ⊤ b ⊤ c ⊤ d (Tschirnhaus)	
a2, a3 …	a², a³ … (Ozanam)	a ꝏ b ꝏ c ꝏ d	
□, [2]	Quadrat	a:b :: c:d	
q., Q.	Quadrat	a . b:c . d	
rq., Rq.	Quadratwurzel	a, b,, c, d	
√C, √(3), Rc	Kubikwurzel	a | b ‖ c | d (Hérigone)	
rqq., Rqq.	4. Wurzel	Elementarsymmetrische Funktionen:	
√ⓝ	n-te Wurzel	xy = ab + ac + … + bd …	
#	identisch	vxy = abc + abd + … + bcd + …	
⊓	gleich	∞	Folge
ꝏ	gleich (Descartes)	●	ausfallende Glieder
ꝏ	gleich (Tschirnhaus-Variante)	*	ausfallende Glieder
~	gleich (Ozanam)		S. 34: Multiplikation
	S. 57: minus (Hérigone)	|	Kürzung eines Bruches
⊏	größer als	∫	facit
⊐	kleiner als	✕	Neunerprobenkreuz

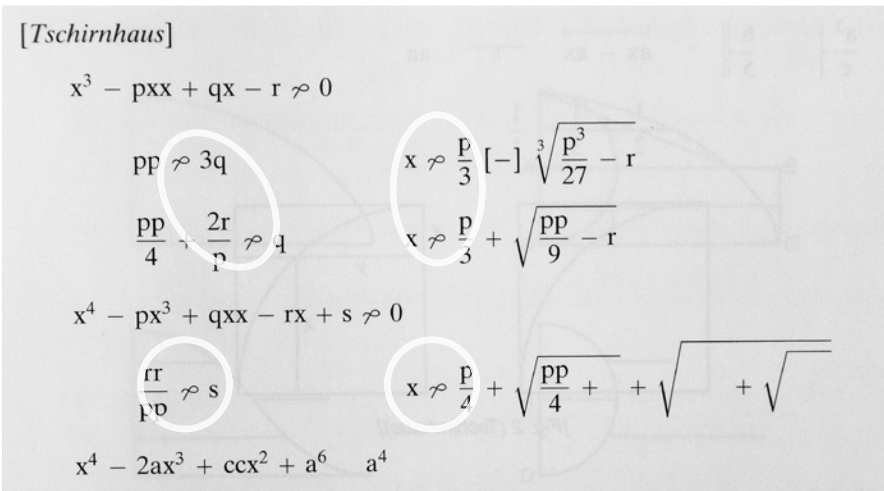
ꝏ CARTESIAN EQUAL – key to symbols, LAA VII-1

German natural scientist Ehrenfried Walther von Tschirnhaus (1651–1708) adopted Descartes' symbol ꝏ for *equal,* but wrote it in a more sloppy version with a straight downwards going line. This led the editors of the Leibniz Akademie-Ausgabe (LAA) to decide to distinguish the two variants, and so these two came into use for many decades. Initially we proposed a second character:

ꝏ TSCHIRNHAUS EQUAL

which reflects this typographic convention. In certain situations it is desirable to maintain the distinction for historiographical reasons, to trace different authors and writing habits. On the other hand, ꝏ and ꝏ actually bear the same meaning: *equal*. Therefore we propose to encode ꝏ as a new character but to encode the Tschirnhaus variant as a variation sequence:

```
xb21;CARTESIAN EQUAL;Sm;0;ON;;;;;N;;;;;
xb21 FE00; with straight descender; # CARTESIAN EQUAL
```



[*Tschirnhaus*]

$x^3 - pxx + qx - r ꝏ 0$

$pp ꝏ 3q \qquad x ꝏ \frac{p}{3} [-] \sqrt[3]{\frac{p^3}{27} - r}$

$\frac{pp}{4} + \frac{2r}{p} ꝏ q \qquad x ꝏ \frac{p}{3} + \sqrt{\frac{pp}{9} - r}$

$x^4 - px^3 + qxx - rx + s ꝏ 0$

$\frac{rr}{pp} ꝏ s \qquad x ꝏ \frac{p}{4} + \sqrt{\frac{pp}{4} +} + \sqrt{\quad} + \sqrt{\quad}$

$x^4 - 2ax^3 + ccx^2 + a^6 \quad a^4$

ꝏ CARTESIAN EQUAL (Tschirnhaus variant)
LAA VII-2 p. 715

N. 26 ARITHMETISCHE KREISQUADRATUR 1673–1676 301

kan sien daer, AB is $\frac{1}{8}$ van AC dat het differ. ontrent is $\frac{1}{2}$ sec: soude dan diff: van de geheele AB. ontrent 3 secunden.

Maer soo men de $\angle ACB$, 2 mahl, in 2 gelijcke deelen deelt, dan is AB, een weijnig kleijnder als $\frac{1}{5}$ deel van AC (wen AB is $\propto AC$) en de $\angle$en differ. als men kan sien in de wercking bouen, daer AB is $\frac{1}{5}$ deel van AC, dat de differentie is ontrent 12 sec.

Daerom wen de sijde AB is $\propto AC$ ofte een wenig kleijnder, het is genoeg om de $\angle ACB$, te deelen in 2 mahl, in 2 gelijcke deel, de $\angle$ sal ontrent $\frac{4}{5}$ deel, van 1 minut differen (als men met de 2 eerste termen, als $\frac{b}{1} - \frac{b^3}{3} \propto$ de arcus ADE werckt) van de Tab. sinus; ende hoe naeder het kombt tot $\frac{1}{3}$ deel van AC, hoeweeniger het verschiet.

Soo AB is $\frac{1}{3}$ deel van AC ofte een wenig groter soo heeft men van nooden de $\angle ACB$

∝ CARTESIAN EQUAL (Tschirnhaus variant)
LAA VII-6 p. 301

66 EHRENFRIED WALTHER VON TSCHIRNHAUS AN LEIBNIZ, 17. IV. 1677 N. 30

sive cubica $x^3 - pxx + qx - r \propto o$ etc.; si jam saltem unicus terminus debeat auferri supponatur $x \propto a + y$ et transmutatur aequatio in qua unicus terminus debet auferri; ope $x \propto a + y$ in aliam; ubi y incognita radix, in qua ponatur ille terminus auferendus $\propto o$ atque sic inveniemus qua ratione a sit assumenda ad terminum illum auferendum. Sit eg. in hac aequatione $xx - px + q \propto o$ auferendus secundus terminus fiat $x \propto y + a$ jam vero

$$\begin{array}{llll} xx \propto yy & + 2ay & + aa \propto o & \text{adeoque ponendo } 2ay - py \propto o \text{ erit } 2a - p \propto o \\ -px \propto & - py & - pa & \\ + q \propto & & q & \end{array}$$

et $a \propto \frac{p}{2}$, hinc patet debere fieri $x \propto y + \frac{p}{2}$ ad secundum terminum in aequatione quadratica

∝ CARTESIAN EQUAL (Tschirnhaus variant)
LAA III-2 p. 66; III-2 p. 285 (below)

incognitae potestates ordine per divisionem inserendo ac assumendo semper quotientes aequaliter compositas, quarum omnium possibilium modorum determinatus semper numerus facile exhibetur; hanc vero Methodum in praesentia abunde declaravi et specimina exhibui; sed non ita pridem ad majorem perfectionem deduxi. 2[da] est supponendo formulas omnes possibiles radicalium $x \propto \sqrt{a} + \sqrt{b}$, $x \propto \sqrt[3]{a} + b$, $x \propto \sqrt{a + \sqrt{b + \sqrt{c}}}$ quae facile omnes quot esse possunt numero determinantur et tunc liberandae sunt ab signis radicalibus atque comparatio instituenda. Specimen Tibi exhibebo ad formulas Cardanicas obtinendas sit $x \propto \sqrt[3]{a} + \sqrt[3]{b}$ supponatur jam $\sqrt[3]{a} \propto c$ et $\sqrt[3]{b} \propto d$ et habebimus has tres aequationes $x \propto c + d$, $a \propto c^3$ et $b \propto d^3$ quibus reductis inveniemus aequationem absque signo radicali (ut Tibi jam notum erit juxta Methodum D. de Beaune radicalia signa auferendi, quaeque

N. 46 NOTAE AD TRIANGULA NUMERORUM ET AD ALGEBRAM, Erste Hälfte Mai 1676 287

[*Vierter Teil*]

$$\cancel{a} + \cancel{b} \propto \cancel{c}c + \cancel{2cd} + dd$$

$$a \propto cc \qquad\qquad b \propto 2cd$$

$$\cancel{a^2} + \cancel{2ab} + b^2 \propto \cancel{c^3} + \cancel{3cd^3} + 3c^3d + d^3$$

$$a^2 \propto c^3 \qquad\qquad 2ab \propto 3c^2d \qquad\qquad b^2 \propto 3cd^2 + d^3$$

$$a \propto \sqrt{c^3} \qquad\qquad b \propto \frac{3c^2d}{2a} \qquad\qquad \frac{9c^4dd}{4\cancel{c^3}} \propto 3c^3d + d^3$$

$$\frac{9cdd}{4}$$

$$9cdd \propto 12c^3d + d^3$$

$$9cd \propto 12c^3 + dd$$

$$dd \propto 9cd - 12c^3$$

$$d \propto 3c + \sqrt{9cc - 12c^3}$$

$$d \propto 3c + c\sqrt{9 - 12c}$$

$\propto$ CARTESIAN EQUAL (Tschirnhaus variant)
LAA VII-8 p. 287; III-2 p. 380 (below)

380 EHRENFRIED WALTHER VON TSCHIRNHAUS AN LEIBNIZ, 10. IV. 1678 N. 154

ratione determinentur. Atque sic haec porro sese ita in infinitum habere; sed prolixioribus non opus, cum operanti juxta ea quae diximus haec sese statim manifestabunt. Attamen ut omni ex parte satisfaciam, Demonstratio possibilitatis poterat universalius et facilius sic absolvi; aequationes seu quaestiones ex aequaliter compositis primis et simplicissimis quantitatibus $x + y \propto a$ et $xy \propto b$ reducuntur ad quadraticam $yy - ay + b \propto o$; $x + y + z \propto a$, $xy + xz + yz \propto b$, $xyz \propto c$ ad Cubicam $y^3 - ayy + by - c \propto o$; $x + y + z + t \propto a$, $xy + xz + xt + yz + yt + zt \propto b$, $xyz + xyt + xzt + yzt \propto c$, $xyzt \propto d$ ad quadrato-quadraticam $y^4 - ay^3 + byy - cy + d \propto o$ atque sic porro ubi jam notum et facillime demonstratur.

Jam vero 2$^{\text{do}}$ aequationes

$xx + yy \propto a$, $xy \propto b$ possunt reduci ad $xx + yy \propto a$ et $xxyy \propto bb$ etc.
$x^3 + y^3 \propto a$ $\qquad x^3 + y^3 \propto a$ $\quad x^3y^3 \propto b^3$
$x^4 + y^4 \propto a$ $\qquad x^4 + y^4 \propto a$ $\quad x^4y^4 \propto b^4$

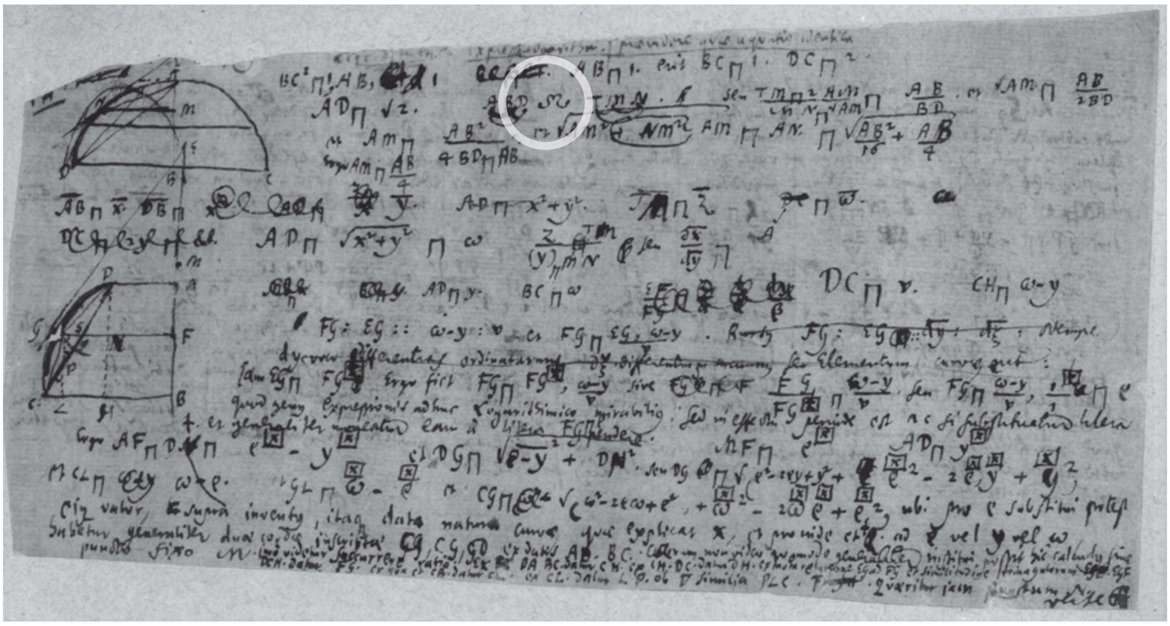
item per superiora Theoremata aequationes

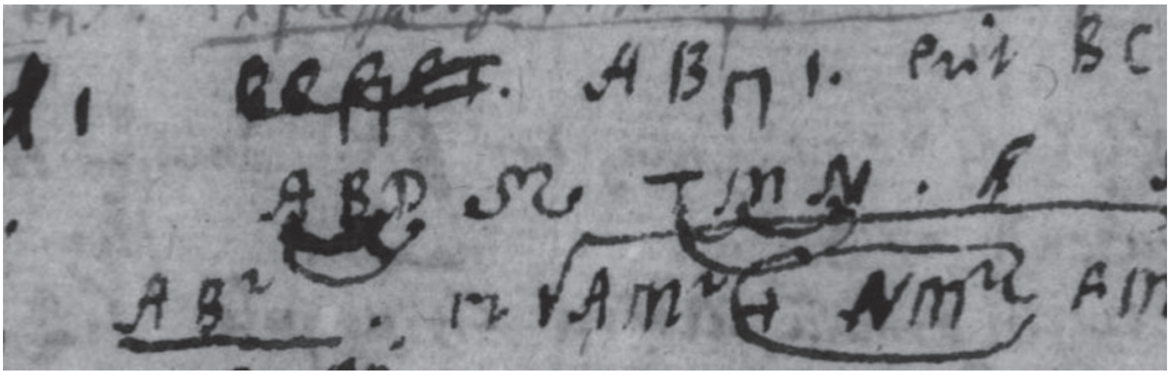
$xx + yy + zz \propto a$, $xy + xz + yz \propto b$, $xyz \propto c$
$x^3 + y^3 + z^3 \propto a$
$x^4 + y^4 + z^4 \propto a$

reducuntur ad aequationes

$xx + yy + zz \propto a$, $\quad xxyy + yyzz + xxzz \propto$ cognitae $\quad xxyyzz \propto cc$
$x^3 + y^3 + z^3 \propto a$ $\quad x^3y^3 + y^3z^3 + x^3z^3$ quantitati $\quad x^3y^3z^3 \propto c^3$
$x^4 + y^4 + z^4 \propto a$ $\quad x^4y^4 + y^4z^4 + x^4z^4$ $\quad x^4y^4z^4 \propto c^4$

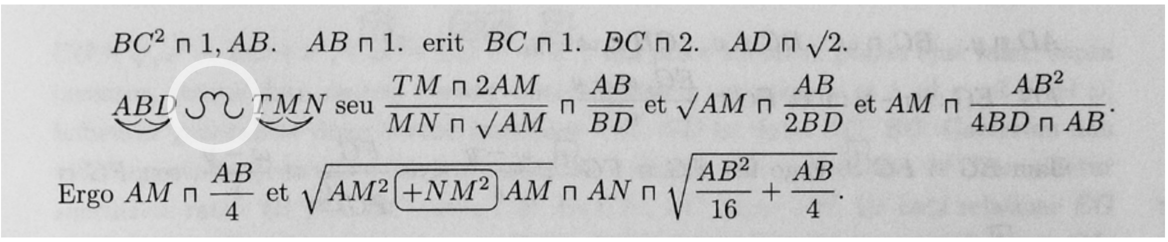
Leibniz used a variety of symbols to denote *similarity:* ∽, [LEIBNIZIAN SIMILARITY-1] and [similarity sign]. Of these, we regard the variant ∽ suitably represented by 223D ∽ REVERSED TILDE. Two other, considerably different *similarity* signs remain for encoding: [LEIBNIZIAN SIMILARITY-1] and [similarity sign].





[LEIBNIZIAN SIMILARITY-1] LEIBNIZIAN SIMILARITY-1
LH 35 XII 1, fol. 343v;

– this is the same text in the LAA edition:



$BC^2 \sqcap 1, AB.$ $AB \sqcap 1.$ erit $BC \sqcap 1.$ $DC \sqcap 2.$ $AD \sqcap \sqrt{2}.$

ABD [LEIBNIZIAN SIMILARITY-1] TMN seu $\frac{TM \sqcap 2AM}{MN \sqcap \sqrt{AM}} \sqcap \frac{AB}{BD}$ et $\sqrt{AM} \sqcap \frac{AB}{2BD}$ et $AM \sqcap \frac{AB^2}{4BD \sqcap AB}$.

Ergo $AM \sqcap \frac{AB}{4}$ et $\sqrt{AM^2 \boxed{+NM^2}}\, AM \sqcap AN \sqcap \sqrt{\frac{AB^2}{16} + \frac{AB}{4}}$.

[LEIBNIZIAN SIMILARITY-1] LEIBNIZIAN SIMILARITY-1
LAA VII-7 p. 595

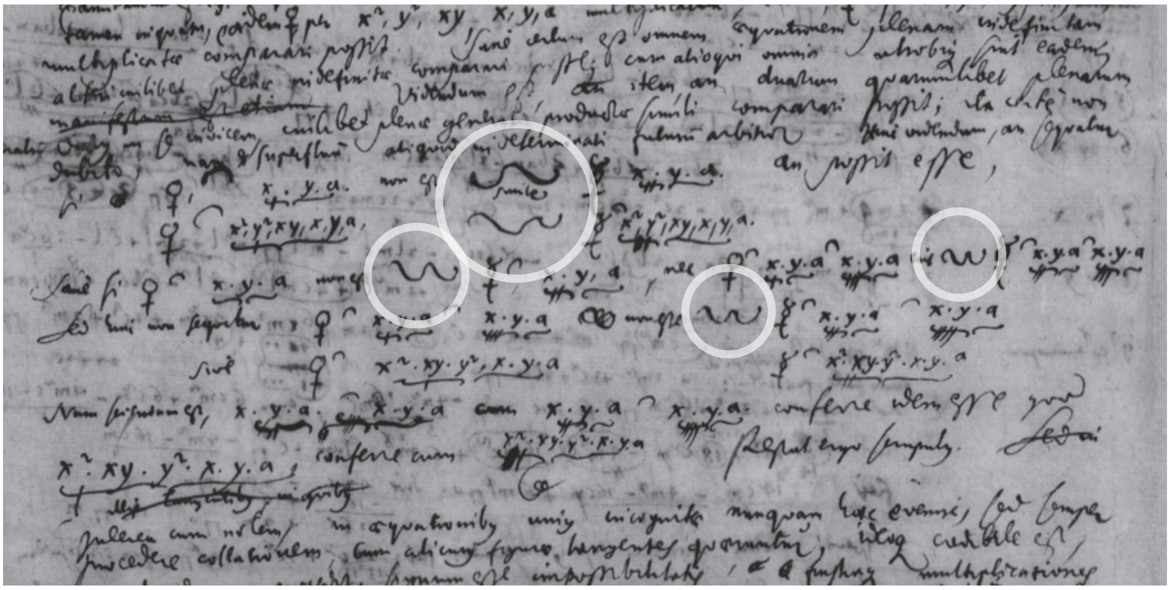
(10) Weitere neue Notationen

Wohl im April 1676 verwendet Leibniz mit ∽ ein neues Symbol für die Ähnlichkeit von Dreiecken. Ob er es auch andernorts einsetzt, ist bislang nicht bekannt. Das Beispiel:

$\underbrace{ABD}$ ∽ $\underbrace{TMN}$ (N. 66)

Im gleichen Stück entwickelt er schrittweise eine neue Notation für die eindeutige Zuordnung bestimmter geometrischer Größen zueinander. Er geht von einer Kurve aus,

∽ LEIBNIZIAN SIMILARITY-1
LAA VII-7 p. LIII



∾ LEIBNIZIAN SIMILARITY-2
LH 35 V 1 fol. 4v;
the same part in the edition:

Hinc videndum, an sequatur si

♀ ⌢ $\underbrace{x.y.a.}_{/}$ non est ∾ ☿ ⌢ $\underbrace{x.y.a.}_{//}$ an possit esse,

♀ ⌢ $\underbrace{x^2, y^2, xy, x, y, a.}_{/}$ ∾ ☿ ⌢ $\underbrace{x^2, y^2, xy, x, y, a.}_{//}$

Sane si ♀ ⌢ $\underbrace{x.y.a.}_{/}$ non est ∾ ☿ ⌢ $\underbrace{x.y.a,}_{//}$ nec ♀ ⌢ $\underbrace{x.y.a}_{/}$ ⌢ $\underbrace{x.y.a}_{///}$ erit ∾ ☿ ⌢ $\underbrace{x.y.a}_{//}$ ⌢ $\underbrace{x.y.a.}_{///}$ Sed hinc non sequitur

♀ ⌢ $\underbrace{x.y.a.}_{/}$ ⌢ $\underbrace{x.y.a}_{///}$ non esse ∾ ☿ ⌢ $\underbrace{x.y.a.}_{//}$ ⌢ $\underbrace{x.y.a.}_{////}$

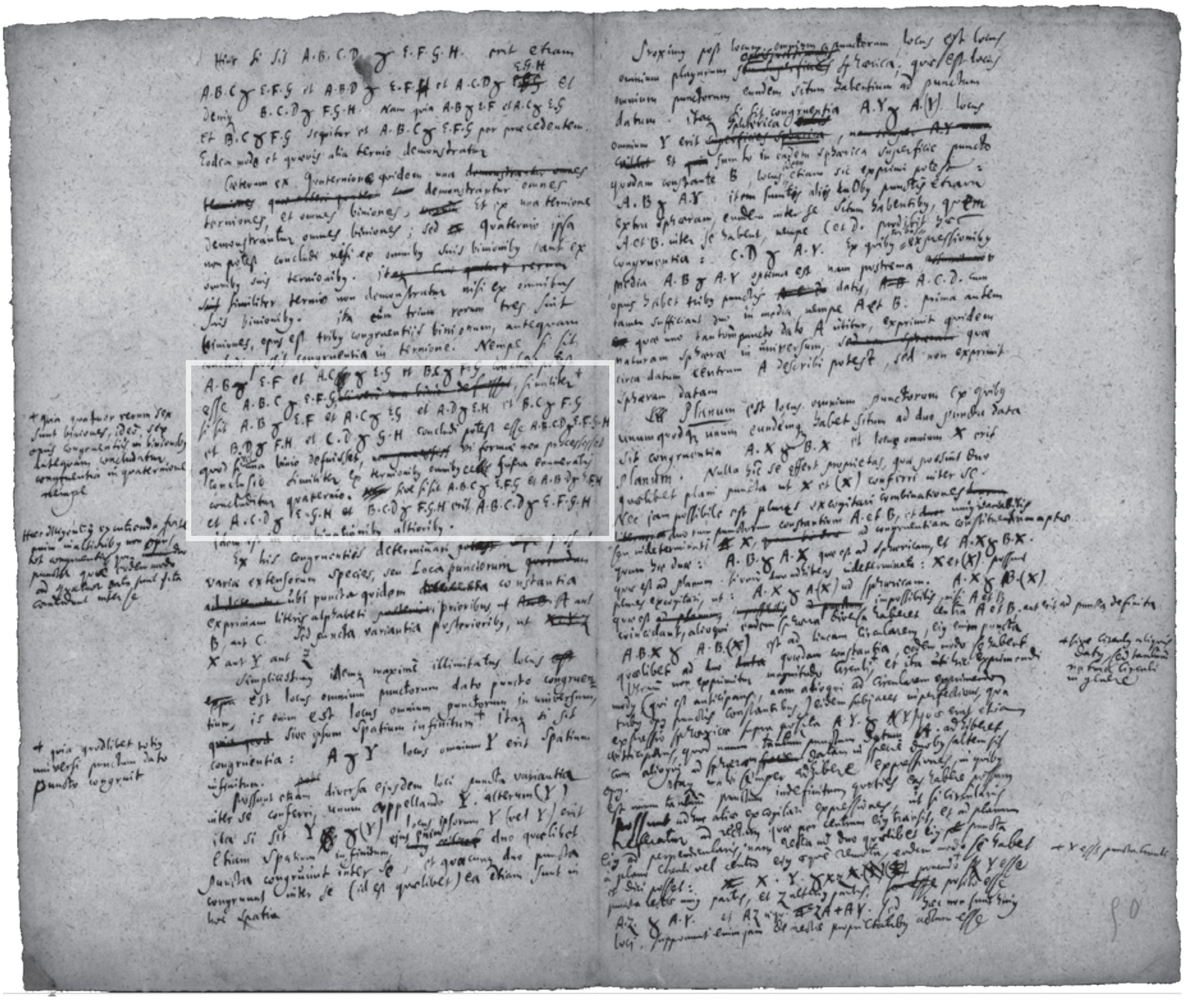
sive ♀ ⌢ $\underbrace{x^2.xy.y^2.x.y.a}_{/}$ ☿ ⌢ $\underbrace{x^2.xy.y^2.x.y.a}_{//}$

---

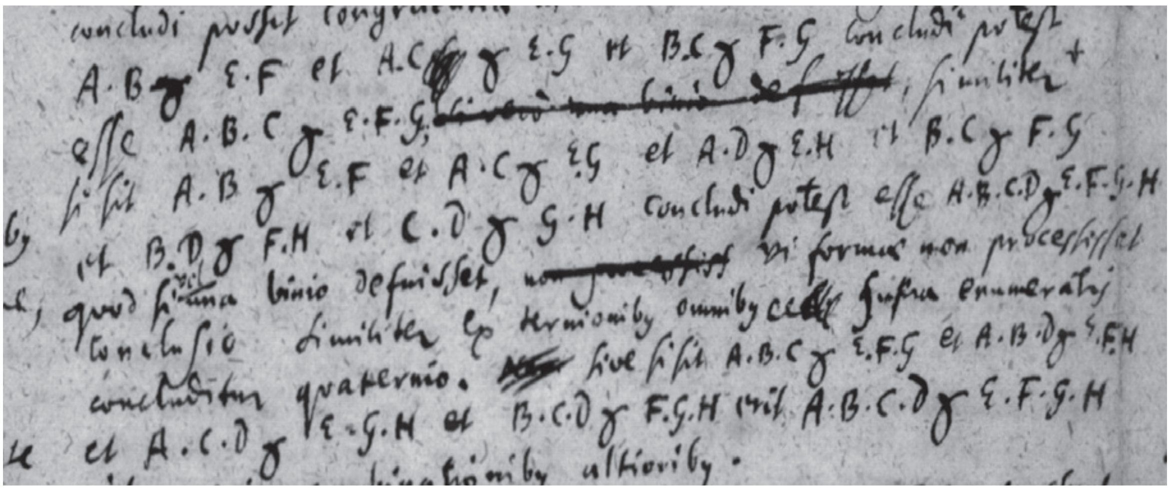
19 *Zu* ∾: simile

∾ LEIBNIZIAN SIMILARITY-2
LAA VII-3 p. 75

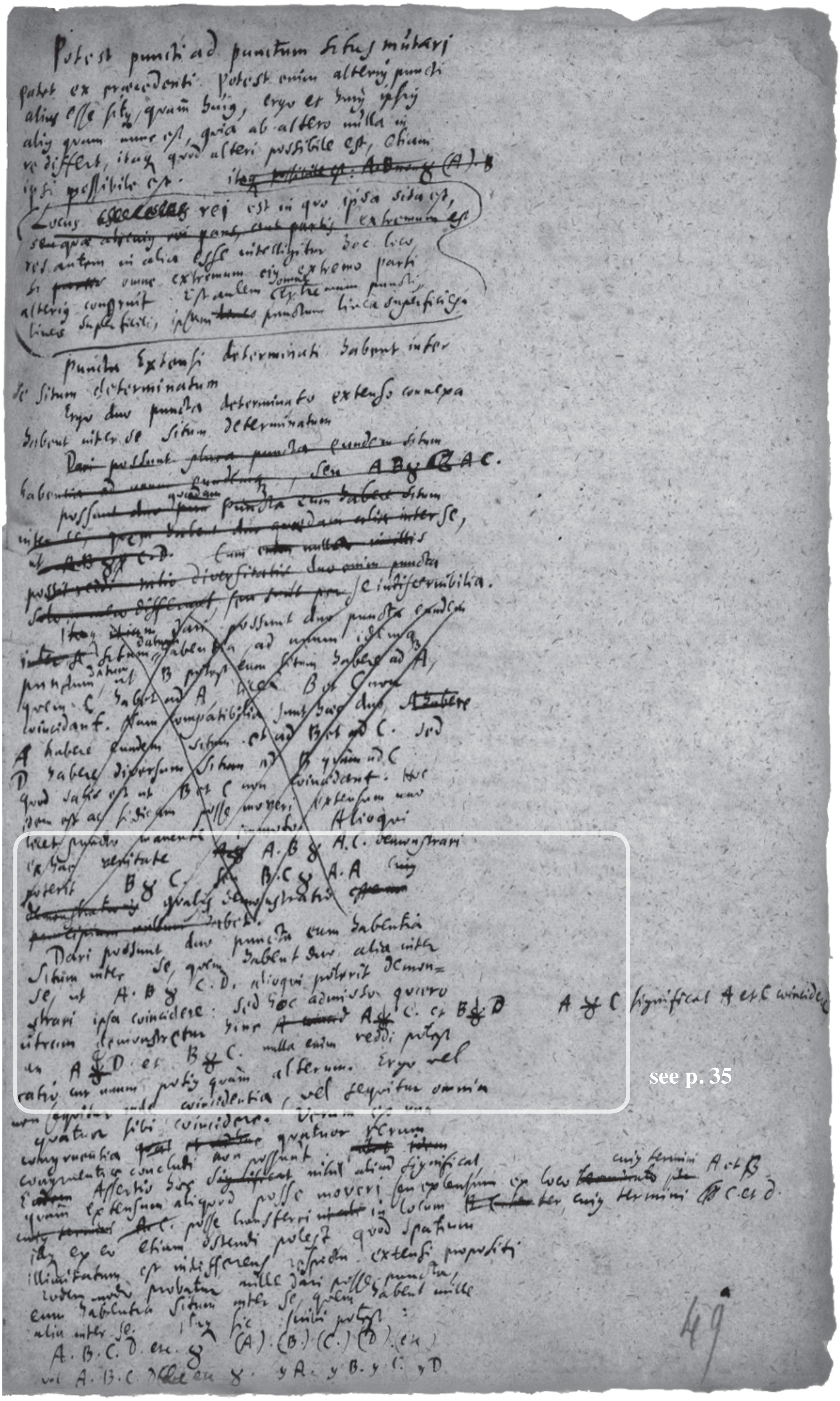
Leibniz used an even greater variety of symbols for *congruence:* ≃, ⩳, ꝏ, ɣ , ɣ̶ and ɣ̶̵ . In this set, ꝏ is a form derived from the letter *c*. The symbols ɣ , ɣ̶ and ɣ̶̵ , used very frequently, form a group in which the base character (ɣ) gets differentiated in terms of the aspect of coincidence (with or without).
First, a few examples from Leibniz's manuscripts.



LH 35 I 11, fol. 47v-46r

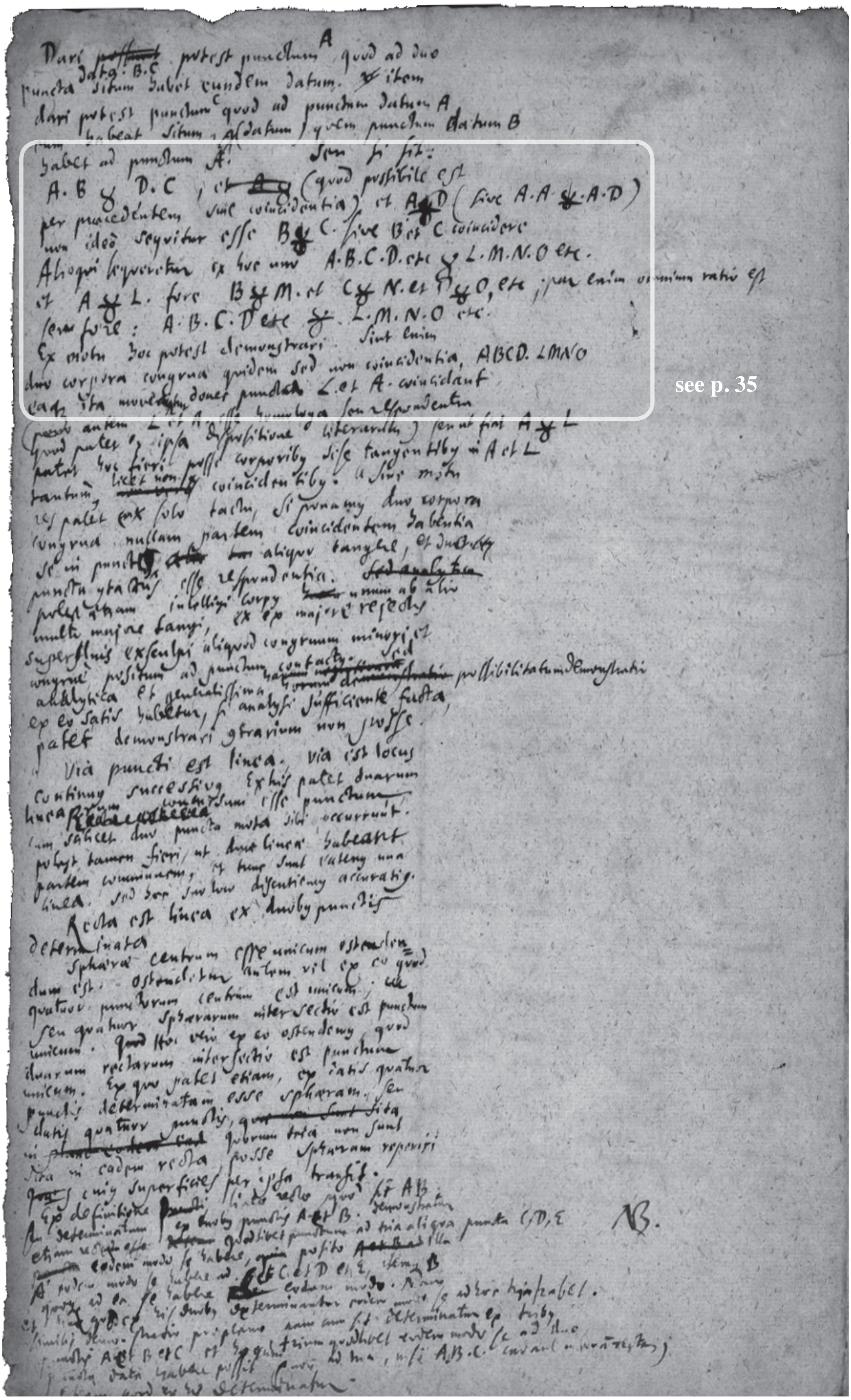


ɣ LEIBNIZIAN CONGRUENCE-4
LH 35 I 11, part of fol. 47v



see p. 35

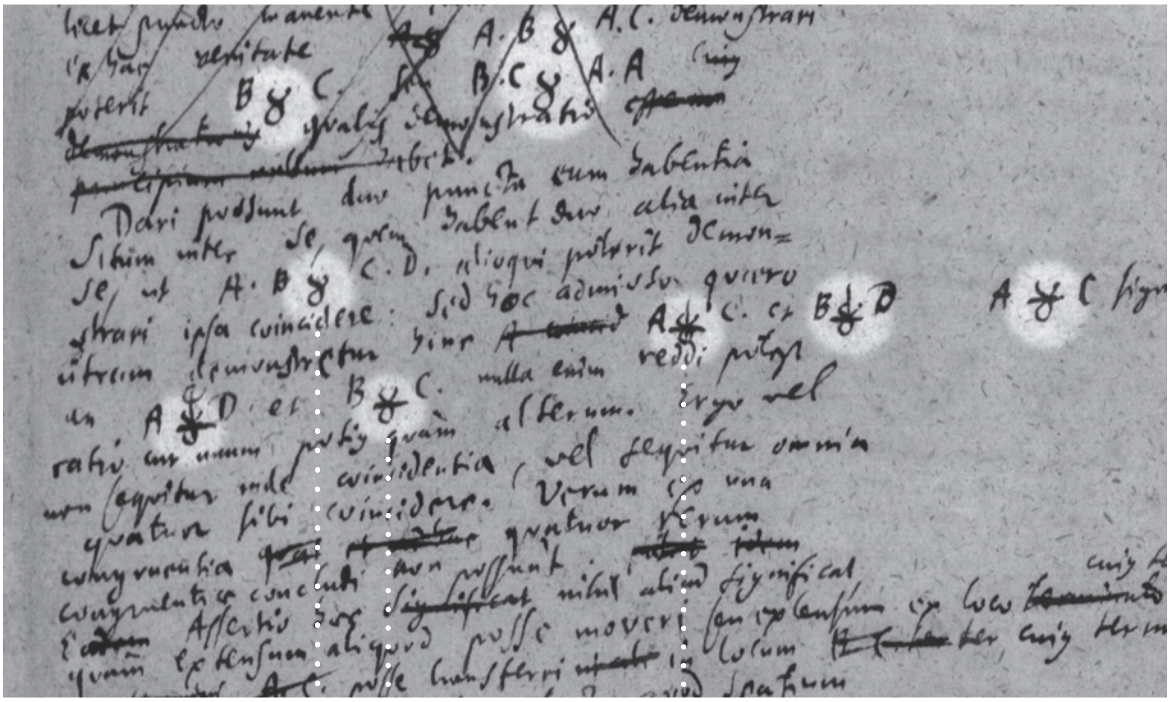
LH 35 I 11, fol. 49r



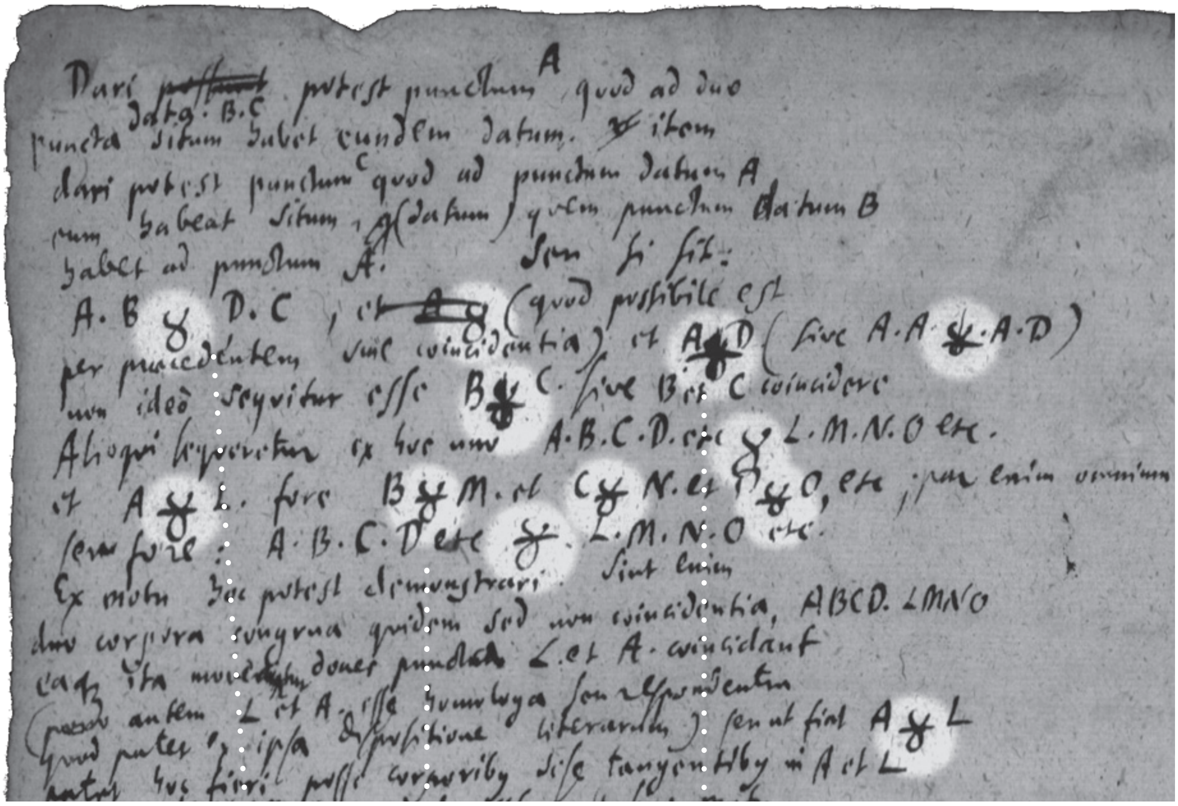
see p. 35

LH 35 I 11, fol. 49v

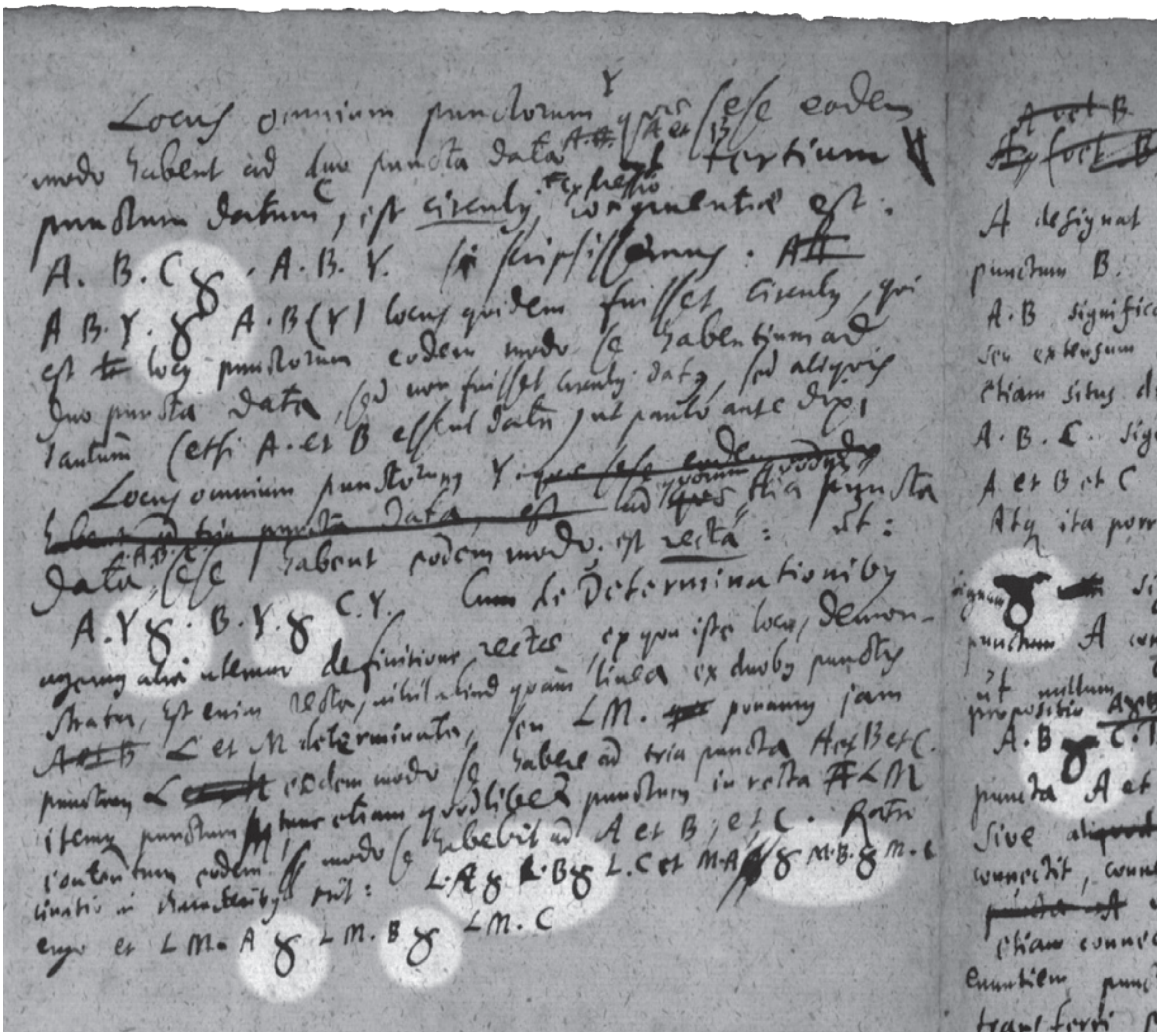
LEIBNIZIAN CONGRUENCE-4, LEIBNIZIAN CONGRUENCE-4 WITH COINCIDENCE, LEIBNIZIAN CONGRUENCE-4 WITHOUT COINCIDENCE



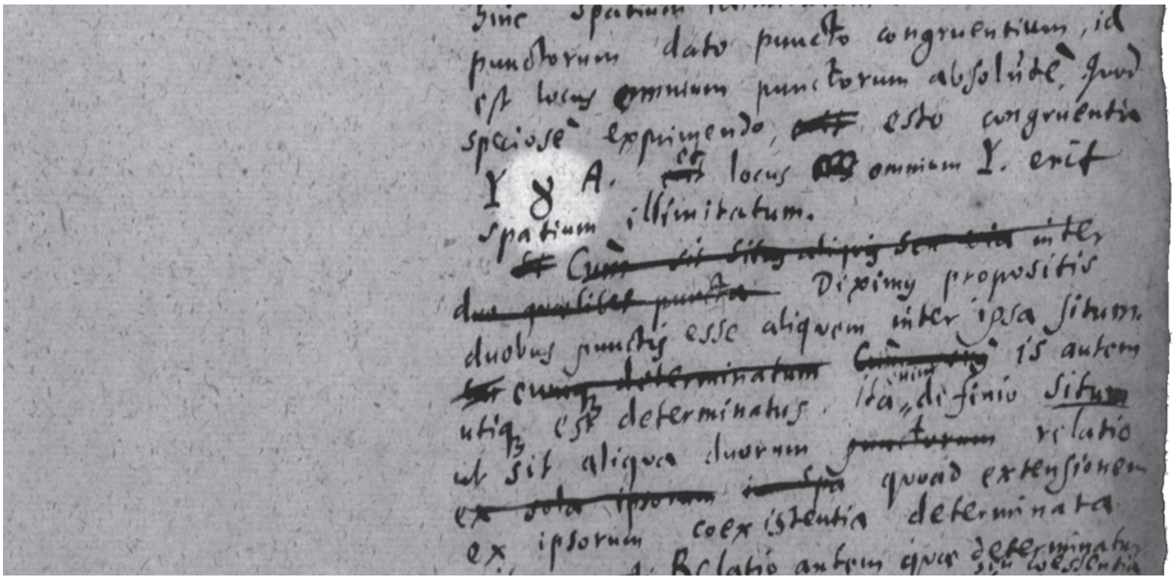
LH 35 I 11, part of fol. 49r



LH 35 I 11, part of fol. 49v



ȣ LEIBNIZIAN CONGRUENCE-4
LH 35 I 11, part of fol. 47r



ȣ LEIBNIZIAN CONGRUENCE-4
LH 35 I 11, part of fol. 49r

inter se, seu omnia puncta esse unum et idem. Nam quod unum punctum A alteri alicui C non coincidat, non potest aliter demonstrari, quam quod aliud quoddam punctum datur, B, cujus respectu diversum habent situm, ita ut $A.B.$ non ɤ $C.B.$

Potest puncti ad punctum situs mutari patet ex praecedenti. Potest enim alterius puncti alius esse situs, quam hujus, ergo et hujus ipsius alius quam nunc est, quia ab altero nulla in re differt, itaque quod alteri possibile est, etiam ipsi possibile est.

> Locus rei est in quo ipsa sita est, res autem in alia esse intelligitur hoc loco, si omne extremum ejus extremo parti alterius congruit. Est autem omne extremum puncti, lineae superficiei, ipsum punctum linea superficies.

Puncta Extensi determinati habent inter se situm determinatum. Ergo duo puncta determinato extenso connexa habent inter se situm determinatum.

Dari possunt duo puncta eum habentia situm inter se, quem habent duo alia inter se, ut $A.B$ ɤ $C.D$. Alioqui poterit demonstrari ipsa coincidere: sed hoc admisso quaero utrum demonstretur hinc A ɤ C et B ɤ D an A ɤ D et B ɤ C. Nulla enim reddi potest ratio cur unum potius quam alterum. Ergo vel non sequitur inde coincidentia, vel sequitur omnia quatuor sibi coincidere. Verum ex una congruentia quatuor rerum congruentiae concludi non possunt. Assertio haec nihil aliud significat, quam extensum aliquod posse moveri seu extensum ex loco cujus termini A et B posse transferri in locum cujus termini C et D.

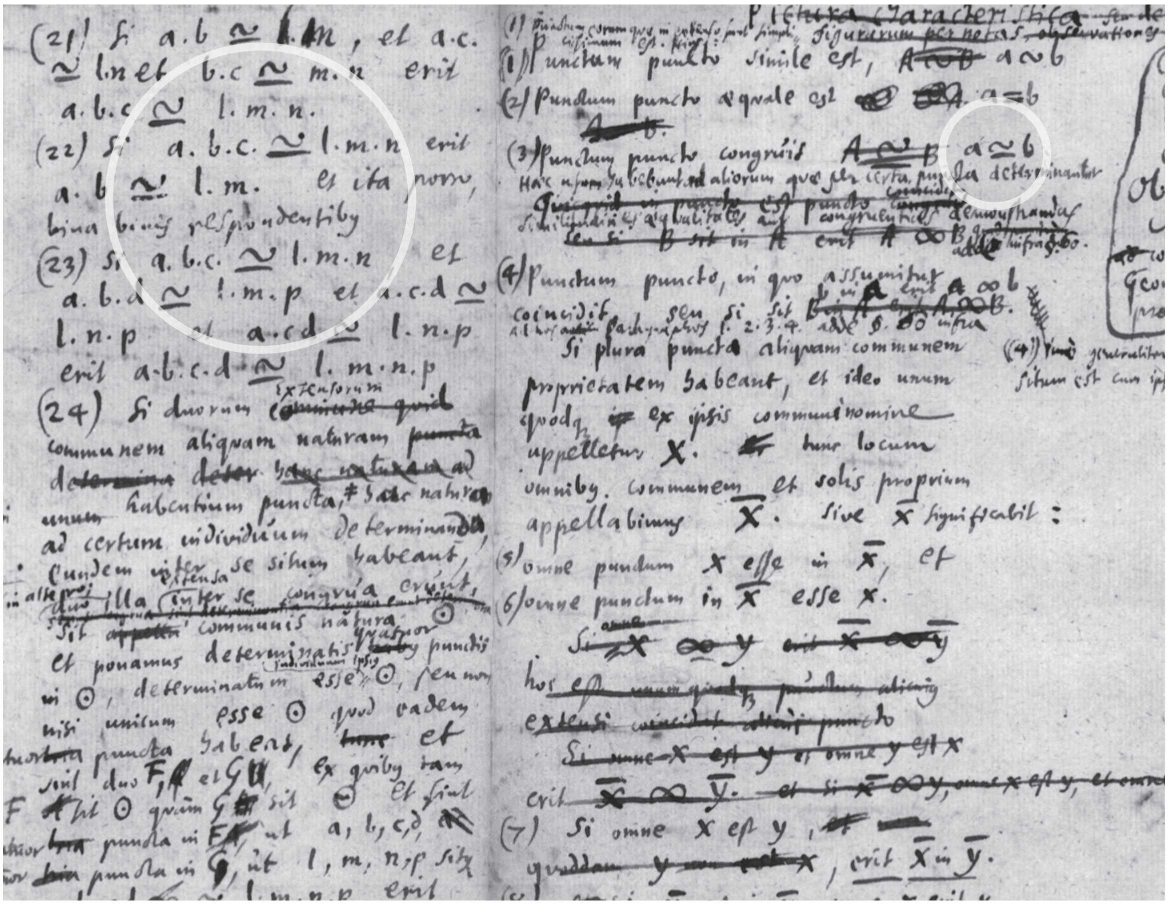
quem habent milla alia inter se. Itaque sic scribi potest: $A.B.C.D.$ etc. ɤ $(A).(B).(C).(D)$. (etc) vel $A.B.C.D$ etc. ɤ $yA.yB.yC.yD$.

Dari potest punctum A, quod ad duo puncta data $B.C$ situm habet eundem datum. Item dari potest punctum C quod ad punctum datum A eum habeat situm (datum), quem punctum datum B habet ad punctum A. Seu si sit: $A.B$ ɤ $D.C$ (quod possibile est per praecedentem sine coincidentia) et A ɤ D (sive $A.A$ ɤ $A.D$) non ideo sequitur esse B ɤ C. sive B et C coincidere. Alioqui sequeretur ex hoc uno $A.B.C.D.$ etc. ɤ $L.M.N.O.$ etc. et A ɤ $L.$ fore B ɤ $M.$ et C ɤ $N.$ et D ɤ O, etc.; par enim omnium ratio est seu fore $A.B.C.D$ etc. ɤ $L.M.N.O$ etc.

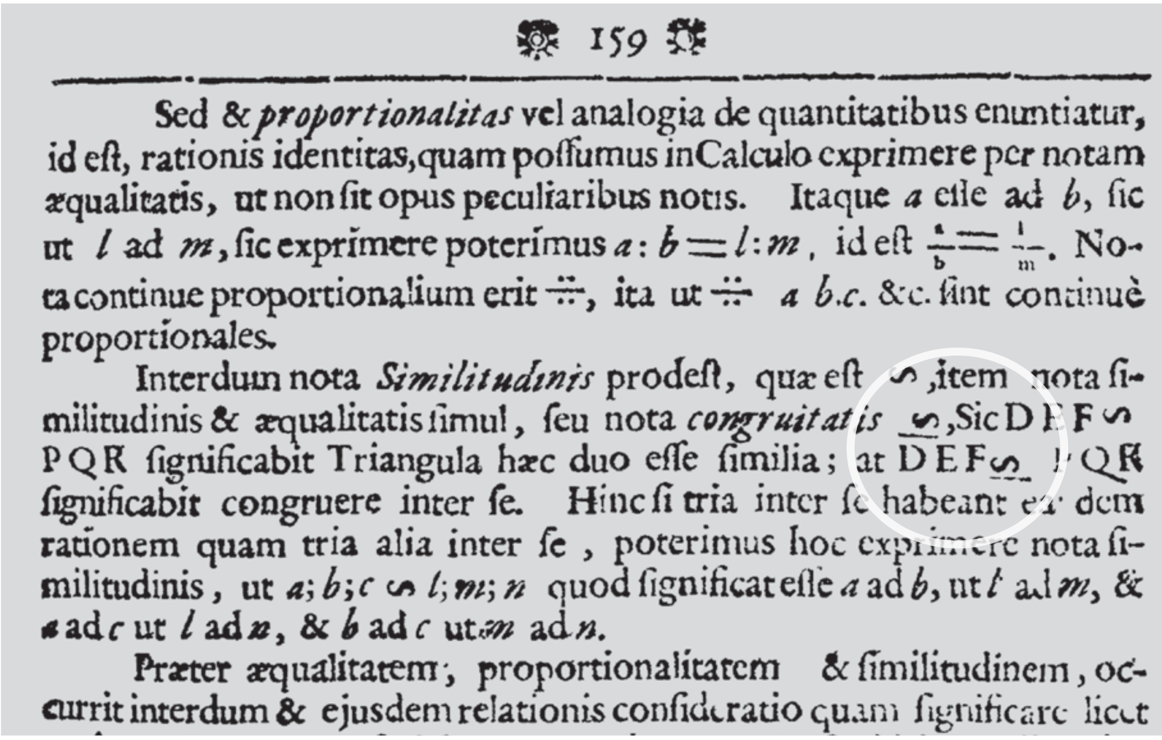
Ex motu hoc potest demonstrari. Sint enim duo corpora congrua quidem sed non coincidentia, $ABCD$. $LMNO$. eaque ita moveantur donec puncta $L.$ et $A.$ coincidant (porro autem $L.$ et $A.$ esse homologa seu respondentia quod patet ex ipsa dispositione literarum) seu ut fiat A ɤ L. Patet hoc fieri posse corporibus sese tangentibus in A et L tantum, licet non coincidentibus. Sine motu res patet ex solo tactu, si ponamus duo corpora congrua nullam partem coincidentem habentia se in puncto aliquo tangere, et duo puncta contactus esse respondentia. Potest etiam intelligi corpus unum ab alio multo majore tangi, et ex majore rejectis superfluis exsculpi aliquod congruum minori et congrue positum ad punctum contactus. Sed analytica et generalissima harum possibilitatum demonstratio ex eo satis habetur, si analysi sufficiente facta, patet demonstrari contrarium non posse.

ɤ LEIBNIZIAN CONGRUENCE-4, ɤ̶ LEIBNIZIAN CONGRUENCE-4 WITH COINCIDENCE

Philiumm vs. 2 (2023), p. 83, 84



≃ LEIBNIZIAN CONGRUENCE-1
LH 35 I 14, fol. 1r



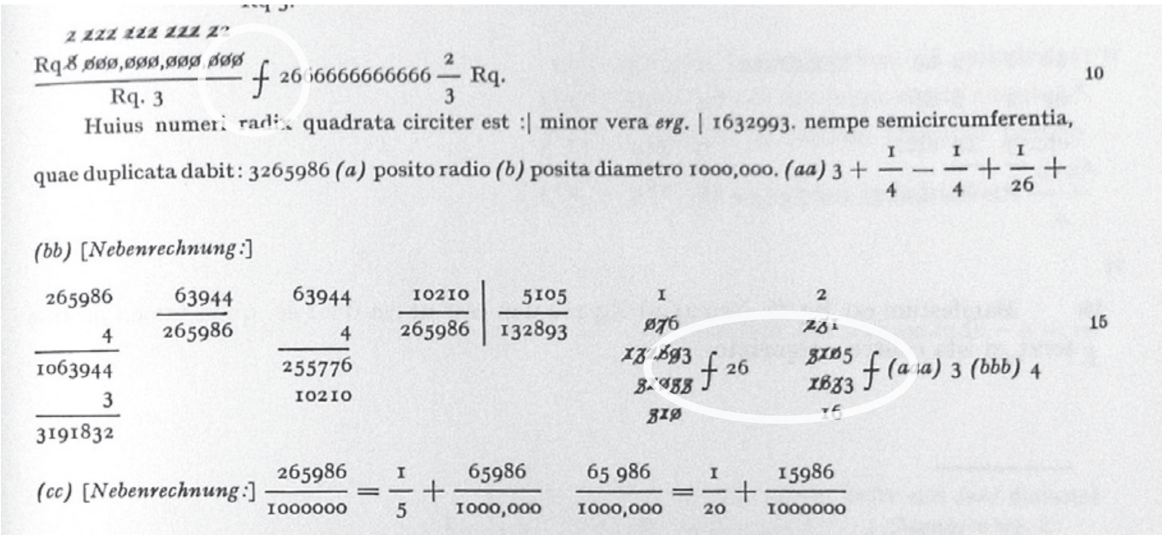
159

Sed & *proportionalitas* vel analogia de quantitatibus enuntiatur, id eſt, rationis identitas,quam poſſumus inCalculo exprimere per notam æqualitatis, ut non ſit opus peculiaribus notis. Itaque *a* eſſe ad *b*, ſic ut *l* ad *m*, ſic exprimere poterimus $a : b = l : m$, id eſt $\frac{a}{b} = \frac{l}{m}$. Nota continue proportionalium erit ∺, ita ut ∺ *a b.c.* &c. ſint continuè proportionales.

Interdum nota *Similitudinis* prodeſt, quæ eſt ∽ ,item nota ſimilitudinis & æqualitatis ſimul, ſeu nota *congruitatis* ≌,Sic DEF ∽ PQR ſignificabit Triangula hæc duo eſſe ſimilia; at DEF ≌ PQR ſignificabit congruere inter ſe. Hinc ſi tria inter ſe habeant eandem rationem quam tria alia inter ſe, poterimus hoc exprimere nota ſimilitudinis, ut *a*;*b*;*c* ∽ *l*;*m*;*n* quod ſignificat eſſe *a* ad *b*, ut *l* ad *m*, & *a* ad *c* ut *l* ad *n*, & *b* ad *c* ut *m* ad *n*.

Præter æqualitatem, proportionalitatem & ſimilitudinem, occurrit interdum & ejusdem relationis conſideratio quam ſignificare licet

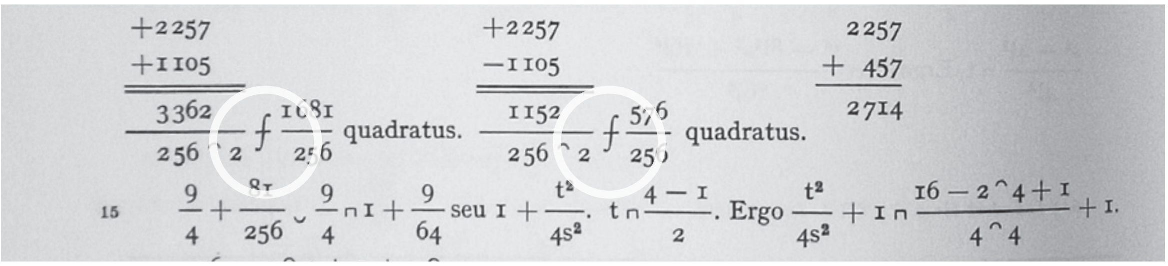
≌ LEIBNIZIAN CONGRUENCE-2
Monitum de Characteribus Algebraica, Miscellanea Berolinensia, 1710, p. 159



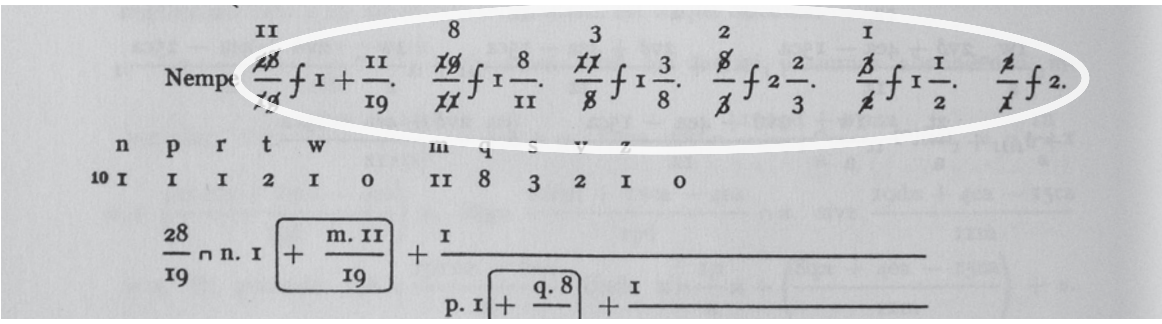
ƒ FACIT SYMBOL – LAA VII-1 p. 65

Leibniz used various script-style forms of the lowercase f for *facit* in his writings. In order to suitably represent them by one unambiguous symbol which make it distinguishable from both the ordinary (upright) f as well as the italic *f;* it is an established practice in the LAA edition for many decades to represent this expression by a specially shaped, “upright cursive” f with a descender and a reversed stress pattern (which not in any case was executed properly).

There is another similar looking character, LATIN SMALL LETTER F WITH HOOK (0192) which is defined as a currency character for *Florin* but which also gets used as an alphabetic character in the Ewe language. Since this unification is rather problematic already, we advise that 0192 not getting further loaded with other meanings. Regardless of a certain optical likeness the reason for including this character is mainly its distinctive purpose and function as an element of mathematical notation. The meaning is also different from that of the modern “function symbol” as which 0192 is annotated, additionally.



ƒ FACIT SYMBOL
LAA VII-1 p. 352



ƒ FACIT SYMBOL
LAA VII-1 p. 508

N. 38_{18} DIFFERENZEN, FOLGEN, REIHEN 1672–1676 551

$$\text{sive (30) } c \sqcap \frac{\begin{array}{l}+9,\ 25fa^2 \\ ǂ\ \ 81\ldots\end{array}}{\begin{array}{l}ǂ3 \frown 125\beta^2 \\ \quad 27\,.. \\ +3 \frown 45\,.. \\ \quad 75\,..\end{array}} \sqcap \frac{\begin{array}{l}+3 \frown 25fa^2 \\ ǂ3 \frown\ 9\ldots\end{array}}{\begin{array}{l}ǂ\,125\,\beta^2 \\ ..\ 27\,.. \\ +\ 45\,.. \\ ..\ 75\,..\end{array}} \sqcap \frac{\begin{array}{l}+3 \frown 25fa^2 \\ ǂ\,..\ \ 9\ldots\end{array}}{\begin{array}{l}ǂ\,[152]\,\beta^2 \\ +\ 120\,..\end{array}}.$$

$$\text{Ac denique erit (31) } b \sqcap \frac{\begin{array}{l}-4,125a^3f \quad -6,3,25a^3f \\ \ldots\ 27\ldots \quad ‡\,..\,..\ 9\ldots \\ ‡\,..\ 45\ldots \\ ..\,..\ 75\ldots\end{array}}{\begin{array}{l}ǂ\ 9,125\beta^3 \\ ..\,..\ 27\,.. \\ +\,..\ 45\,.. \\ ..\,..\ 75\,..\end{array}},\ \text{seu } b \sqcap \frac{\begin{array}{l}‡\ 642fa^3 \\ -\ 802\ldots\end{array}}{\begin{array}{l}ǂ1368\beta^3 \\ +1080\,..\end{array}}.$$

550,15–551,5 *Nebenrechnungen:*

zu Z. 15: 4 ⨍ 25 (225 ⨍ 25), 9; 15 ⌢ 25, 9 ⌢ 9

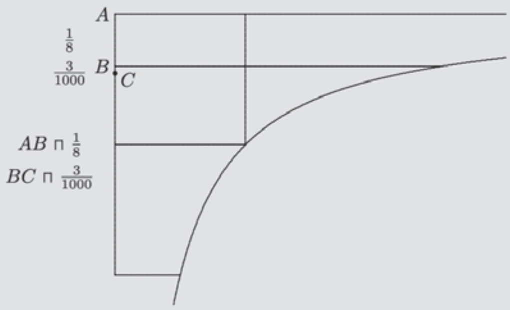
zu Z. 1–5: +9, 25 ǂ99 ǂ3 ⌢ 125 9 ⌢ 15
±18 ǂ3 ⌢ 27 9 ⌢ 25
ǂ81 3 ⌢ ǂ152 3 ⌢ 45
3 ⌢ 75

⨍ FACIT SYMBOL
LAA VII-3 p. 553 (top),
LAA VII-6 p. 449 (right)

These samples demonstrate the intentional use of a specific character for “facit” in order to distinguish it from the the ordinary italic *f*.

N. 43 ARITHMETISCHE KREISQUADRATUR 1673–1676 449

Quaeritur log. a 10. Inveniamus a 250 id est a 25 in 10. Habebimus et a 10 ex dato a 2. Est enim 5^3 in 2. Inveniemus a 250. si habeamus a $\frac{1}{250}$. Est autem notus log. ab $\frac{1}{256}$. Quaeratur differentia inter $\frac{1}{250}$ et $\frac{1}{256}$. Ea est $\frac{256-250}{250,256} \Big| \frac{6}{64000} \Big| \frac{3}{32000}$ eritque $\frac{1}{250} \sqcap \frac{1}{256} + \frac{3}{32000}$ vel $\sqcap \frac{1}{8} + \frac{3}{1000} \sqcap \frac{1024}{8000} \sqcap \frac{16}{125}$. Nam si hoc dividas per 32. habebis $\frac{1}{250}$ nam fit $\frac{1024}{8000}$ in $\frac{1}{32}$ dat $\frac{1024}{256000}$. Ergo quaerenda quantitas $\frac{d}{f} - \frac{d^2}{2f^2} + \frac{d^3}{3f^3}$ etc. ita ut d sit $\frac{3}{1000}$. et f. $\frac{1}{8}$.



[*Fig. 2*]

1–5 *Nebenbetrachtung:* $\frac{1}{250} - \frac{1}{256} \sqcap \frac{6}{64000} \Big| \frac{3}{32000}$. Ergo $\frac{1}{250} \sqcap \frac{1}{256} + \frac{3}{32000}$ cujus quaeritur logarithmus.

256
250
12800
512
64000

0
1
22
256000 ⨍ 250
10244
1022
10

5. Unicode Character Properties

```
xb01;LEIBNIZIAN EQUAL;Sm;0;ON;;;;;N;;;;;
xb02;LEIBNIZIAN EQUAL WITH DOUBLE VERTICALS;Sm;0;ON;;;;;N;;;;;
xb03;LEIBNIZIAN EQUAL WITH SMALL S;Sm;0;ON;;;;;N;;;;;
xb04;LEIBNIZIAN GREATER;Sm;0;ON;;;;;N;;;;;
xb05;LEIBNIZIAN LESS;Sm;0;ON;;;;;N;;;;;
xb06;LEIBNIZIAN GREATER WITH SMALL P;Sm;0;ON;;;;;N;;;;;
xb07;LEIBNIZIAN LESS WITH SMALL P;Sm;0;ON;;;;;N;;;;;
xb08;LEIBNIZIAN GREATER-LESS;Sm;0;ON;;;;;N;;;;;
xb09;RECTANGULAR GREATER OPEN RIGHT;Sm;0;ON;;;;;N;;;;;
xb10;RECTANGULAR GREATER OPEN LEFT;Sm;0;ON;;;;;N;;;;;
xb11;RECTANGULAR LESS OPEN RIGHT;Sm;0;ON;;;;;N;;;;;
xb12;RECTANGULAR LESS OPEN LEFT;Sm;0;ON;;;;;N;;;;;
xb13;TWO-LINE GREATER;Sm;0;ON;;;;;N;;;;;
xb14;TWO-LINE LESS;Sm;0;ON;;;;;N;;;;;
xb15;COMMENSURABILITY;Sm;0;ON;;;;;N;;;;;
xb16;INCOMMENSURABILITY;Sm;0;ON;;;;;N;;;;;
xb17;COMMENSURABILITY IN SQUARE;Sm;0;ON;;;;;N;;;;;
xb18;INCOMMENSURABILITY IN SQUARE;Sm;0;ON;;;;;N;;;;;
xb19;HORIZONTAL EQUAL TO OR GREATER-THAN;Sm;0;ON;;;;;N;;;;;
xb20;HORIZONTAL EQUAL TO OR LESS-THAN;Sm;0;ON;;;;;N;;;;;
xb21;CARTESIAN EQUAL;Sm;0;ON;;;;;N;;;;;
xb21 FE00; with straight descender; # CARTESIAN EQUAL
xb22;LEIBNIZIAN CONGRUENCE-1;Sm;0;ON;;;;;N;;;;;
xb23;LEIBNIZIAN CONGRUENCE-2;Sm;0;ON;;;;;N;;;;;
xb24;LEIBNIZIAN CONGRUENCE-3;Sm;0;ON;;;;;N;;;;;
xb25;LEIBNIZIAN CONGRUENCE-4;Sm;0;ON;;;;;N;;;;;
xb26;LEIBNIZIAN CONGRUENCE-4 WITH COINCIDENCE;Sm;0;ON;;;;;N;;;;;
xb27;LEIBNIZIAN CONGRUENCE-4 WITHOUT COINCIDENCE;Sm;0;ON;;;;;N;;;;;
xb28;LEIBNIZIAN SIMILARITY-1;Sm;0;ON;;;;;N;;;;;
xb29;LEIBNIZIAN SIMILARITY-2;Sm;0;ON;;;;;N;;;;;
xb30;FACIT SYMBOL;Sm;0;ON;;;;;N;;;;;
```

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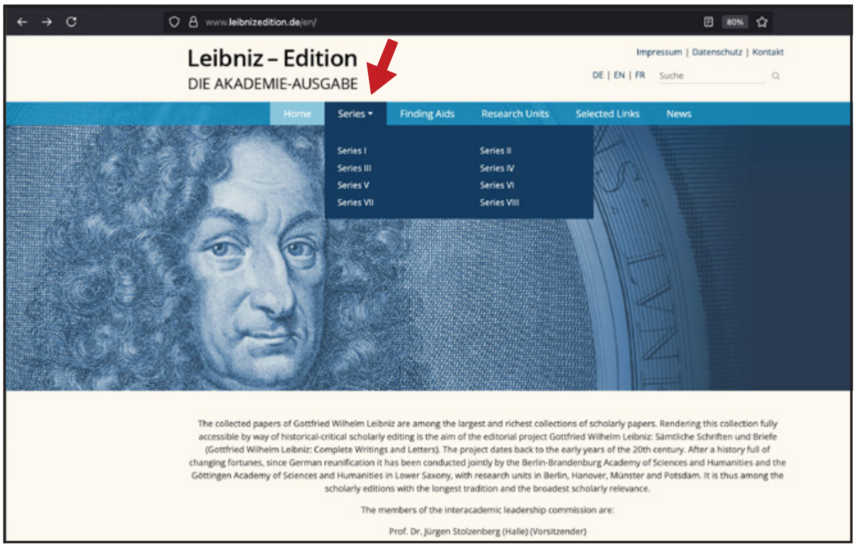
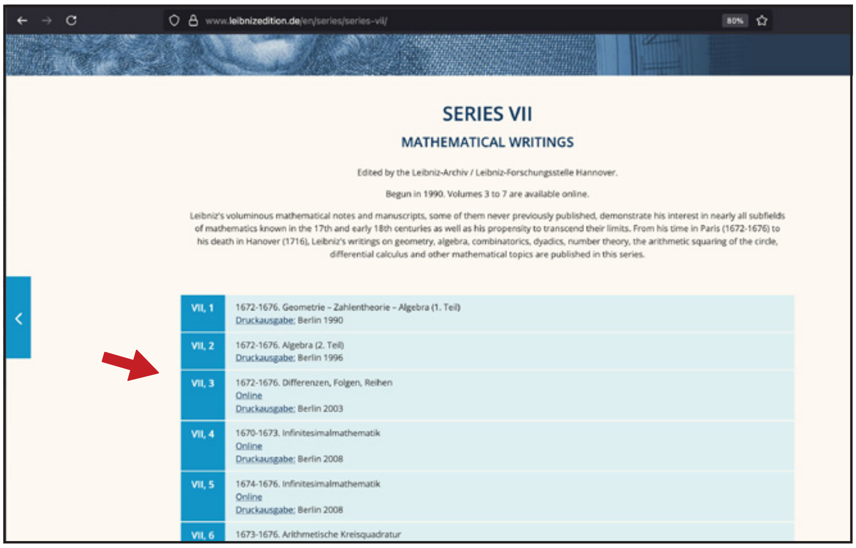
Leibniz-Forschungsstelle Hannover der Akademie der Wissenschaften zu Göttingen: PHILIUMM. Transkriptionen und Vorauseditionen mathematischer Schriften für die Leibniz-Akademie-Ausgabe. Version 2. Hannover 2023
— : Monitum de Characteribus Algebraica. In: Königliche Akademie der Wissenschaften (Berlin): Miscellanea Berolinensia. Berlin 1710
Oughtred, William: Guilelmi Oughtred Aetonensis, quondam Collegii Regalis in Cantabrigia Socii, Clavis Mathematicae … Oxford 1667
Parsons, John and Wastell, Thomas: Clavis Arithmeticae. Or, A Key to Arithmetick in Numbers & Species … 1705
Probst, Siegmund: Édition des symboles de Leibniz. PDF. Hanover 2023 (presentation Paris 2023)
Rinner, Elisabeth: List of glyphs in Leib.mf. PDF. Hanover 2022
Wallis, John: De sectionibus conicis nova methodo expositis tractatus. Oxford 1655
— : Operum mathematicorum, Oxford 1657
— : Treatise of Algebra. London 1685

When a source is referenced by e.g. **LAA VII-3**, that means: Leibniz-Edition, Akademie-Ausgabe, series VII, volume 3. For mathematical topics series III and VII are relevant in the first place. Currently, of series III volumes 5 to 10 and of series VII volumes 3 to 8 are accessible online (PDF). Go to **leibnizedition.de** to select a series and a volume:





ISO/IEC JTC 1/SC 2/WG 2
PROPOSAL SUMMARY FORM TO ACCOMPANY SUBMISSIONS
FOR ADDITIONS TO THE REPERTOIRE OF ISO/IEC 10646[1]
Please fill all the sections A, B and C below.
Please read Principles and Procedures Document (P & P) from http://std.dkuug.dk/JTC1/SC2/WG2/docs/principles.html for guidelines and details before filling this form.
Please ensure you are using the latest Form from http://std.dkuug.dk/JTC1/SC2/WG2/docs/summaryform.html.
See also http://std.dkuug.dk/JTC1/SC2/WG2/docs/roadmaps.html for latest *Roadmaps.*

A. Administrative

1. **Title:** **Proposal to encode historical mathematical relations**
2. Requester's name: Uwe Mayer, Siegmund Probst, David Rabouin, Elisabeth Rinner, Andreas Stötzner, Achim Trunk, Charlotte Wahl
3. Requester type (Member body/Liaison/Individual contribution): Individual (work group)
4. Submission date: 2025-05.30.
5. Requester's reference (if applicable): LUCP L-2519
6. Choose one of the following:
   This is a complete proposal: Yes
   (or) More information will be provided later:

B. Technical – General

1. Choose one of the following:
   a. This proposal is for a new script (set of characters): No
      Proposed name of script:
   b. The proposal is for addition of character(s) to an existing block: No
      Name of the existing block:
2. Number of characters in proposal: 31
3. Proposed category (select one from below - see section 2.2 of P&P document):

A-Contemporary		B.1-Specialized (small collection)	Yes	B.2-Specialized (large collection)	
C-Major extinct		D-Attested extinct		E-Minor extinct	
F-Archaic Hieroglyphic or Ideographic		G-Obscure or questionable usage symbols			

4. Is a repertoire including character names provided? Yes
   a. If YES, are the names in accordance with the "character naming guidelines" in Annex L of P&P document? Yes
   b. Are the character shapes attached in a legible form suitable for review? Yes
5. Fonts related:
   a. Who will provide the appropriate computerized font to the Project Editor of 10646 for publishing the standard?
      Andreas Stötzner
   b. Identify the party granting a license for use of the font by the editors (include address, e-mail, ftp-site, etc.):
      Andreas Stötzner Gestaltung, Klauflügelweg 21, 88400 Biberach/R., Germany, as@signographie.de
6. References:
   a. Are references (to other character sets, dictionaries, descriptive texts etc.) provided? Yes
   b. Are published examples of use (such as samples from newspapers, magazines, or other sources) of proposed characters attached? Yes
7. Special encoding issues:
   Does the proposal address other aspects of character data processing (if applicable) such as input, presentation, sorting, searching, indexing, transliteration etc. (if yes please enclose information)? No

8. Additional Information:
Submitters are invited to provide any additional information about Properties of the proposed Character(s) or Script that will assist in correct understanding of and correct linguistic processing of the proposed character(s) or script. Examples of such properties are: Casing information, Numeric information, Currency information, Display behaviour information such as line breaks, widths etc., Combining behaviour, Spacing behaviour, Directional behaviour, Default Collation behaviour, relevance in Mark Up contexts, Compatibility equivalence and other Unicode normalization related information. See the Unicode standard at http://www.unicode.org for such information on other scripts. Also see Unicode Character Database (http://www.unicode.org/reports/tr44/) and associated Unicode Technical Reports for information needed for consideration by the Unicode Technical Committee for inclusion in the Unicode Standard.

[1] Form number: N4502-F (Original 1994-10-14; Revised 1995-01, 1995-04, 1996-04, 1996-08, 1999-03, 2001-05, 2001-09, 2003-11, 2005-01, 2005-09, 2005-10, 2007-03, 2008-05, 2009-11, 2011-03, 2012-01)

C. Technical - Justification

1. Has this proposal for addition of character(s) been submitted before? Yes
   - If YES explain: *see N5277 (L-2402n)*
2. Has contact been made to members of the user community (for example: National Body, user groups of the script or characters, other experts, etc.)? Yes
   - If YES, with whom? Leibniz-Archiv, Forschungsstelle der Leibniz-Edition, Niedersächsische Landesbibliothek (GWLB), Hanover, Göttingen Academy of Science and Humanities in Lower Saxony (DE), Philiumm research group of CNRS (UMR 7219, laboratoire SPHERE) / Université de Paris VII; general: scholars, researchers, authors and editors working in the field of science history and upon editions of historic text corpora (e.g. of G. W. Leibniz, but also many others)
   - If YES, available relevant documents: L-2409, L-2410
3. Information on the user community for the proposed characters (for example: size, demographics, information technology use, or publishing use) is included? Yes
   - Reference:
4. The context of use for the proposed characters (type of use; common or rare) Common
   - Reference: mainly specialist usage, scholarly, worldwide
5. Are the proposed characters in current use by the user community? Yes
   - If YES, where? Reference: mainly Europe, Americas; other countries
6. After giving due considerations to the principles in the P&P document must the proposed characters be entirely in the BMP? No
   - If YES, is a rationale provided?
   - If YES, reference:
7. Should the proposed characters be kept together in a contiguous range (rather than being scattered)? No
8. Can any of the proposed characters be considered a presentation form of an existing character or character sequence? No
   - If YES, is a rationale for its inclusion provided?
   - If YES, reference:
9. Can any of the proposed characters be encoded using a composed character sequence of either existing characters or other proposed characters? No
   - If YES, is a rationale for its inclusion provided?
   - If YES, reference:
10. Can any of the proposed character(s) be considered to be similar (in appearance or function) to, or could be confused with, an existing character? No
    - If YES, is a rationale for its inclusion provided?
    - If YES, reference:
11. Does the proposal include use of combining characters and/or use of composite sequences? No
    - If YES, is a rationale for such use provided?
    - If YES, reference:
    - Is a list of composite sequences and their corresponding glyph images (graphic symbols) provided? No
    - If YES, reference:
12. Does the proposal contain characters with any special properties such as control function or similar semantics? No
    - If YES, describe in detail (include attachment if necessary)
13. Does the proposal contain any Ideographic compatibility characters? No
    - If YES, are the equivalent corresponding unified ideographic characters identified?
    - If YES, reference: