ISO/IEC JTC1/SC2/WG2 **N5334R**
LUCP L-2530

Universal Multiple-Octet Coded Character Set
International Organization for Standardization
Internationale Standardisierungs-Organisation
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Διεθνής Οργανισμός Τυποποίησης
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Title: Proposal to encode historical mathematical relations

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Requester’s reference: LUCP L-2530

1. Mathematical relation symbols in historic sources

The topic of this proposal is a group of symbols for relations, like *equal, congruence, greater-than* or *commensurability*. They are testified in works of G. W. Leibniz and many other authors, mainly of the 17th century. Some of the proposed characters basically represent the same meaning as e.g. 003D = EQUAL SIGN or 003E > GREATER-THAN SIGN. However, for the purpose of historiographically exact transcriptions and editions it is neccessary to encode the difference between such modern symbols and historic ones, since either of them may occur in the very same edition.
In character names we left out the component ‘SIGN’ as we see this in line with most of comparable names of symbols already encoded. In some cases we propose personal identifiers as name parts (‘LEIBNIZIAN’, ‘CARTESIAN’) because we regard this as a suitable means of clarification.

2. Revision of Proposal

This 2nd revision of the *Relations* proposal is a significant update and an extended version. After expert discussion a couple of changes have been implemented. RECTANGULAR GREATER OPEN RIGHT ⫍ and RECTANGULAR GREATER OPEN RIGHT ⫎ have been unified with 2ACD and 2ACE. The remaining characters ⊐ and ⊏ have been given new names in accordance to 2ACD and 2ACE. HORIZONTAL EQUAL TO OR LESS-THAN ⋜ and HORIZONTAL EQUAL TO OR GREATER-THAN ⋝ are now proposed as variation sequences based on 22DC and 22DD.
The subset of the *congruence* characters has been re-ordered and in the course of recent research work a number of additional characters has been identified. Some of these characters can be defined as historic precedences of modern symbols like ≃, ≌, ≈ or ≋; in these cases they are proposed as “lazy-S” variation sequences. The two characters |∾| and ∾ do not have “tilde” equivalents because their usage did not make it into modern math notation, they are therefore proposed as new (historical) characters.
The Unicode Character Properties entries have been updated accordingly. This version also contains more demonstration samples from MS sources as well as from printed editions.

3.a New Characters

If this proposal gets accepted, the following 29 new characters will exist:

 LEIBNIZIAN EQUAL

 LEIBNIZIAN EQUAL WITH DOUBLE VERTICALS

 LEIBNIZIAN EQUAL WITH SMALL S

 LEIBNIZIAN GREATER

 LEIBNIZIAN LESS

 LEIBNIZIAN GREATER WITH SMALL P

 LEIBNIZIAN LESS WITH SMALL P

 LEIBNIZIAN GREATER-LESS

 INVERTED SQUARE LEFT OPEN BOX OPERATOR

 INVERTED SQUARE RIGHT OPEN BOX OPERATOR

 TWO-LINE GREATER

 TWO-LINE LESS

 COMMENSURABILITY

 INCOMMENSURABILITY

 COMMENSURABILITY IN SQUARE

 INCOMMENSURABILITY IN SQUARE

 CARTESIAN EQUAL

 LEIBNIZIAN CONGRUENCE

 LEIBNIZIAN CONGRUENCE WITH VERTICAL BAR

 LEIBNIZIAN CONGRUENCE-2

 LEIBNIZIAN CONGRUENCE-2 INVERTED

 LEIBNIZIAN CONGRUENCE-2 WITH HORIZONTAL BAR

 LEIBNIZIAN CONGRUENCE-2 WITH HORIZONTAL AND VERTICAL BAR

 LEIBNIZIAN COINCIDENCE

 INVERTED LAZY S OVER LAZY S

 LEIBNIZIAN SIMILARITY

 LEIBNIZIAN SIMILARITY-2

 LEIBNIZIAN DISSIMILARITY

 FACIT SYMBOL

3.b New Variation sequences

For these characters we propose new standardized variation sequences:

variation sequence to CARTESIAN EQUAL ∞

variation sequence to 223D – REVERSED TILDE ∽

variation sequence to 2243 – ASYMPTOTICALLY EQUAL TO ≃

variation sequence to 22CD – REVERSED TILDE EQUALS ⋍

variation sequence to 2242 – MINUS TILDE ≂

variation sequence to 2248 – ALMOST EQUAL TO ≈

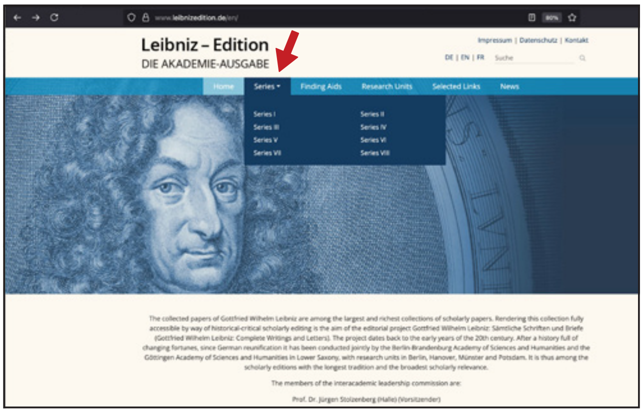
variation sequence to 2A6C – SIMILAR MINUS SIMILAR ⩬

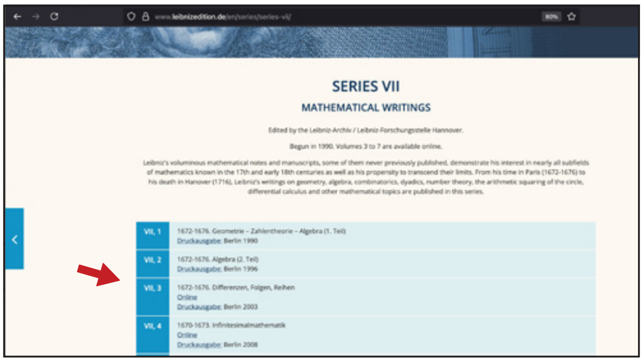
variation sequence to 22DC – EQUAL TO OR LESS-THAN ⋜

variation sequence to 22DD – EQUAL TO OR GREATER-THAN ⋝

The character is a historically significant variant of the widely used CARTESIAN EQUAL sign ∞, see further explanations on p. 37 f.
The other 8 variations relate to existing characters.

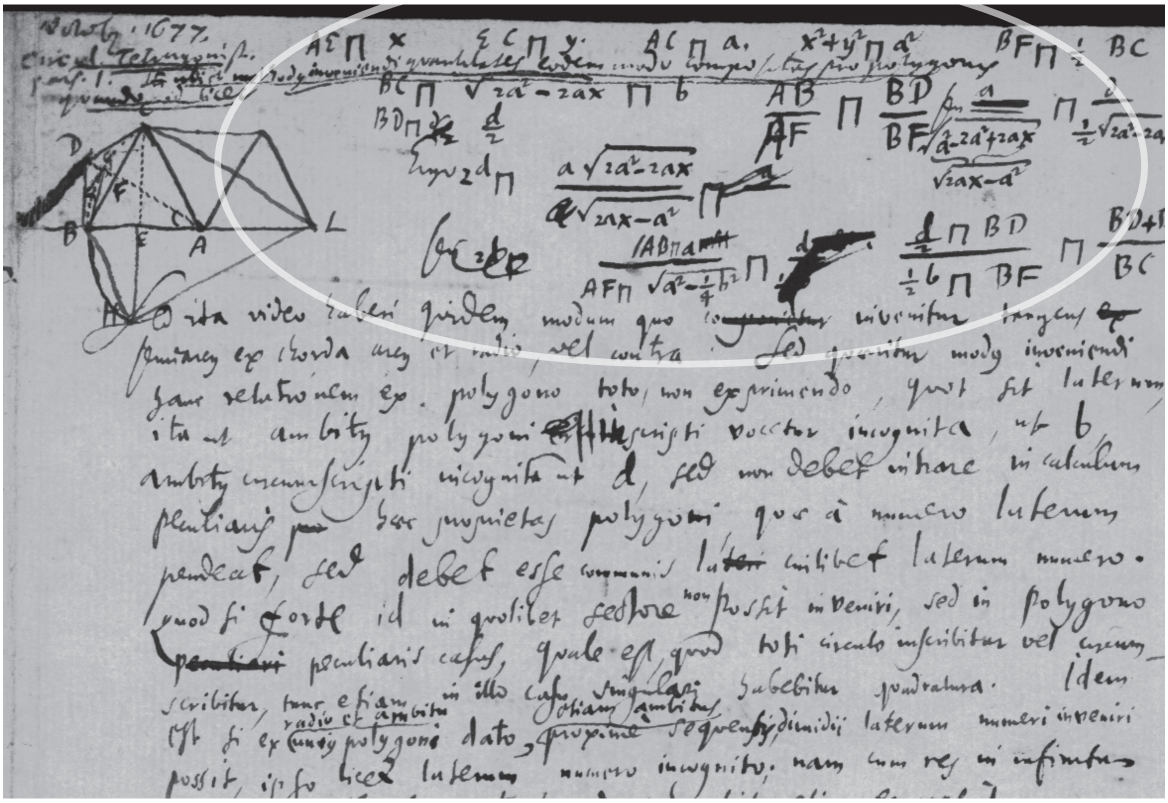
When a source is referenced by e.g. **LAA VII-3**, that means: Leibniz-Edition, Akademie-Ausgabe, series VII, volume 3. For mathematical topics series III and VII are relevant in the first place. Currently, of series III volumes 5 to 10 and of series VII volumes 3 to 8 are accessible online (PDF). Go to **leibnizedition.de** to select a series and a volume:



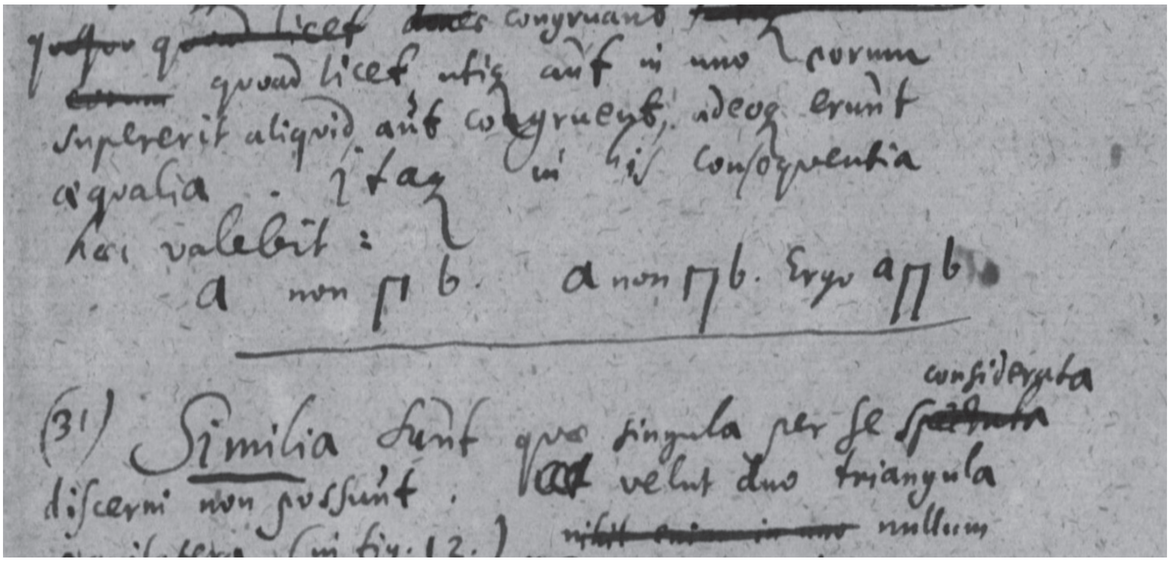


4. Figures and explanations

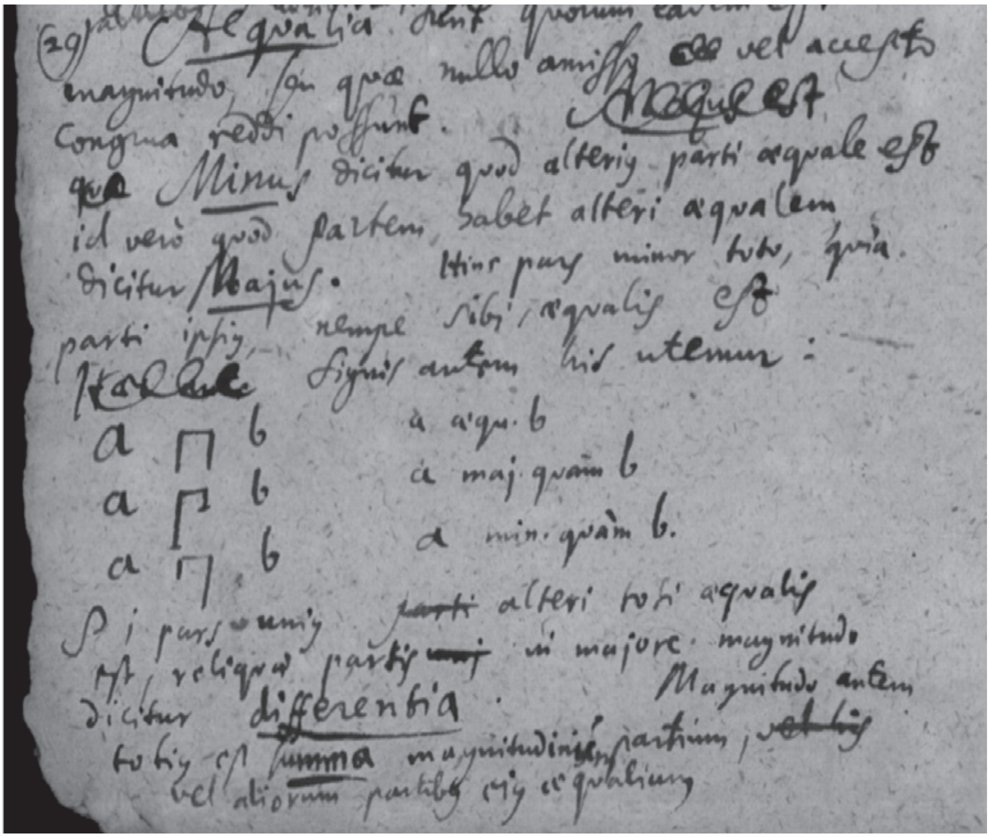
Leibniz made use of a fine differentiation of notions of equality and inequality in his mathematical writings. The character ⊓ LEIBNIZIAN EQUAL signifies in many of his mathematical works *equality* in the common meaning as it denotes the equality of two things with regard to some property.



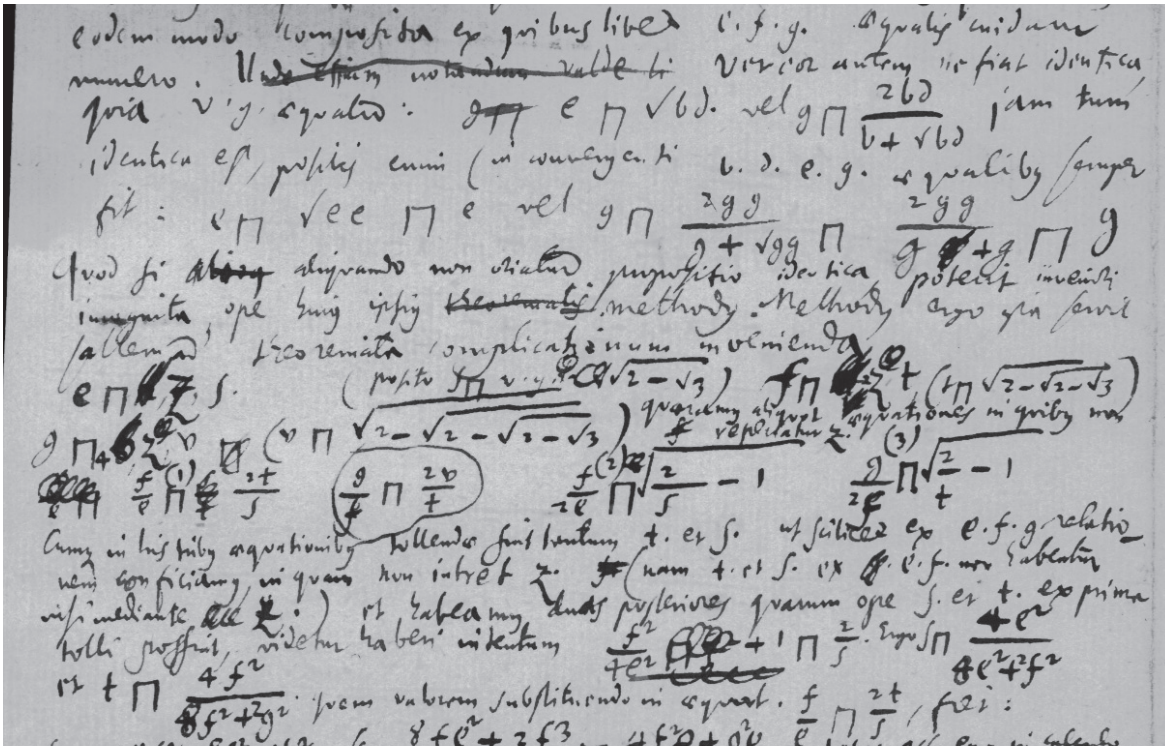
⊓ LEIBNIZIAN EQUAL
LH 35 XIII 3, fol. 72r



⊓ LEIBNIZIAN EQUAL, ⊓ LEIBNIZIAN GREATER, ⊓ LEIBNIZIAN LESS
LH 35 I 11, fol. 8r

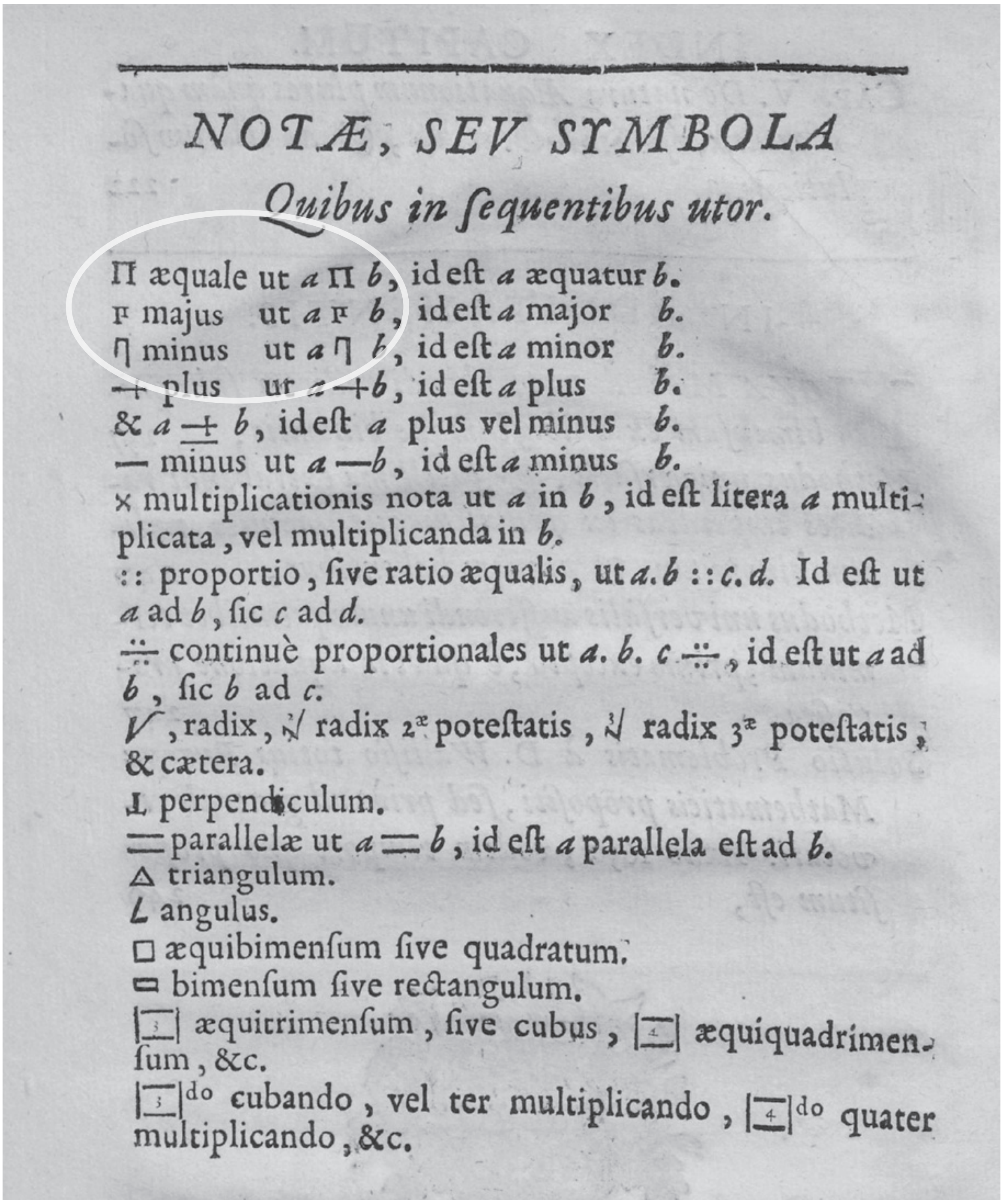


⊓ LEIBNIZIAN EQUAL, ⊓ LEIBNIZIAN GREATER, ⊓ LEIBNIZIAN LESS
LH 35 I 11, fol. 7v



⊓ LEIBNIZIAN EQUAL
LH 35 XIII 3, fol. 73v

Leibniz adopted the symbol (as well as the related symbols for “greater than” and “less than”) probably in 1674, after reading François Dulaurens: Specimina Mathematica Duobus Libris Comprehensa, Paris, 1667.

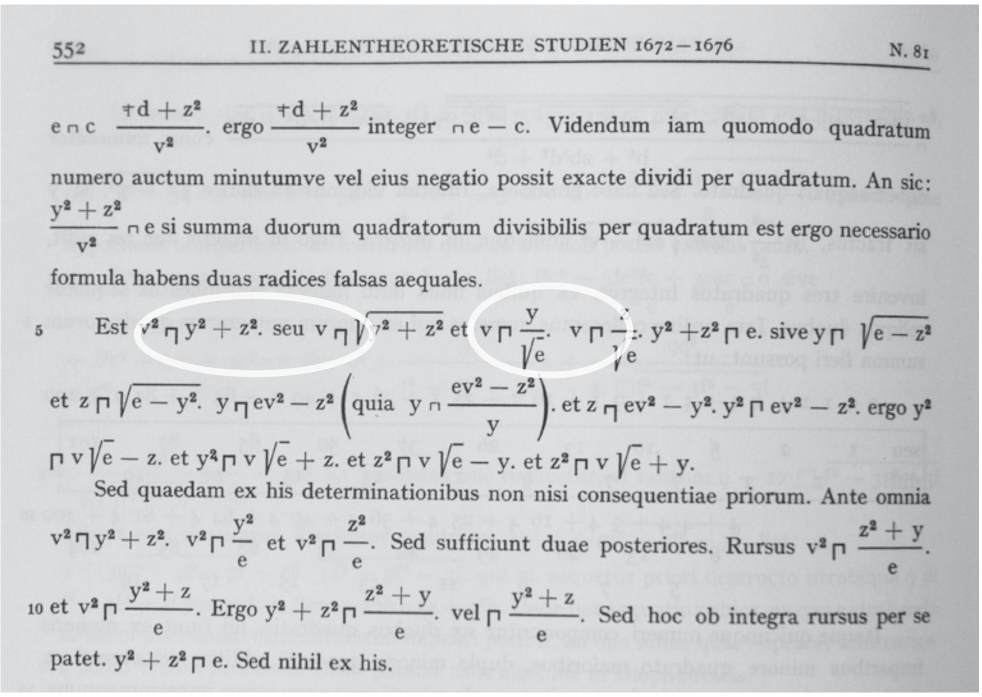


NOTÆ, SEV SYMBOLA

Quibus in ſequentibus utor.

Π æquale ut *a* Π *b*, id eſt *a* æquatur *b*.
ꟼ majus ut *a* ꟼ *b*, id eſt *a* major *b*.
ꞁ minus ut *a* ꞁ *b*, id eſt *a* minor *b*.
+ plus ut *a* + *b*, id eſt *a* plus *b*.
& *a* ± *b*, id eſt *a* plus vel minus *b*.
— minus ut *a* — *b*, id eſt *a* minus *b*.
× multiplicationis nota ut *a* in *b*, id eſt litera *a* multiplicata, vel multiplicanda in *b*.
:: proportio, ſive ratio æqualis, ut *a*. *b* :: *c*. *d*. Id eſt ut *a* ad *b*, ſic *c* ad *d*.
∺ continuè proportionales ut *a*. *b*. *c* ∺, id eſt ut *a* ad *b*, ſic *b* ad *c*.
√, radix, √2 radix 2æ poteſtatis, √3 radix 3æ poteſtatis, & cætera.
⊥ perpendiculum.
= parallelæ ut *a* = *b*, id eſt *a* parallela eſt ad *b*.
△ triangulum.
∠ angulus.
□ æquibimenſum ſive quadratum.
▭ bimenſum ſive rectangulum.
[3] æquitrimenſum, ſive cubus, [4] æquiquadrimenſum, &c.
[3]do cubando, vel ter multiplicando, [4]do quater multiplicando, &c.

⊓ LEIBNIZIAN EQUAL, ⊓ LEIBNIZIAN GREATER, ⊓ LEIBNIZIAN LESS
Dulaurens, Specimina Mathematica, 1667. Note the typesetter’s makeshift solution, he borrowed two different greek Π-characters for *æquale* and *majus*.



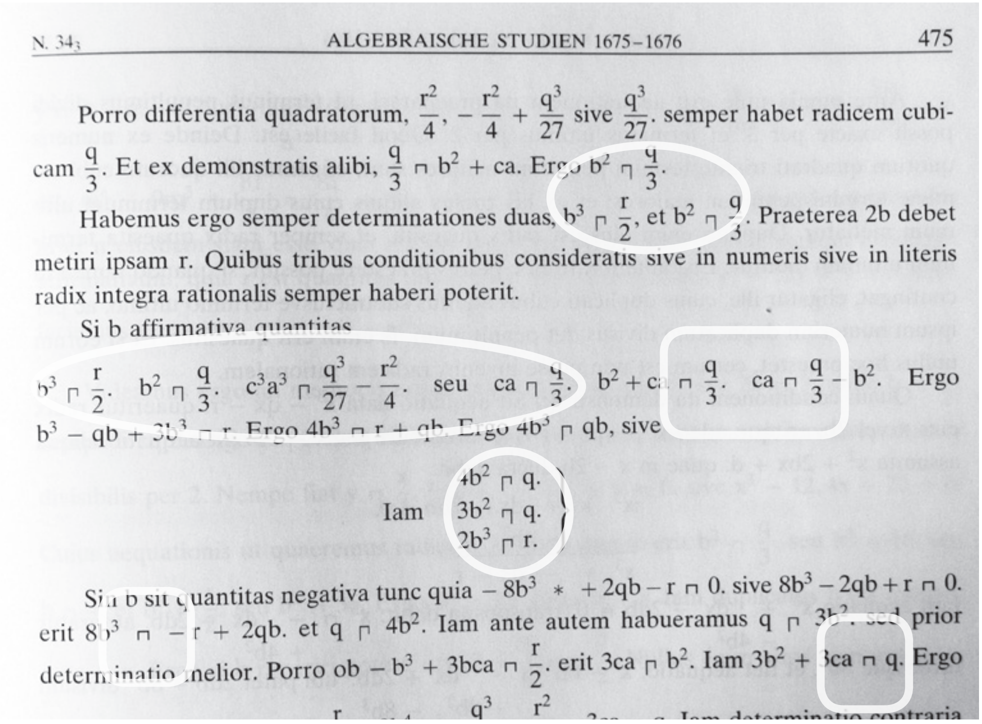
552 II. ZAHLENTHEORETISCHE STUDIEN 1672–1676 N. 81

e ⊓ c $\frac{\mp d + z^2}{v^2}$. ergo $\frac{\mp d + z^2}{v^2}$ integer ⊓ e − c. Videndum iam quomodo quadratum numero auctum minutumve vel eius negatio possit exacte dividi per quadratum. An sic: $\frac{y^2 + z^2}{v^2}$ ⊓ e si summa duorum quadratorum divisibilis per quadratum est ergo necessario formula habens duas radices falsas aequales.

Est v^2 ⊓ $y^2 + z^2$. seu v ⊓ $\sqrt{y^2 + z^2}$ et v ⊓ $\frac{y}{\sqrt{e}}$. v ⊓ $\frac{z}{\sqrt{e}}$. $y^2 + z^2$ ⊓ e. sive y ⊓ $\sqrt{e - z^2}$ et z ⊓ $\sqrt{e - y^2}$. y ⊓ $ev^2 - z^2$ (quia y ⊓ $\frac{ev^2 - z^2}{y}$). et z ⊓ $ev^2 - y^2$. y^2 ⊓ $ev^2 - z^2$. ergo y^2 ⊓ $v\sqrt{e} - z$. et y^2 ⊓ $v\sqrt{e} + z$. et z^2 ⊓ $v\sqrt{e} - y$. et z^2 ⊓ $v\sqrt{e} + y$.

Sed quaedam ex his determinationibus non nisi consequentiae priorum. Ante omnia v^2 ⊓ $y^2 + z^2$. v^2 ⊓ $\frac{y^2}{e}$ et v^2 ⊓ $\frac{z^2}{e}$. Sed sufficiunt duae posteriores. Rursus v^2 ⊓ $\frac{z^2 + y}{e}$. et v^2 ⊓ $\frac{y^2 + z}{e}$. Ergo $y^2 + z^2$ ⊓ $\frac{z^2 + y}{e}$. vel ⊓ $\frac{y^2 + z}{e}$. Sed hoc ob integra rursus per se patet. $y^2 + z^2$ ⊓ e. Sed nihil ex his.

⊓ LEIBNIZIAN GREATER, ⊓ LEIBNIZIAN LESS
LAA VII-1 p. 552



N. 34₃ ALGEBRAISCHE STUDIEN 1675–1676 475

Porro differentia quadratorum, $\frac{r^2}{4}$, $-\frac{r^2}{4} + \frac{q^3}{27}$ sive $\frac{q^3}{27}$. semper habet radicem cubicam $\frac{q}{3}$. Et ex demonstratis alibi, $\frac{q}{3}$ ⊓ $b^2 + ca$. Ergo b^2 ⊓ $\frac{q}{3}$.

Habemus ergo semper determinationes duas, b^3 ⊓ $\frac{r}{2}$, et b^2 ⊓ $\frac{q}{3}$. Praeterea 2b debet metiri ipsam r. Quibus tribus conditionibus consideratis sive in numeris sive in literis radix integra rationalis semper haberi poterit.

Si b affirmativa quantitas

b^3 ⊓ $\frac{r}{2}$. b^2 ⊓ $\frac{q}{3}$. c^3a^3 ⊓ $\frac{q^3}{27} - \frac{r^2}{4}$. seu ca ⊓ $\frac{q}{3}$. $b^2 + ca$ ⊓ $\frac{q}{3}$. ca ⊓ $\frac{q}{3} - b^2$. Ergo $b^3 - qb + 3b^3$ ⊓ r. Ergo $4b^3$ ⊓ r + qb. Ergo $4b^3$ ⊓ qb, sive

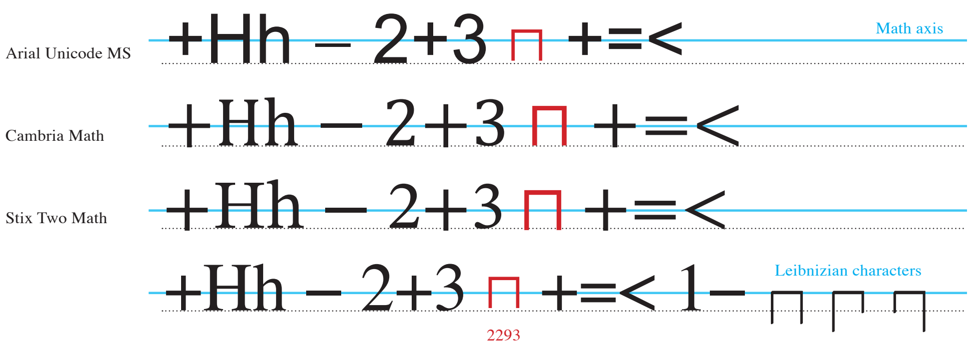
Iam $4b^2$ ⊓ q. $3b^2$ ⊓ q. $2b^3$ ⊓ r.

Si b sit quantitas negativa tunc quia $-8b^3 * +2qb - r$ ⊓ 0. sive $8b^3 - 2qb + r$ ⊓ 0. erit $8b^3$ ⊓ $-r + 2qb$. et q ⊓ $4b^2$. Iam ante autem habueramus q ⊓ $3b^2$. sed prior determinatio melior. Porro ob $-b^3 + 3bca$ ⊓ $\frac{r}{2}$. erit 3ca ⊓ b^2. Iam $3b^2 + 3ca$ ⊓ q. Ergo

⊓ LEIBNIZIAN EQUAL, ⊓ LEIBNIZIAN GREATER, ⊓ LEIBNIZIAN LESS
LAA VII-2 p. 475

Ideally these character's glyphs are adjusted with their horizontal parts to the *math axis*, like e.g. + and –

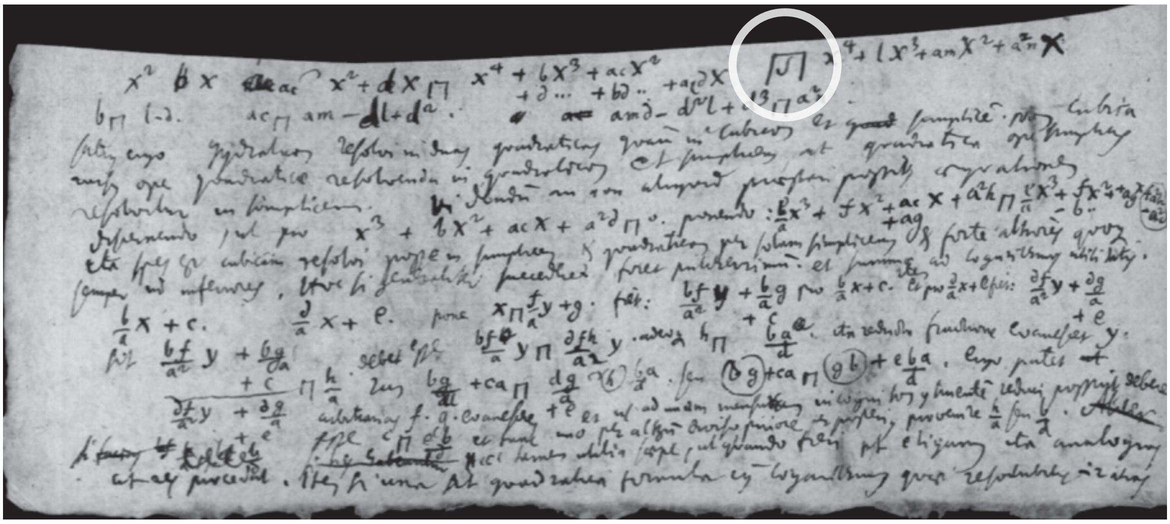
Whereas the printer of Dulaurens' book (mis-)used capital Greek Pi types as stand-ins for *equality* and *greater,* thus getting the representations of *greater* and *less* inconsistent; in Leibniz's manuscripts we encounter a well-considered coordination of these signs: The *equals* sign represents, as it were, a balance beam with two equal weights symbolized by the vertical strokes. For *greater* and *less,* respectively, vertical strokes of unequal length are used. These symbols have to be aligned vertically with their horizontal parts to the *math axis* which is usually represented by the vertical centres of + and – (*plus, minus*). This graphosystemic requirement together with different semantics exclude ⊓ LEIBNIZIAN EQUAL from being united with the (visually similar) character 2293 ⊓ SQUARE CAP.



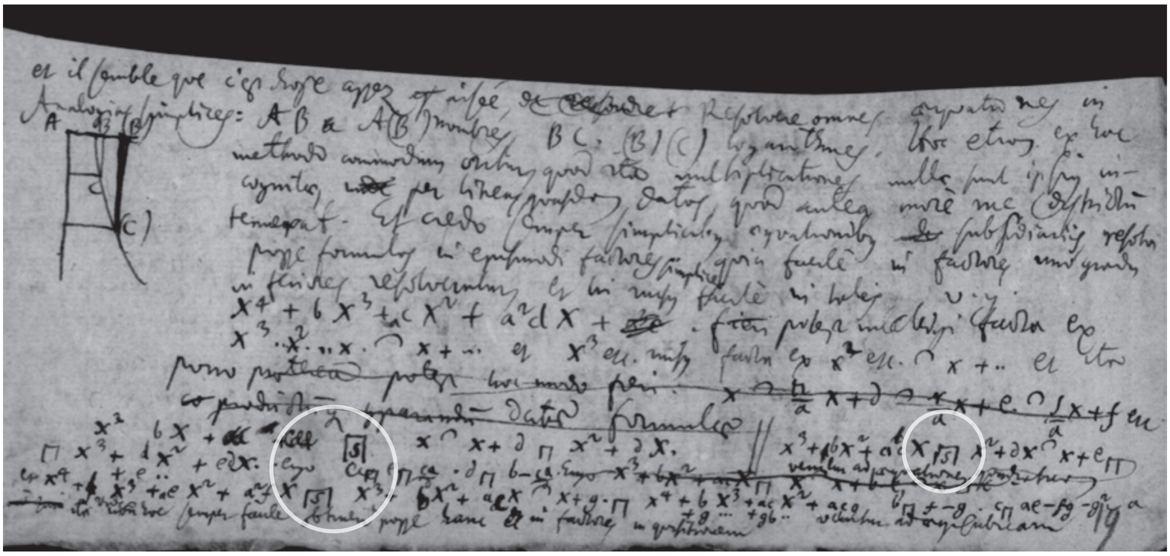
Due to their semantical connections, the 2293 ⊓ SQUARE CAP, 2229 ∩ INTERSECTION, 222A ∪ UNION and 2294 ⊔ SQUARE CUP characters need a strong consistency in their visual representation. The same is needed for ⊓ LEIBNIZIAN EQUAL, ⊓ LEIBNIZIAN EQUAL WITH DOUBLE VERTICALS, ⊓ LEIBNIZIAN EQUALITY WITH S, ┌ LEIBNIZIAN GREATER, ┐ LEIBNIZIAN LESS, ┌ LEIBNIZIAN GREATER WITH SMALL P, ┐ LEIBNIZIAN LESS WITH SMALL P and ┌┐ LEIBNIZIAN GREATER-LESS.

This is how the glyphs of the new characters may be integrated into a Roman-style typeface:

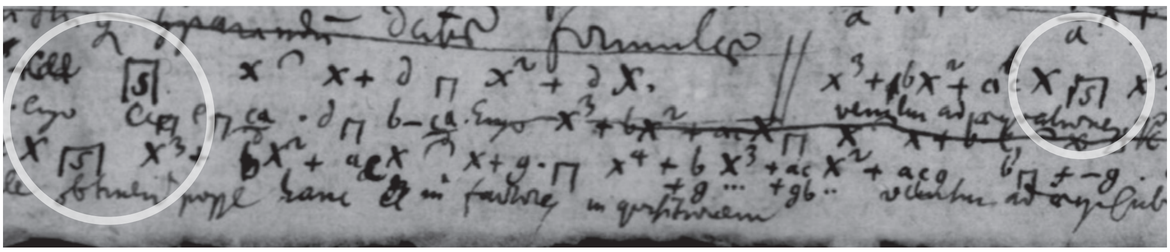
Leibniz derived the configurations of several other symbols from ⊓ LEIBNIZIAN EQUAL:
⊡ LEIBNIZIAN EQUALITY WITH S denotes a kind of equality by definition that originates from equating two expressions with each other as in the phrase “let *a* be equal to *b*”. Unlike the definition sign in modern mathematics, there is no specific direction in Leibniz’s sign. The “s” in the sign is an abbreviation of the Latin word “sit”.
Combining both ⊓ and ⊓ into ⊓̅ LEIBNIZIAN GREATER-LESS leads to an ambiguous inequality sign that denotes “greater than in the first case and less than in the second case”.

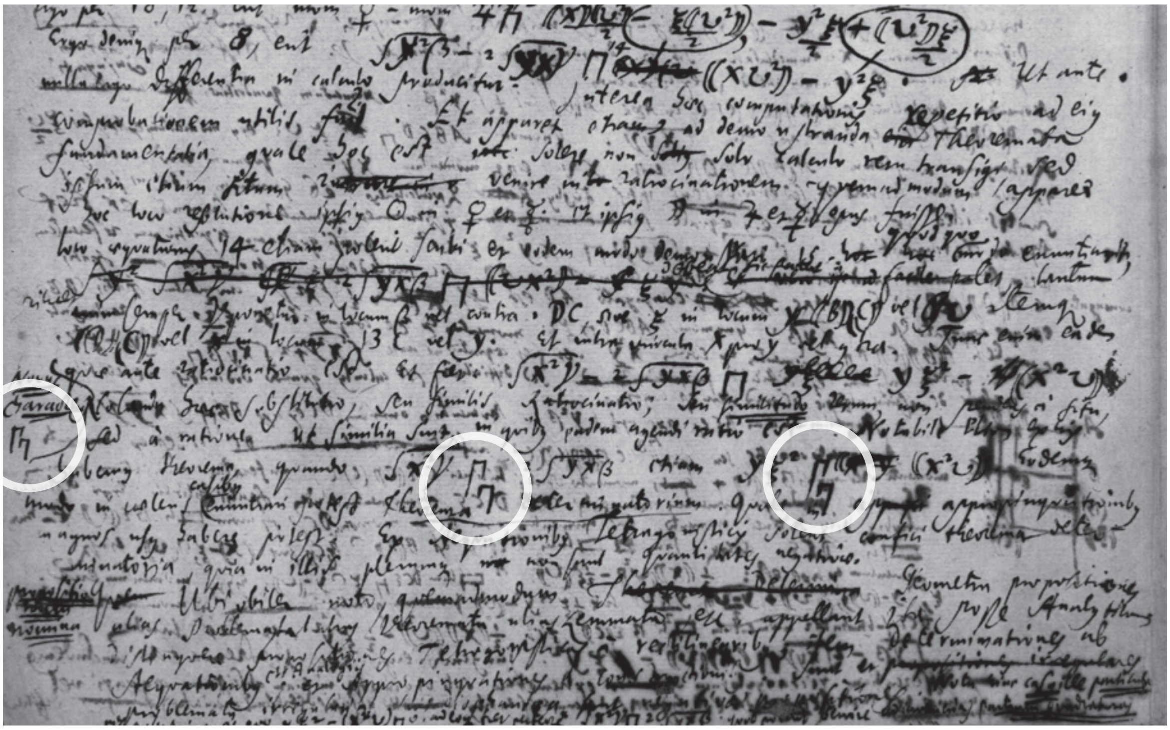


⊡ LEIBNIZIAN EQUALITY WITH S
LH 35 V 14, fol. 18r. *The edition of this manuscript is currently in progress.*



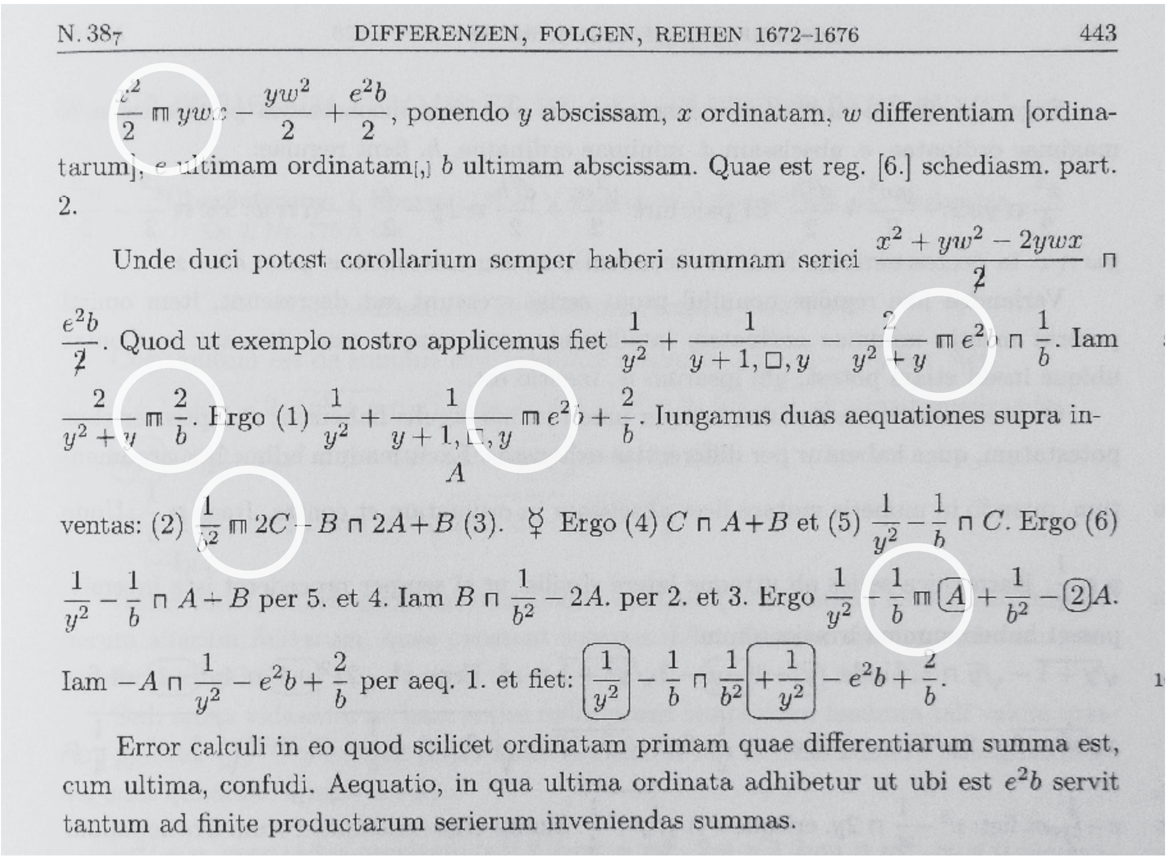
⊡ LEIBNIZIAN EQUALITY WITH S
LH 35 V 14, fol. 19r. *The edition of this manuscript is currently in progress.*





⊓⊓ LEIBNIZIAN GREATER-LESS

LH 35 XIII 3, fol. 150v. *The edition of this manuscript is currently in progress*.



N. 38₇ DIFFERENZEN, FOLGEN, REIHEN 1672–1676 443

$\frac{x^2}{2} \mathbin{\sqcap\!\!\!\sqcap} ywx - \frac{yw^2}{2} + \frac{e^2 b}{2}$, ponendo y abscissam, x ordinatam, w differentiam [ordinatarum], e ultimam ordinatam[,] b ultimam abscissam. Quae est reg. [6.] schediasm. part. 2.

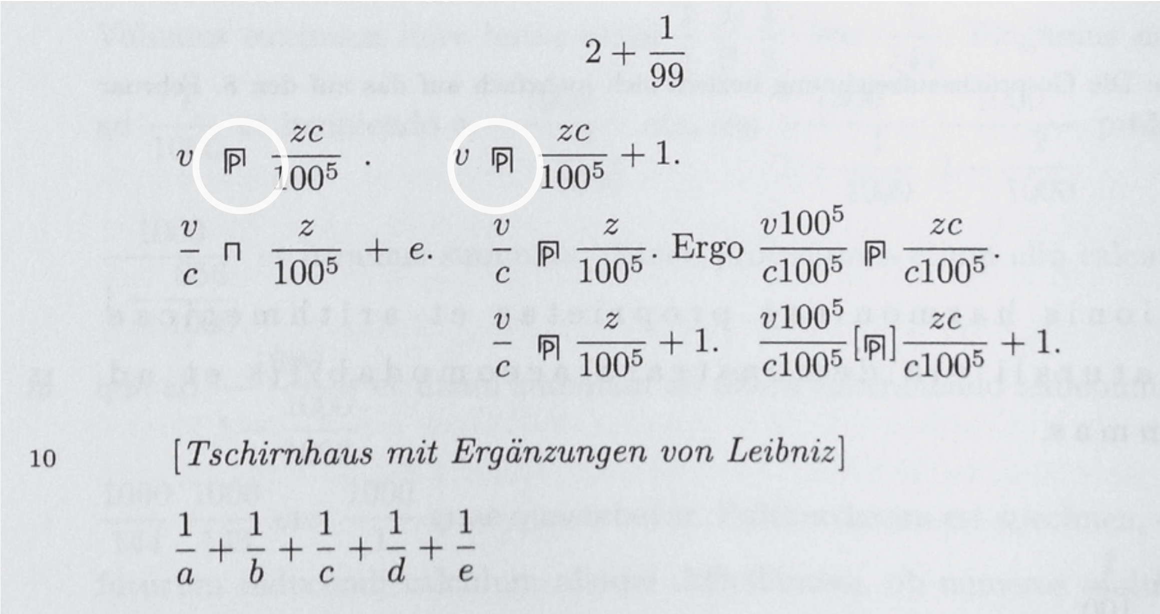
Unde duci potest corollarium semper haberi summam seriei $\frac{x^2 + yw^2 - 2ywx}{2} \sqcap \frac{e^2 b}{2}$. Quod ut exemplo nostro applicemus fiet $\frac{1}{y^2} + \frac{1}{y+1, \square, y} - \frac{2}{y^2+y} \mathbin{\sqcap\!\!\!\sqcap} e^2 b \sqcap \frac{1}{b}$. Iam $\frac{2}{y^2+y} \mathbin{\sqcap\!\!\!\sqcap} \frac{2}{b}$. Ergo (1) $\frac{1}{y^2} + \underbrace{\frac{1}{y+1, \square, y}}_{A} \mathbin{\sqcap\!\!\!\sqcap} e^2 b + \frac{2}{b}$. Iungamus duas aequationes supra inventas: (2) $\frac{1}{y^2} \mathbin{\sqcap\!\!\!\sqcap} 2C - B \sqcap 2A + B$ (3). ☿ Ergo (4) $C \sqcap A + B$ et (5) $\frac{1}{y^2} - \frac{1}{b} \sqcap C$. Ergo (6) $\frac{1}{y^2} - \frac{1}{b} \sqcap A + B$ per 5. et 4. Iam $B \sqcap \frac{1}{b^2} - 2A$. per 2. et 3. Ergo $\frac{1}{y^2} - \frac{1}{b} \mathbin{\sqcap\!\!\!\sqcap} \boxed{A} + \frac{1}{b^2} - \boxed{2}A$. Iam $-A \sqcap \frac{1}{y^2} - e^2 b + \frac{2}{b}$ per aeq. 1. et fiet: $\boxed{\frac{1}{y^2}} - \frac{1}{b} \sqcap \frac{1}{b^2} \boxed{+\frac{1}{y^2}} - e^2 b + \frac{2}{b}$.

Error calculi in eo quod scilicet ordinatam primam quae differentiarum summa est, cum ultima, confudi. Aequatio, in qua ultima ordinata adhibetur ut ubi est $e^2 b$ servit tantum ad finite productarum serierum inveniendas summas.

⊓⊓ LEIBNIZIAN EQUAL WITH DOUBLE VERTICALS

Leibniz uses this symbol for “equality of sums”.

LAA VII-3 p. 443



$$2+\frac{1}{99}$$

$$v \;\ulcorner\!p\; \frac{zc}{100^5}. \qquad v \;p\!\urcorner\; \frac{zc}{100^5}+1.$$

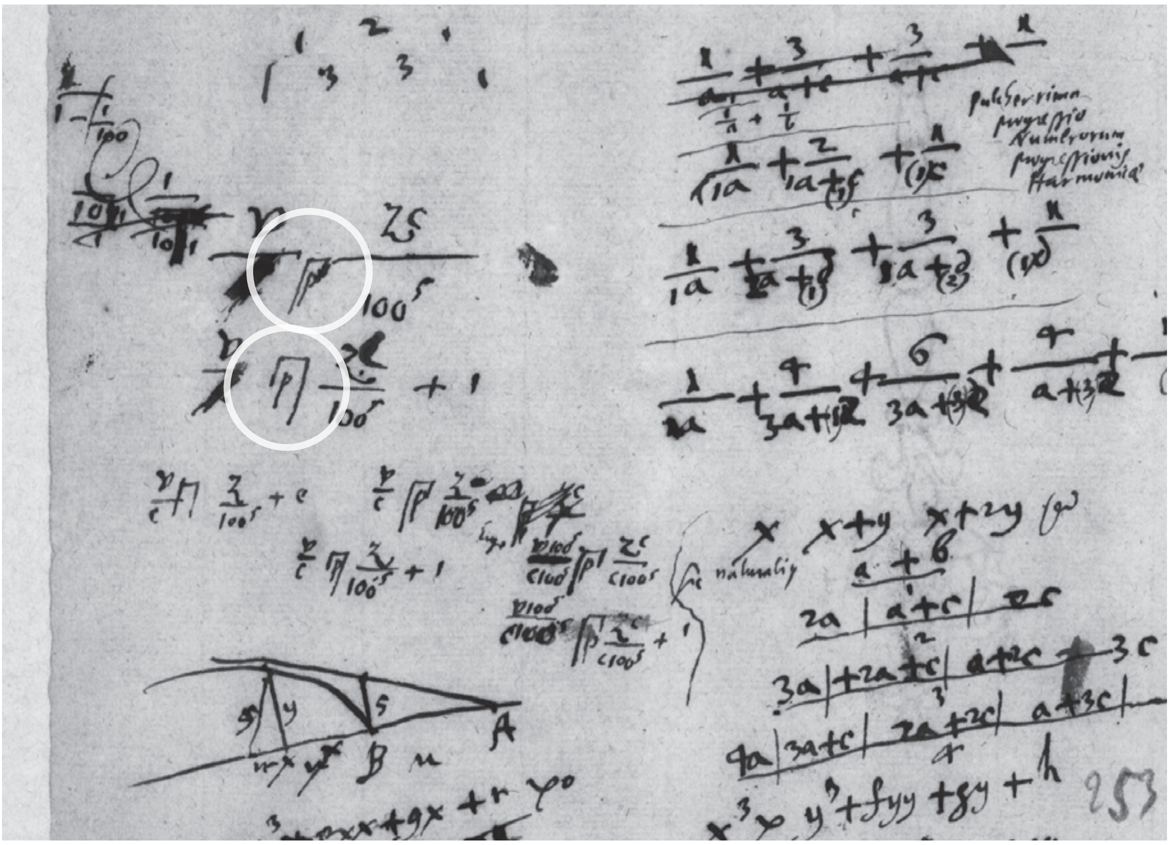
$$\frac{v}{c} \sqcap \frac{z}{100^5}+e. \quad \frac{v}{c} \;\ulcorner\!p\; \frac{z}{100^5}. \text{ Ergo } \frac{v100^5}{c100^5} \;\ulcorner\!p\; \frac{zc}{c100^5}.$$

$$\frac{v}{c} \;p\!\urcorner\; \frac{z}{100^5}+1. \quad \frac{v100^5}{c100^5}\,[p\!\urcorner]\,\frac{zc}{c100^5}+1.$$

[*Tschirnhaus mit Ergänzungen von Leibniz*]

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}+\frac{1}{e}$$

$\ulcorner\!p$ LEIBNIZIAN GREATER WITH SMALL P, $p\!\urcorner$ LEIBNIZIAN LESS WITH SMALL P
These symbols denote “a little bit greater” and “a little bit less”, the letter “p” abbreviating the Latin word “paulo” (little). – *Corresponding Ms.: see below.*
LAA VII-3 p. 732



$\ulcorner\!p$ LEIBNIZIAN GREATER WITH SMALL P, $p\!\urcorner$ LEIBNIZIAN LESS WITH SMALL P
The handwriting shows that a lowercase p was intended by the author, so the representation of these symbols in the printed edition is not accurate in this respect.
LH 35 XII 1, fol. 253r

(7) Ungleichungen:

Zusätzlich zu den üblichen Symbolen ⊓ für „größer“ und ⊓ für „kleiner“ (N. 66) führt Leibniz noch Zeichen für „ein wenig größer“ (⊓p) bzw. „ein wenig kleiner“ (⊓p) ein (N. 54).

⊓p LEIBNIZIAN GREATER WITH SMALL P, ⊓p LEIBNIZIAN LESS WITH SMALL P
LAA VII-3 p. XXXI

Demonstr. Productae sint particulae curvae in tangentes $DBCL$, ECG, eritque $LBI = DBA = BAC + BCA = BMC$ $(2BFC) + BCA = 2ECD + BCA = ECD + ECA = LCG + GCH = LCH$. Ergo BI parallela CH. Quod si a sit intra circulum, erit DBa ⊏ $DBA = LBI$, quare divaricabitur a CH. Sin α sit extra circulum, erit $DB\alpha$ ⊐ $DBA = LBI$, quare coibit cum CH. Q. E. D.

Coroll. Hinc possunt inveniri puncta Causticae: Nam quia $BF = 2BM$; et

⊐ INVERTED SQUARE RIGHT OPEN BOX OPERATOR, here bearing the meaning of “less”, alongside with ⊏ SQUARE LEFT OPEN BOX OPERATOR 2ACD
LAA III-6 p. 688; corresponding manuscript part (below)

α P F

Demonstr. Productae sint particulae curvae in tangentes DBCL, ECG, eritque LBI = DBA = BAC + BCA = BMC (2BFC) + BCA = 2ECD + BCA = ECD + ECA = LCG + GCH = LCH. Ergo BI parallela CH. Quod si autem sit intra circulum, erit DBa ⊏ DBA = LBI, quare divaricabitur à CH. Sin α sit extra Circulum, erit DBα ⊐ DBA = LBI. quare coibit cum CH. Q. E. D.

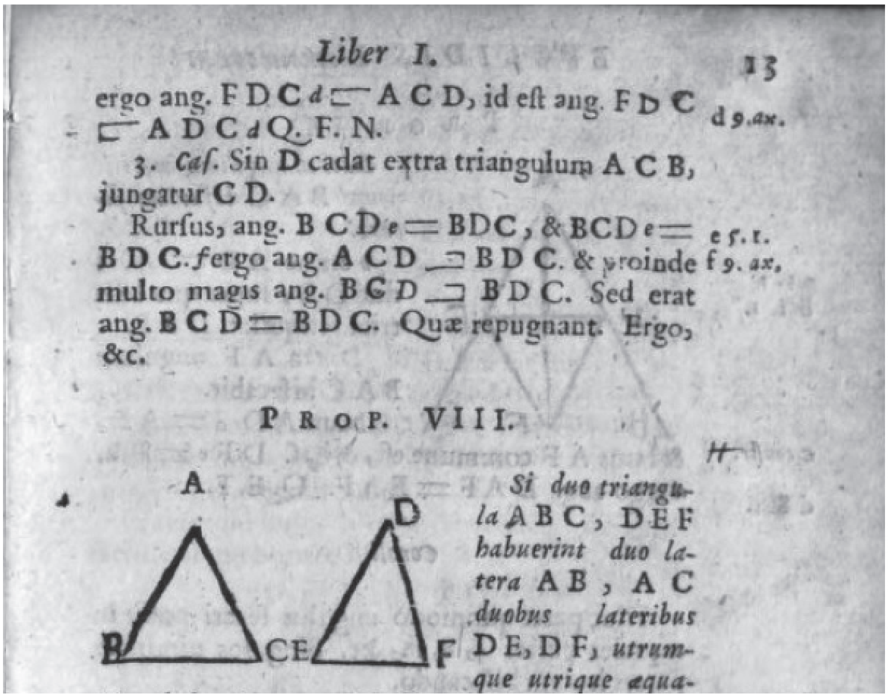
... Caustica: Nam quia BF = 2BM.

Distinct from the above signs are these two greater / less signs, which lack the vertical part:

Et pro $\frac{\sqrt{aa+b\sqrt{cc+dd}}}{e+\sqrt{f\sqrt{gg+hh}+kk}}$ scribi poterit $\sqrt{(aa+b\sqrt{(cc+dd)})}:,e+\sqrt{(f\sqrt{(gg+hh)}+kk)}$

Hactenus notas exposuimus, quibus termini, id est numeri vel quantitates formantur, tanquam subjecta aut prædicata in veritatibus. Sequuntur notæ quæ explicant modum prædicationis, seu quomodo quantitates quæ Terminos constituunt in propositiones conjungantur, potissimum autem de iis enuntiatur, *Aequales* esse, vel *majores*, aut *minores* aliis, itaque $a = b$ significat, a, esse æquale ipsi b, & a ⩵ b significat a esse majus quàm b, & a ⩵ b significat a esse minus quàm b. Sed

⩵ TWO-LINE GREATER, ⩵ TWO-LINE LESS
Monitum de Characteribus Algebraica, Miscellanea Berolinensia, 1710, p. 158



Liber I. 13

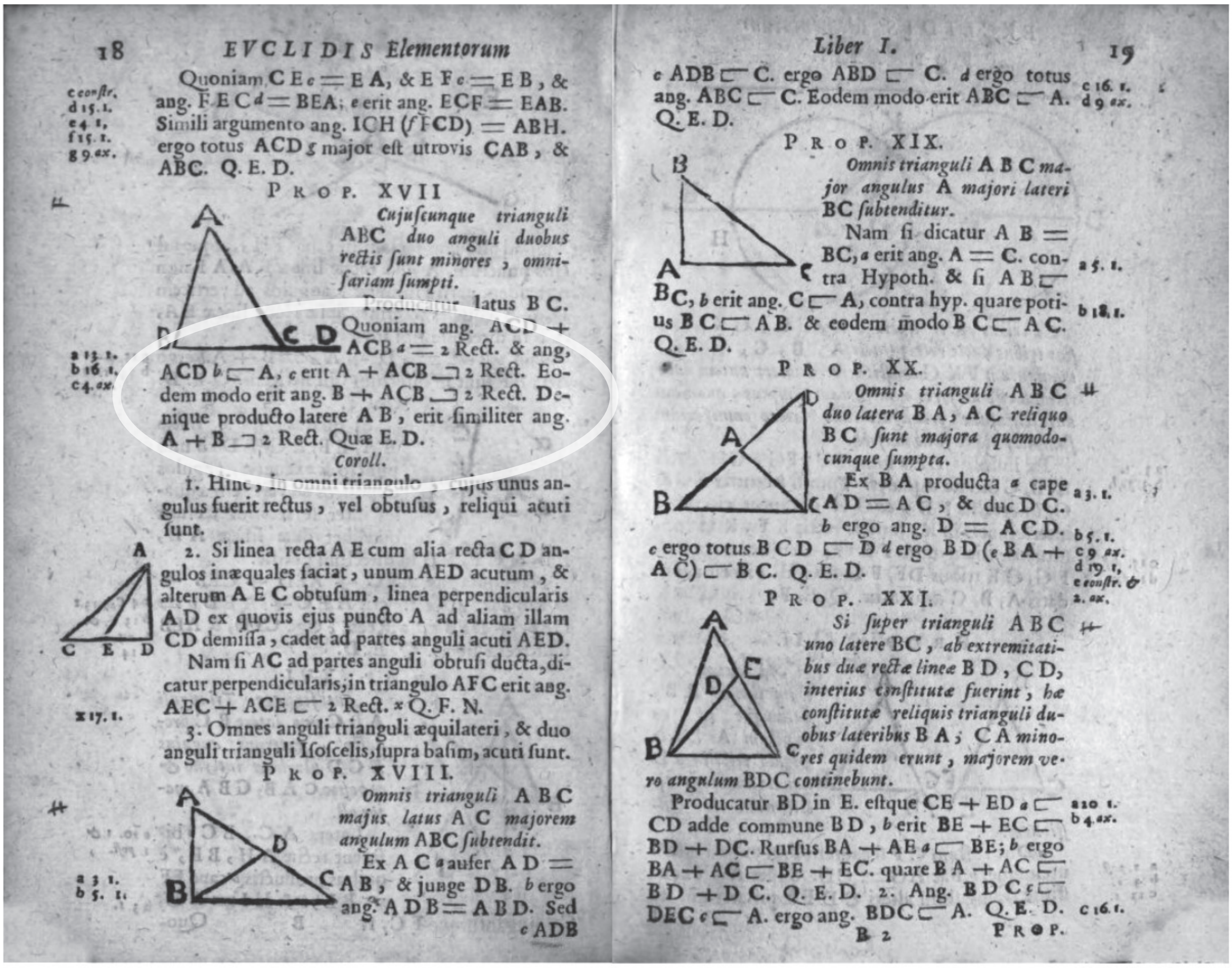
ergo ang. FDC d ⊏ ACD, id eſt ang. FDC ⊏ ADC d Q. F. N. d 9. ax.

3. *Caſ.* Sin D cadat extra triangulum ACB, jungatur CD.

Rurſus, ang. BCD e = BDC, & BCD e = BDC. f ergo ang. ACD ⊐ BDC. & proinde multo magis ang. BCD ⊐ BDC. Sed erat ang. BCD = BDC. Quæ repugnant. Ergo, &c. e 5. 1. f 9. ax.

PROP. VIII.

Si duo triangula ABC, DEF habuerint duo latera AB, AC duobus lateribus DE, DF, utrumque utrique æqua-



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c conſtr. d 15. 1. e 4. 1. f 15. 1. g 9. ax. Quoniam CE c = EA, & EF c = EB, & ang. FEC d = BEA; e erit ang. ECF = EAB. Simili argumento ang. ICH (f FCD) = ABH. ergo totus ACD g major eſt utrovis CAB, & ABC. Q. E. D.

PROP. XVII

Cujuſcunque trianguli ABC duo anguli duobus rectis ſunt minores, omnifariam ſumpti.

Producatur latus BC. Quoniam ang. ACD + ACB a = 2 Rect. & ang. ACD b ⊏ A, c erit A + ACB ⊐ 2 Rect. Eodem modo erit ang. B + ACB ⊐ 2 Rect. Denique producto latere AB, erit ſimiliter ang. A + B ⊐ 2 Rect. Quæ E. D. a 13. 1. b 16. 1. c 4. ax.

Coroll.

1. Hinc, in omni triangulo, cujus unus angulus fuerit rectus, vel obtuſus, reliqui acuti ſunt.

2. Si linea recta AE cum alia recta CD angulos inæquales faciat, unum AED acutum, & alterum AEC obtuſum, linea perpendicularis AD ex quovis ejus puncto A ad aliam illam CD demiſſa, cadet ad partes anguli acuti AED.

Nam ſi AC ad partes anguli obtuſi ducta, dicatur perpendicularis, in triangulo AFC erit ang. AEC + ACE ⊏ 2 Rect. x Q. F. N. x 17. 1.

3. Omnes anguli trianguli æquilateri, & duo anguli trianguli Iſoſcelis, ſupra baſim, acuti ſunt.

PROP. XVIII.

Omnis trianguli ABC majus latus AC majorem angulum ABC ſubtendit.

a 3. 1. b 5. 1. Ex AC a aufer AD = AB, & junge DB. b ergo ang. ADB = ABD. Sed c ADB

Liber I. 19

c ADB ⊏ C. ergo ABD ⊏ C. d ergo totus ang. ABC ⊏ C. Eodem modo erit ABC ⊏ A. Q. E. D. c 16. 1. d 9 ax.

PROP. XIX.

Omnis trianguli ABC major angulus A majori lateri BC ſubtenditur.

Nam ſi dicatur AB = BC, a erit ang. A = C. contra Hypoth. & ſi AB ⊏ BC, b erit ang. C ⊏ A, contra hyp. quare potius BC ⊏ AB. & eodem modo BC ⊏ AC. Q. E. D. a 5. 1. b 18. 1.

PROP. XX.

Omnis trianguli ABC duo latera BA, AC reliquo BC ſunt majora quomodocunque ſumpta.

Ex BA producta a cape AD = AC, & duc DC. b ergo ang. D = ACD. c ergo totus BCD ⊏ D d ergo BD (e BA + AC) ⊏ BC. Q. E. D. a 3. 1. b 5. 1. c 9 ax. d 19. 1. e conſtr. & 2. ax.

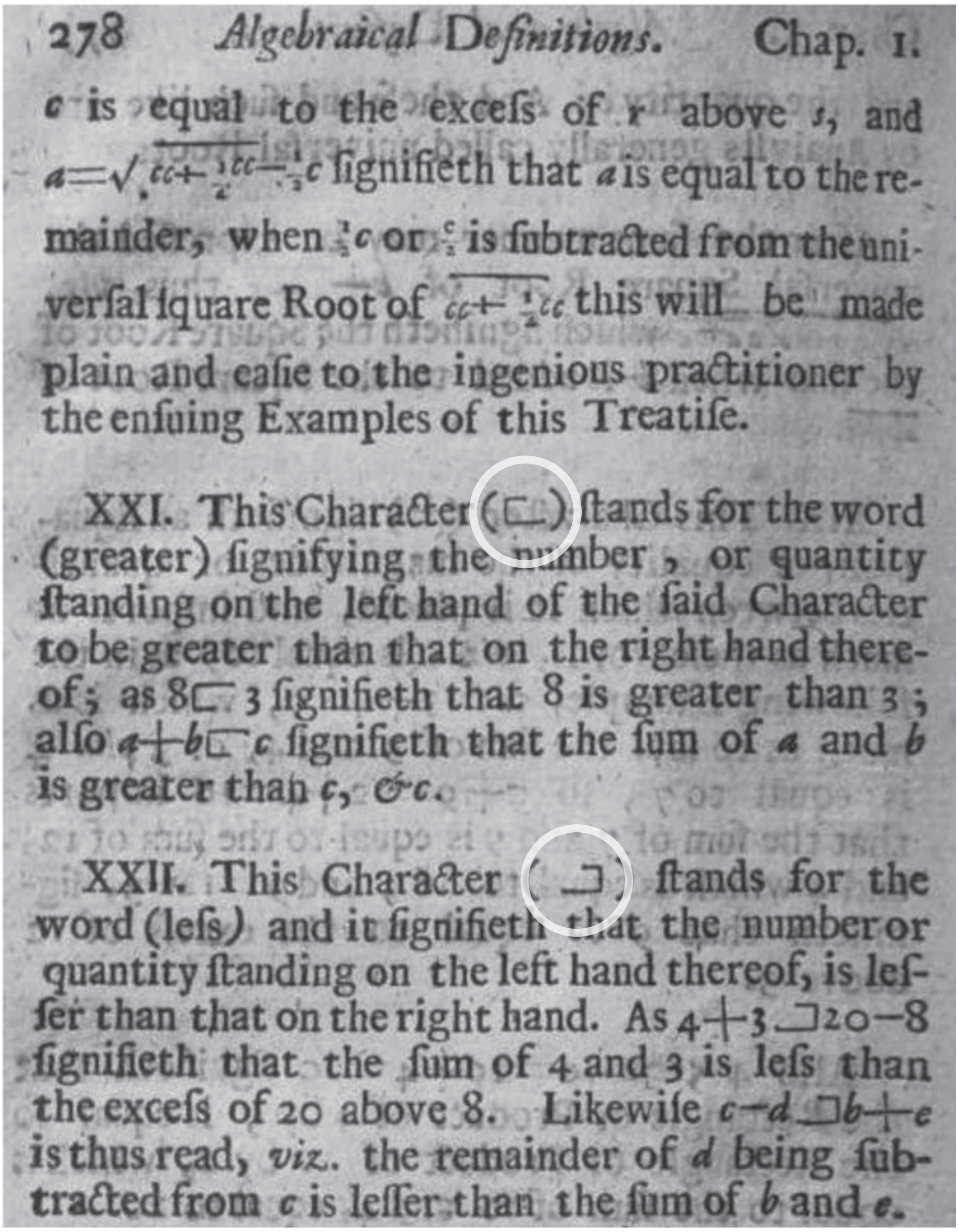
PROP. XXI.

Si ſuper trianguli ABC uno latere BC, ab extremitatibus duæ rectæ lineæ BD, CD, interius conſtitutæ fuerint, hæ conſtitutæ reliquis trianguli duobus lateribus BA, CA minores quidem erunt, majorem vero angulum BDC continebunt.

Producatur BD in E. eſtque CE + ED a ⊏ CD adde commune BD, b erit BE + EC ⊏ BD + DC. Rurſus BA + AE a ⊏ BE; b ergo BA + AC ⊏ BE + EC. quare BA + AC ⊏ BD + DC. Q. E. D. 2. Ang. BDC c ⊏ DEC c ⊏ A. ergo ang. BDC ⊏ A. Q. E. D. a 20. 1. b 4. ax. c 16. 1.

B 2 PROP.

⊐ INVERTED SQUARE RIGHT OPEN BOX OPERATOR
Barrow 1659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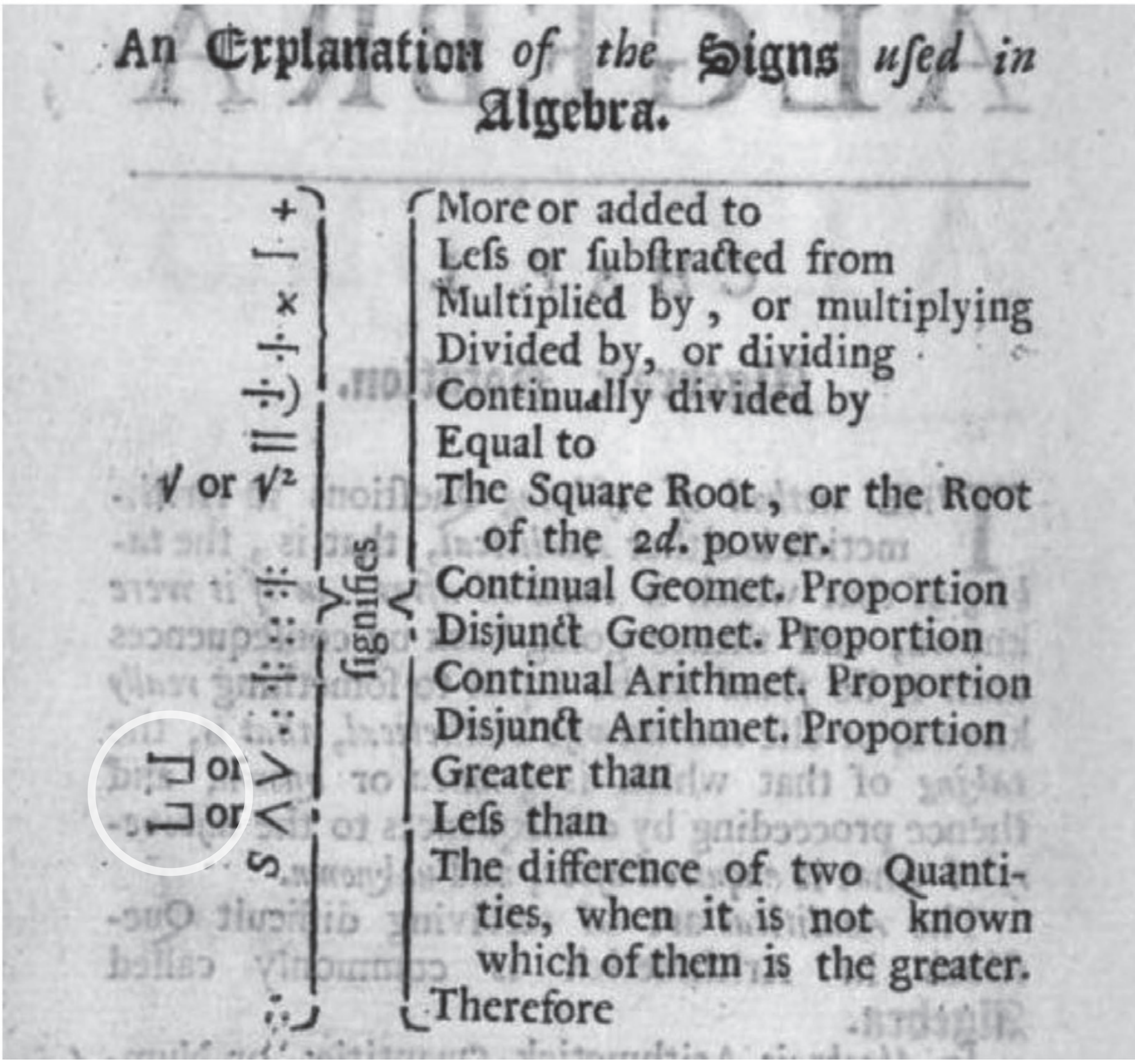
c is equal to the exceſs of *r* above *s*, and $a=\sqrt{.\,cc+\frac{1}{4}cc}-\frac{1}{2}c$ ſignifieth that *a* is equal to the remainder, when $\frac{1}{2}c$ or $\frac{c}{2}$ is ſubtracted from the univerſal ſquare Root of $\overline{cc+\frac{1}{4}cc}$ this will be made plain and eaſie to the ingenious practitioner by the enſuing Examples of this Treatiſe.

XXI. This Character (⊏) ſtands for the word (greater) ſignifying the number, or quantity ſtanding on the left hand of the ſaid Character to be greater than that on the right hand thereof; as 8⊏3 ſignifieth that 8 is greater than 3; alſo *a*+*b*⊏*c* ſignifieth that the ſum of *a* and *b* is greater than *c*, &c.

XXII. This Character (⊐) ſtands for the word (leſs) and it ſignifieth that the number or quantity ſtanding on the left hand thereof, is leſſer than that on the right hand. As 4+3⊐20−8 ſignifieth that the ſum of 4 and 3 is leſs than the exceſs of 20 above 8. Likewiſe *c*−*d*⊐*b*+*e* is thus read, *viz.* the remainder of *d* being ſubtracted from *c* is leſſer than the ſum of *b* and *e*.

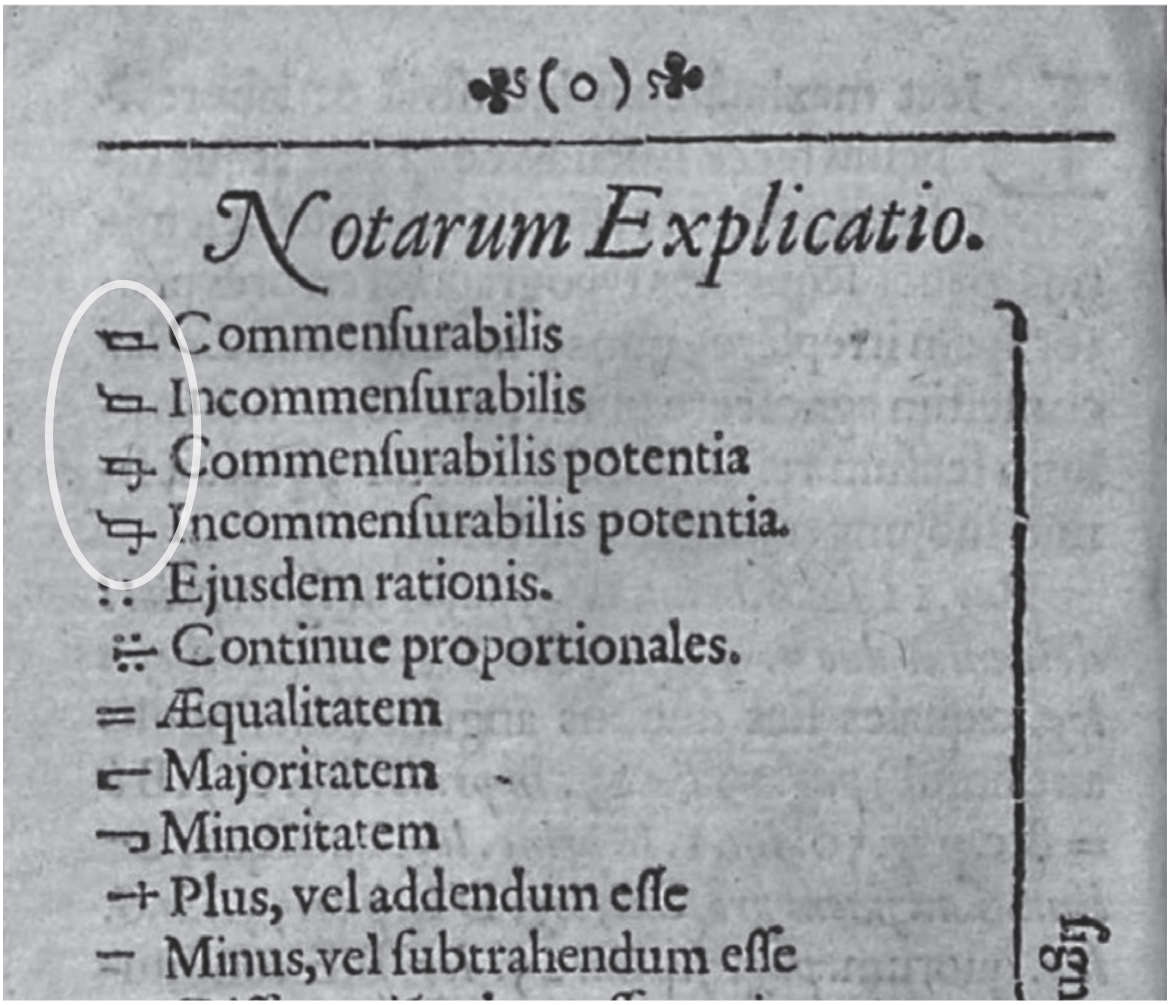
In this 1685 edition of Edward Cocker's Decimal Arithmetick ⊏ INVERTED SQUARE LEFT OPEN BOX OPERATOR is used to denote *greater*, whereas ⊐ INVERTED SQUARE RIGHT OPEN BOX OPERATOR stands for *less*. Source: Google books



An Explanation of the Signs used in Algebra.

+	signifies	More or added to
—		Less or substracted from
×		Multiplied by, or multiplying
÷		Divided by, or dividing
÷)		Continually divided by
=		Equal to
√ or √²		The Square Root, or the Root of the 2d. power.
∺		Continual Geomet. Proportion
::		Disjunct Geomet. Proportion
÷		Continual Arithmet. Proportion
∴		Disjunct Arithmet. Proportion
⫎ or >		Greater than
⊐ or <		Less than
∽		The difference of two Quantities, when it is not known which of them is the greater.
∴		Therefore

⫎ 2ACE for *greater than,* ⊐ INVERTED SQUARE RIGHT OPEN BOX OPERATOR for *less than*. John Parsons, Thomas Wastell: Clavis Arithmeticae, 1705. Source: Google books

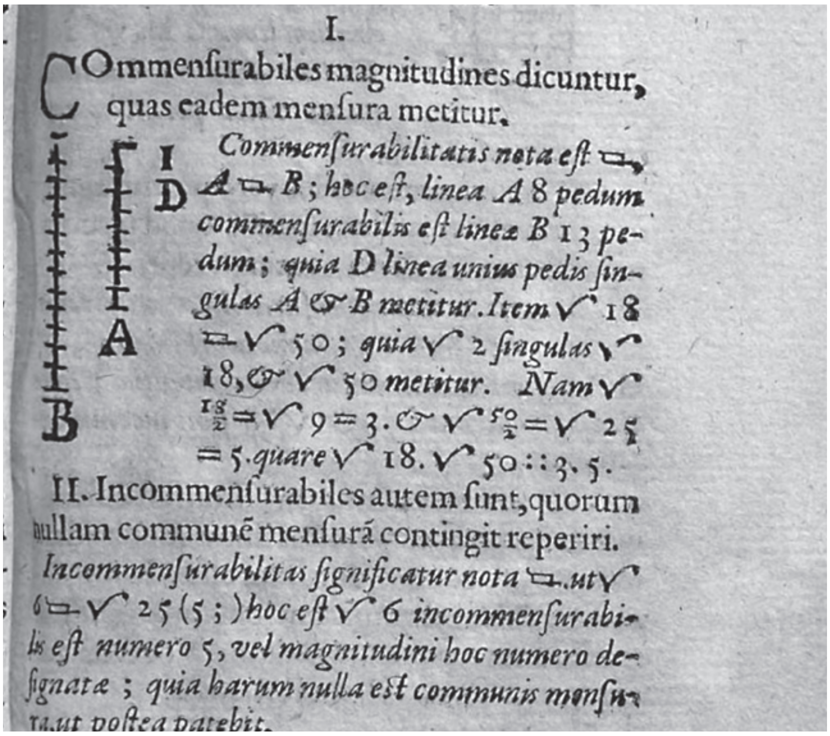


(o)

Notarum Explicatio.

Commenſurabilis
Incommenſurabilis
Commenſurabilis potentia
Incommenſurabilis potentia.
∷ Ejusdem rationis.
∺ Continue proportionales.
= Æqualitatem
Majoritatem
Minoritatem
+ Plus, vel addendum eſſe
− Minus, vel ſubtrahendum eſſe

COMMENSURABILITY, INCOMMENSURABILITY, COMMENSURABILITY IN SQUARE, INCOMMENSURABILITY IN SQUARE
Barrow 1676



I.

Commensurabiles magnitudines dicuntur, quas eadem mensura metitur.

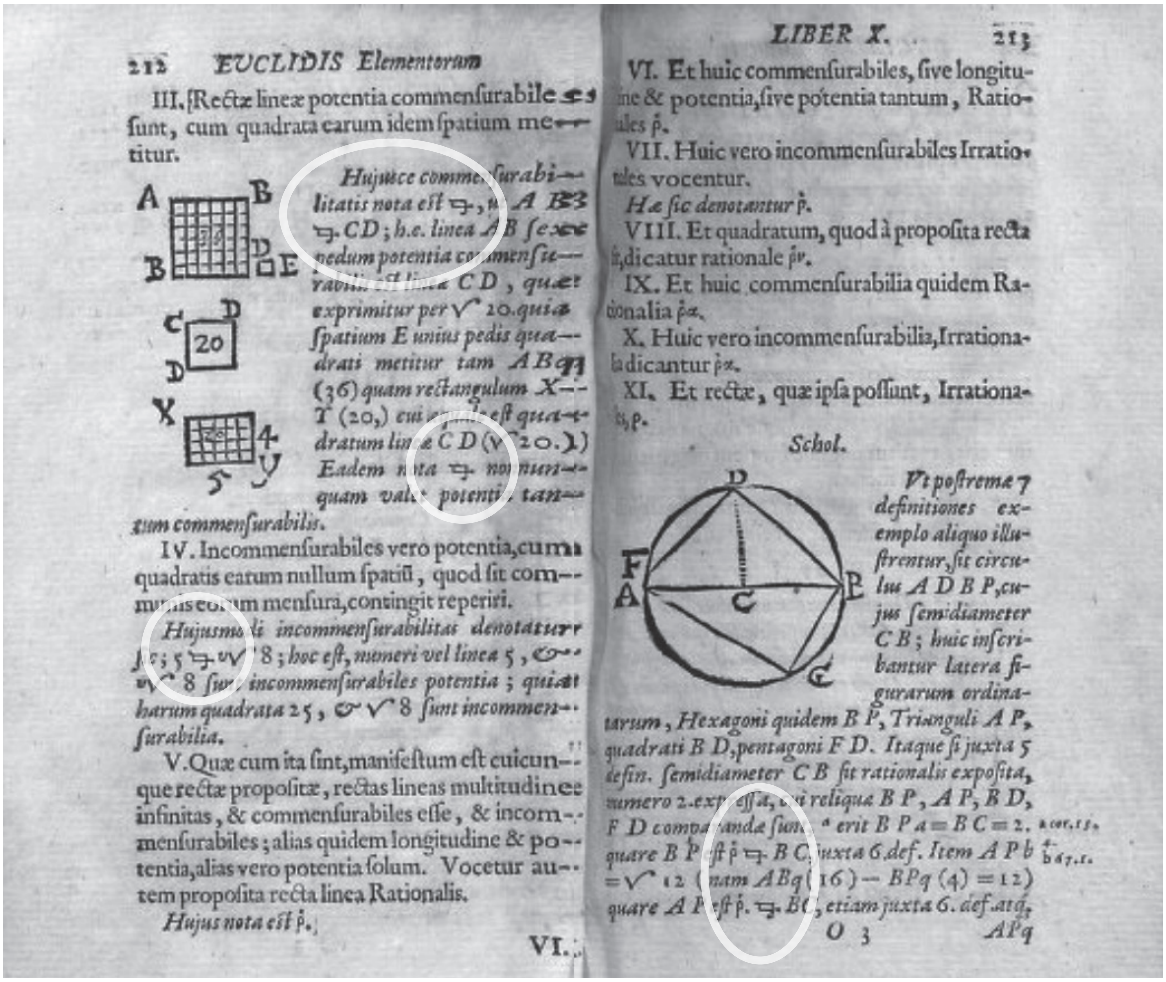
Commensurabilitatis nota est ⊏, *A* ⊏ *B; hoc est, linea A 8 pedum commensurabilis est lineæ B 13 pedum; quia D linea unius pedis singulas A & B metitur. Item* √18 ⊏ √50; *quia* √2 *singulas* √18, & √50 *metitur. Nam* √(18/2) = √9 = 3. & √(50/2) = √25 = 5. *quare* √18. √50 :: 3. 5.

II. Incommensurabiles autem sunt, quorum nullam communẽ mensurã contingit reperiri.

Incommensurabilitas significatur nota ⊐. *ut* √6 ⊐ √25 (5;) *hoc est* √6 *incommensurabilis est numero 5, vel magnitudini hoc numero designatæ; quia harum nulla est communis mensura, ut postea patebit.*

⊏ COMMENSURABILITY, ⊐ INCOMMENSURABILITY, ⊏ COMMENSURABILITY IN SQUARE, ⊐ INCOMMENSURABILITY IN SQUARE

Barrow 1676



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III. Rectæ lineæ potentia commensurabiles sunt, cum quadrata earum idem spatium metitur.

Hujusce commensurabilitatis nota est ⊏, *ut A B* ⊏ *C D; h.e. linea A B sex pedum potentia commensurabilis est lineæ C D, quæ exprimitur per* √20. *quia spatium E unius pedis quadrati metitur tam A Bq (36) quam rectangulum X T (20,) cui æquale est quadratum lineæ C D* (√20.) *Eadem nota* ⊏ *nonnunquam valet potentia tantum commensurabilis.*

IV. Incommensurabiles vero potentia, cum quadratis earum nullum spatiũ, quod sit communis eorum mensurã, contingit reperiri.

Hujusmodi incommensurabilitas denotatur; 5 ⊐ √8; *hoc est, numeri vel linea 5, &* √8 *sunt incommensurabiles potentia; quia harum quadrata 25, &* √8 *sunt incommensurabilia.*

V. Quæ cum ita sint, manifestum est cuicunque rectæ propositæ, rectas lineas multitudine infinitas, & commensurabiles esse, & incommensurabiles; alias quidem longitudine & potentia, alias vero potentia solum. Vocetur autem proposita recta linea Rationalis.

Hujus nota est ρ̇.

VI.

LIBER X. 213

VI. Et huic commensurabiles, sive longitudine & potentia, sive potentia tantum, Rationales ρ̇.

VII. Huic vero incommensurabiles Irrationales vocentur.

Hæ sic denotantur ρ̇.

VIII. Et quadratum, quod à proposita recta fit, dicatur rationale ρ̇ν.

IX. Et huic commensurabilia quidem Rationalia ρ̇α.

X. Huic vero incommensurabilia, Irrationalia dicantur ρ̇α.

XI. Et rectæ, quæ ipsa possunt, Irrationales, ρ̇.

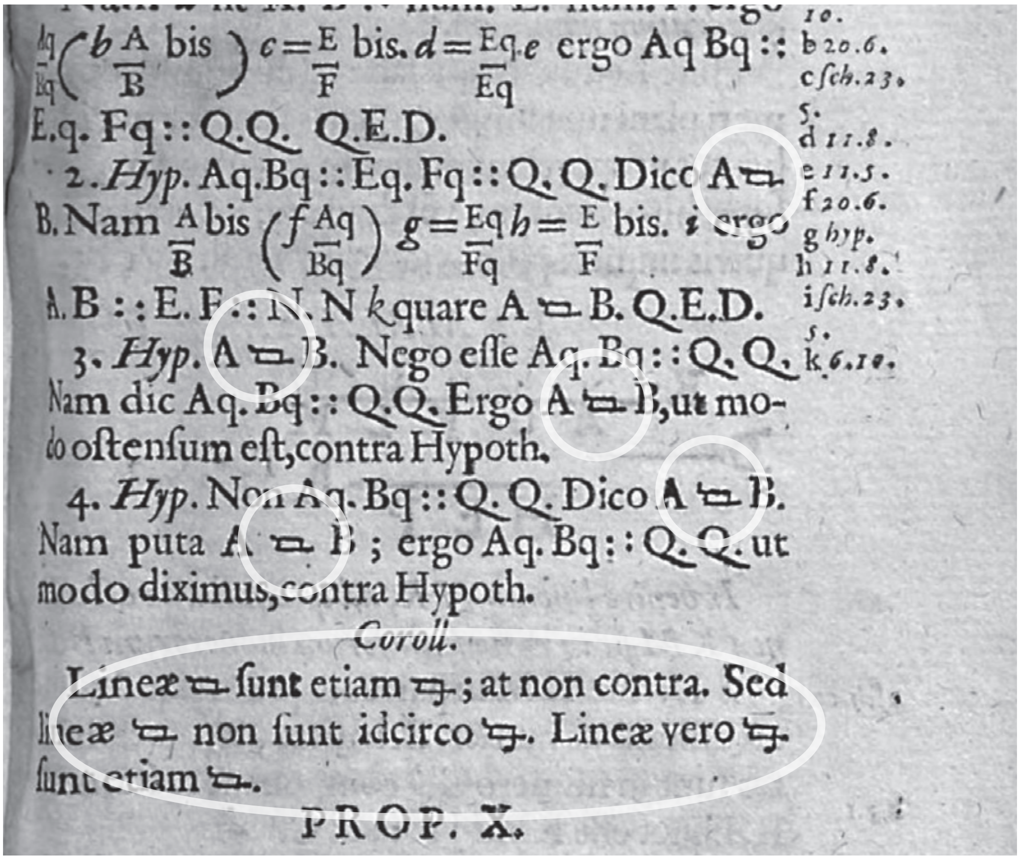
Schol.

Ut postremæ 7 definitiones exemplo aliquo illustrentur, sit circulus A D B P, cujus semidiameter C B; huic inscribantur latera figurarum ordinatarum, Hexagoni quidem B P, Trianguli A P, quadrati B D, pentagoni F D. Itaque si juxta 5 defin. semidiameter C B sit rationalis exposita, numero 2. expressa, cui reliqua B P, A P, B D, F D comparanda sunt; a erit B P a = B C = 2. a cor. 15. 4.

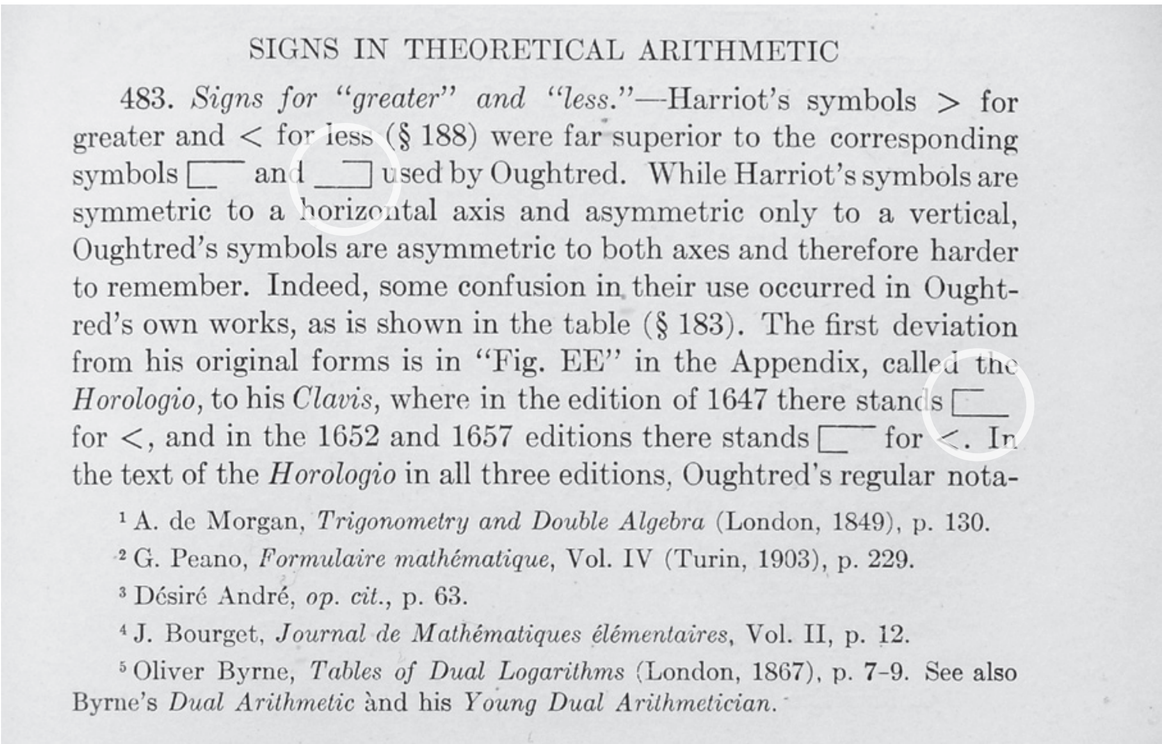
quare B P est ρ̇ ⊏ *B C, juxta 6. def. Item A P b* = √12 *(nam A Bq (16) − B Pq (4) = 12)* b 47. 1.

quare A P est ρ̇. ⊏ *B C, etiam juxta 6. def. atq;*

O 3 APq



COMMENSURABILITY, INCOMMENSURABILITY, COMMENSURABILITY IN SQUARE, INCOMMENSURABILITY IN SQUARE
Barrow 1676



SIGNS IN THEORETICAL ARITHMETIC

483. *Signs for "greater" and "less."*—Harriot's symbols > for greater and < for less (§ 188) were far superior to the corresponding symbols and used by Oughtred. While Harriot's symbols are symmetric to a horizontal axis and asymmetric only to a vertical, Oughtred's symbols are asymmetric to both axes and therefore harder to remember. Indeed, some confusion in their use occurred in Oughtred's own works, as is shown in the table (§ 183). The first deviation from his original forms is in "Fig. EE" in the Appendix, called the *Horologio*, to his *Clavis*, where in the edition of 1647 there stands for <, and in the 1652 and 1657 editions there stands for <. In the text of the *Horologio* in all three editions, Oughtred's regular nota-

[1] A. de Morgan, *Trigonometry and Double Algebra* (London, 1849), p. 130.

[2] G. Peano, *Formulaire mathématique*, Vol. IV (Turin, 1903), p. 229.

[3] Désiré André, *op. cit.*, p. 63.

[4] J. Bourget, *Journal de Mathématiques élémentaires*, Vol. II, p. 12.

[5] Oliver Byrne, *Tables of Dual Logarithms* (London, 1867), p. 7–9. See also Byrne's *Dual Arithmetic* and his *Young Dual Arithmetician*.

INVERTED SQUARE LEFT OPEN BOX OPERATOR and INVERTED SQUARE RIGHT OPEN BOX OPERATOR. Cajori vol. II p. 115 (1928)

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tion is adhered to. Isaac Barrow used ⊏ for "majus" and ⊐ for "minus" in his *Euclidis Data* (Cambridge, 1657), page 1, and also in his *Euclid's Elements* (London, 1660), Preface, as do also John Kersey,[1] Richard Sault,[2] and Roger Cotes.[3] In one place John Wallis[4] writes ⊐ for >, ⊐ for <.

Seth Ward, another pupil of Oughtred, writes in his *In Ismaelis Bullialdi astronomiae philolaicae fundamenta inquisitio brevis* (Oxoniae, 1653), page 1, ⊏ for "majus" and ⊏ for "minus." For further notices of discrepancy in the use of these symbols, see *Bibliotheca mathematica*, Volume XII[3] (1911–12), page 64. Harriot's > and < easily won out over Oughtred's notation. Wallis follows Harriot almost exclusively; so do Gibson[5] and Brancker.[6] Richard Rawlinson of Oxford used ⊐ for greater and ⊏ for less (§ 193). This notation is used also by Thomas Baker[7] in 1684, while E. Cocker[8] prefers ⊏ for ⊐. In the arithmetic of S. Jeake,[9] who gives "⊏ greater, ⊏. next greater, ⊐. lesser, ⊐. next lesser, ⊏ not greater, ⊐. not lesser, ⊏. equal or less, ⊐. equal or greater," there is close adherence to Oughtred's original symbols.

Ronayne[10] writes in his *Algebra* ⊏ for "greater than," and ⊐ for "less than." As late as 1808, S. Webber[11] says: ". . . . . we write *a* ⊏ *b*, or *a*>*b*; . . . . *a* ⊐ *b*, or *a*<*b*." In Isaac Newton's *De Analysi per Aequationes*, as printed in the *Commercium Epistolicum* of 1712, page 20, there occurs x ⊐ $\frac{1}{2}$, probably for $x<\frac{1}{2}$; apparently, Newton used here the symbolism of his teacher, I. Barrow, but in Newton's *Opuscula* (Castillion's ed., 1744) and in Lefort's *Commercium Epistolicum* (1856), page 74, the symbol is interpreted as meaning $x>\frac{1}{2}$. Eneström[12]

[1] John Kersey, *Elements of Algebra* (London, 1674), Book IV, p. 177.

[2] Richard Sault, *A New Treatise of Algebra* (London, n.d.).

[3] Roger Cotes, *Harmonia mensurarum* (Cambridge, 1722), p. 115.

[4] John Wallis, *Algebra* (1685), p. 127.

[5] Thomas Gibson, *Syntaxis mathematica* (London, 1655), p. 246.

[6] Thomas Brancker, *Introduction to Algebra* (trans. of Rahn's *Algebra;* London, 1668), p. 76.

[7] Thomas Baker, *Clavis geometrica* (London, 1684), fol. *d* 2 *a*.

[8] Edward Crocker, *Artificial Arithmetick* (London, 1684), p. 278.

[9] Samuel Jeake, Sr., ΛΟΓΙΣΤΙΚΗΛΟΓΙ'Α *or Arithmetick* (London, 1696), p. 12

[10] Philip Ronayne, *Treatise of Algebra* (London, 1727), p. 3.

[11] Samuel Webber, *Mathematics*, Vol. I (Cambridge, Mass., 1808; 2d ed.), p. 233.

[12] G. Eneström, *Bibliotheca mathematica* (3d ser.), Vol. XII (1911–12), p. 74

Cajori vol. II p. 116. In this chapter Cajori discusses the differing use cases of the rectangular symbols for *greater* and *less* in the works of various authors.

argues that Newton followed his teacher Barrow in the use of ¯¯] and actually took $x<\frac{1}{2}$, as is demanded by the reasoning.

In E. Stone's *New Mathematical Dictionary* (London, 1726), article "Characters," one finds [__ or ¯¯] for "greater" and __] or [¯¯ for "less." In the Itàlian translation (1800) of the mathematical part of Diderot's *Encyclopédie*, article "Carattere," the symbols are further modified, so that ⊏ and ¯| stand for "greater than," ⊐ for "less than"; and the remark is added, "but today they are no longer used."

Brook Taylor[1] employed ¯¯] and [¯¯ for "greater" and "less," respectively, while E. Hatton[2] in 1721 used [__ and __], and also > and <. The original symbols of Oughtred are used in Colin Maclaurin's *Algebra*.[3] It is curious that as late as 1821, in an edition of Thomas Simpson's *Elements of Geometry* (London), pages 40, 42, one finds [¯¯ for > and ¯¯] for <.

The inferiority of Oughtred's symbols and the superiority of Harriot's symbols for "greater" and "less" are shown nowhere so strongly as in the confusion which arose in the use of the former and the lack of confusion in employing the latter. The burden cast upon the memory by Oughtred's symbols was even greater than that of double asymmetry; there was difficulty in remembering the distinction between the symbol [¯¯ and the symbol [__ . It is not strange that Oughtred's greatest admirers—John Wallis and Isaac Borrow—differed not only from Oughtred, but also from each other, in the use of these symbols. Perhaps nowhere is there another such a fine example of symbols ill chosen and symbols well chosen. Yet even in the case of Harriot's symbolism, there is on record at least one strange instance of perversion. John Frend[4] defined < as "greater than" and > as "less than."

484. *Sporadic symbols for "greater" or "less."*—A symbol constructed on a similar plan to Oughtred's was employed by Leibniz[5] in 1710, namely, "*a* — significat *a* esse majus quam *b*, et *a* ═ significat *a* esse minus quam *b*." Leibniz borrowed these signs from his teacher Erhard Weigel,[6] who used them in 1693. In the 1749 edition of the *Miscellanea Berolinensia* from which we now quote, these inequality

[1] Brook Taylor, *Phil. Trans.*, Vol. XXX (1717–19), p. 961.

[2] Edward Hatton, *Intire System of Arithmetic* (London, 1721), p. 287.

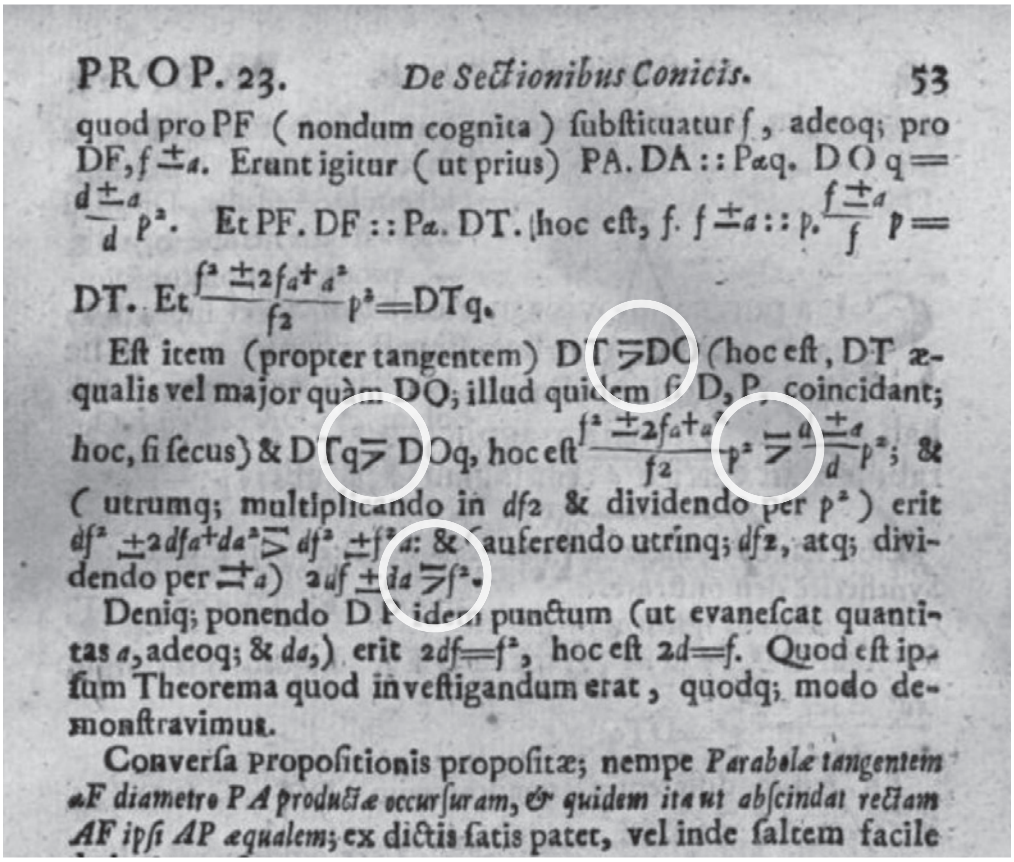
[3] Colin Maclaurin, *A Treatise of Algebra* (3d ed.; London, 1771).

[4] John Frend, *Principles of Algebra* (London, 1796), p. 3.

[5] *Miscellanea Berolinensia* (Berlin, 1710), p. 158.

[6] *Erhardi Weigelii Philosophia mathematica* (Jenae, 1693), p. 135.

Cajori vol. II p. 117. In this chapter Cajori discusses the differing use cases of the rectangular symbols for *greater* and *less* in the works of various authors.



PROP. 23. *De Sectionibus Conicis.* 53

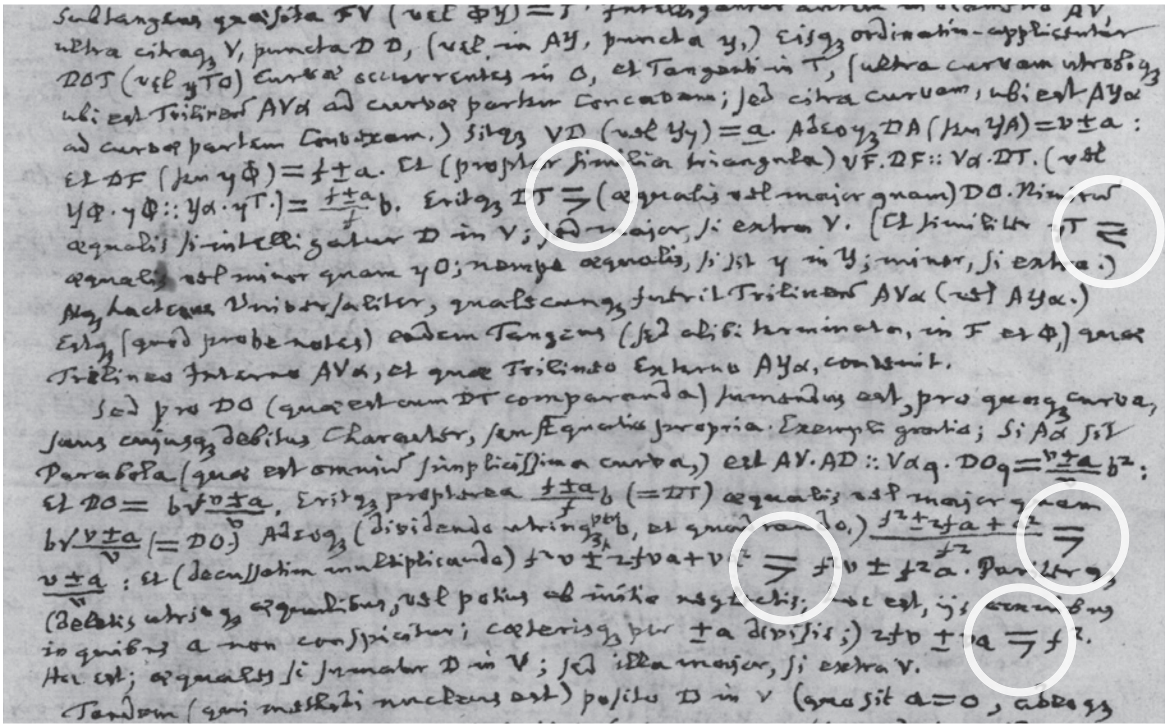
quod pro PF (nondum cognita) substituatur f, adeoq; pro DF, $f \pm a$. Erunt igitur (ut prius) PA. DA :: Paq. DOq $= \frac{d \pm a}{d} p^2$. Et PF. DF :: Pa. DT. (hoc est, $f . f \pm a :: p . \frac{f \pm a}{f} p =$ DT. Et $\frac{f^2 \pm 2fa + a^2}{f2} p^2 =$ DTq.

Est item (propter tangentem) DT ⋝ DO (hoc est, DT æqualis vel major quàm DO; illud quidem si D, P, coincidant; hoc, si secus) & DTq ⋝ DOq, hoc est $\frac{f^2 \pm 2fa + a^2}{f2} p^2$ ⋝ $\frac{d \pm a}{d} p^2$; & (utrumq; multiplicando in $df2$ & dividendo per p^2) erit $df^2 \pm 2dfa + da^2$ ⋝ $df^2 \pm f^2 a$: & auferendo utrinq; $df2$, atq; dividendo per $\pm a$) $2df \pm da$ ⋝ f^2.

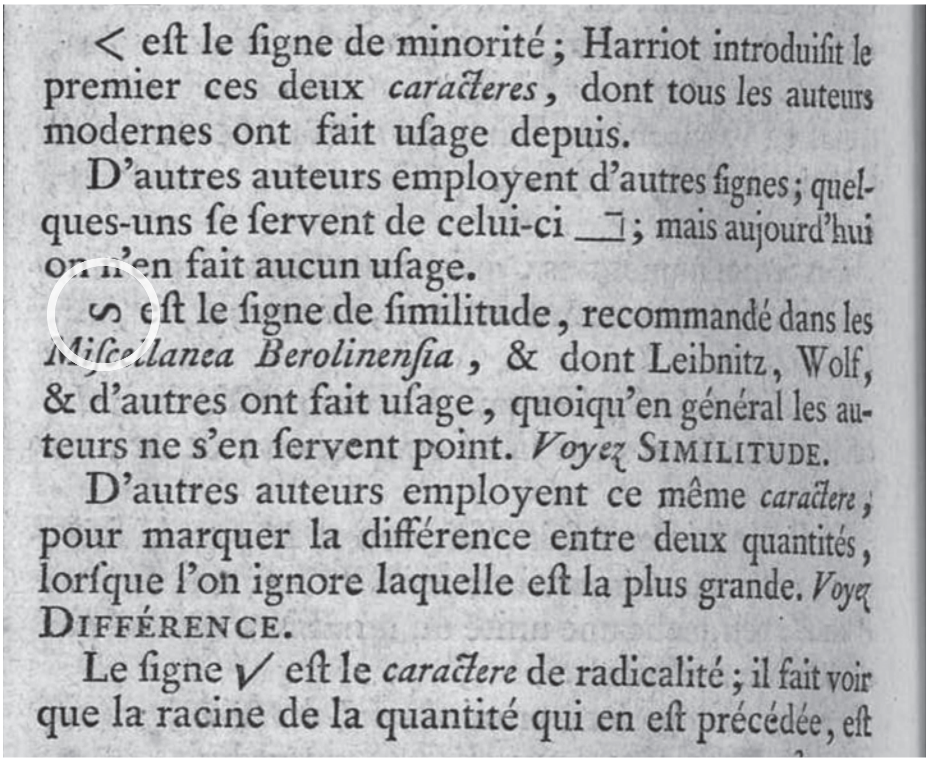
Deniq; ponendo D P idem punctum (ut evanescat quantitas a, adeoq; & da,) erit $2df = f^2$, hoc est $2d = f$. Quod est ipsum Theorema quod investigandum erat, quodq; modo demonstravimus.

Conversa propositionis propositæ; nempe *Parabolæ tangentem aF diametro PA productæ occursuram, & quidem ita ut abscindat rectam AF ipsi AP æqualem;* ex dictis satis patet, vel inde saltem facile

⋝ EQUAL TO OR GREATER-THAN (22DD) in the parallelised form, which we propose as a variation sequence. Wallis, De sectionibus conicis nova methodo expositis tractatus, 1655; p. 53



⋝ EQUAL TO OR GREATER-THAN (22DD) and ⋜ EQUAL TO OR LESS-THAN (22DC) in their parallelised forms, which we propose as variation sequences. Manuscript of J. Wallis, LBr 974, 28v.



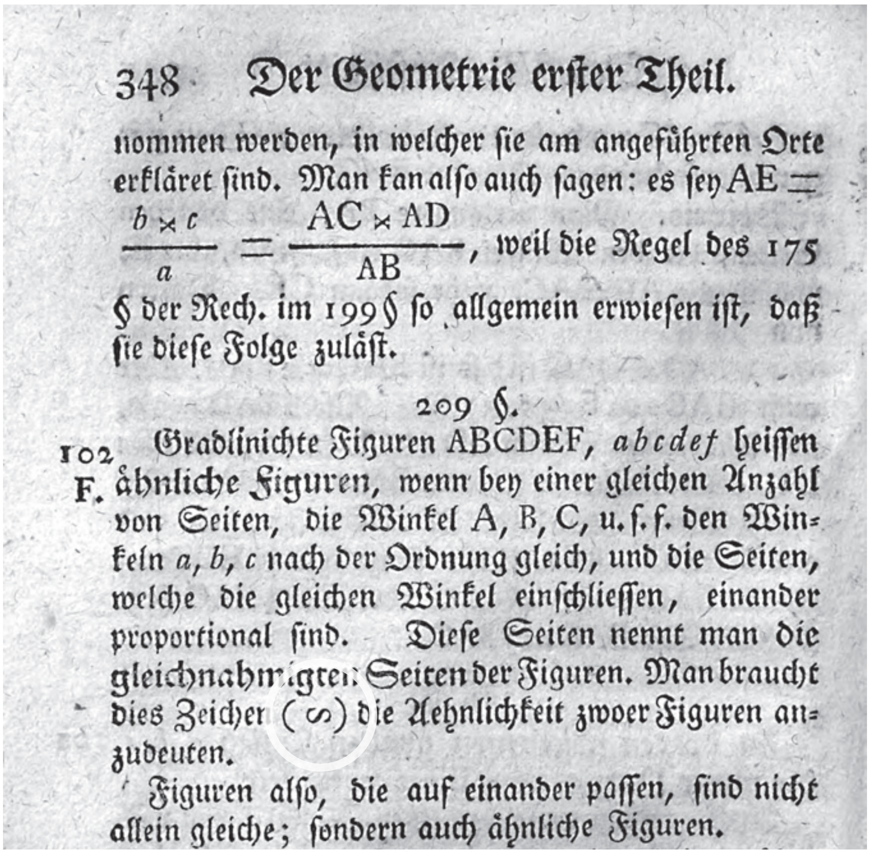
< eſt le ſigne de minorité; Harriot introduiſit le premier ces deux *caracteres*, dont tous les auteurs modernes ont fait uſage depuis.

D'autres auteurs employent d'autres ſignes; quelques-uns ſe ſervent de celui-ci ⌐; mais aujourd'hui on n'en fait aucun uſage.

∽ eſt le ſigne de ſimilitude, recommandé dans les *Miſcellanea Berolinenſia*, & dont Leibnitz, Wolf, & d'autres ont fait uſage, quoiqu'en général les auteurs ne s'en ſervent point. *Voyez* SIMILITUDE.

D'autres auteurs employent ce même *caractere*, pour marquer la différence entre deux quantités, lorſque l'on ignore laquelle eſt la plus grande. *Voyez* DIFFÉRENCE.

Le ſigne √ eſt le *caractere* de radicalité; il fait voir que la racine de la quantité qui en eſt précédée, eſt



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nommen werden, in welcher ſie am angeführten Orte erkläret ſind. Man kan alſo auch ſagen: es ſey AE = $\frac{b \times c}{a} = \frac{AC \times AD}{AB}$, weil die Regel des 175 § der Rech. im 199 § ſo allgemein erwieſen iſt, daß ſie dieſe Folge zuläſt.

209 §.

102 F. Gradlinichte Figuren ABCDEF, *abcdef* heiſſen ähnliche Figuren, wenn bey einer gleichen Anzahl von Seiten, die Winkel A, B, C, u.ſ.f. den Winkeln *a*, *b*, *c* nach der Ordnung gleich, und die Seiten, welche die gleichen Winkel einſchlieſſen, einander proportional ſind. Dieſe Seiten nennt man die gleichnahmigten Seiten der Figuren. Man braucht dies Zeichen (∽) die Aehnlichkeit zwoer Figuren anzudeuten.

Figuren alſo, die auf einander paſſen, ſind nicht allein gleiche; ſondern auch ähnliche Figuren.

∽ *variation sequence to 223D*

Diderot, Encyclopédie, Paris 1751 (top); Karsten 1767 (bottom).

The "lazy S" character is the historic predecessor of what we know in modern math notation as the "reversed tilde", 223D. Originally it was created by simply turning a Latin sort **S** by 90 degrees. It occurs in larger amount of sources, of which we show a selection on the following pages.

findet man a.b = c.d als Bezeichnung einer geometrischen Proportion, nach Saverien a. a. O., aber selten. Ein Verhältniß, welches aus den Verhältnissen a:b, c:d, e:f, u. s. f. zusammengesetzt ist, bezeichnet man durch (a:b) + (c:d) + (e:f) + ... Eine geometrische Progression wird auch bezeichnet durch ∺ 3, 6, 12, 24, 48, ...; eben so eine stetige arithmetische Proportion durch ÷ a.b.c, eine stetige geometrische Proportion durch ∺ a.b.c. Eine arithmetische Progression durch ÷ 2, 6, 10, 14, 18, 22, ...

∽ bedeutet bei einigen englischen und französischen Schriftstellern, wenn es zwischen zwei Größen steht, wie z. B. a ∽ b, den Unterschied der beiden Größen a und b,

1182 Zeichen.

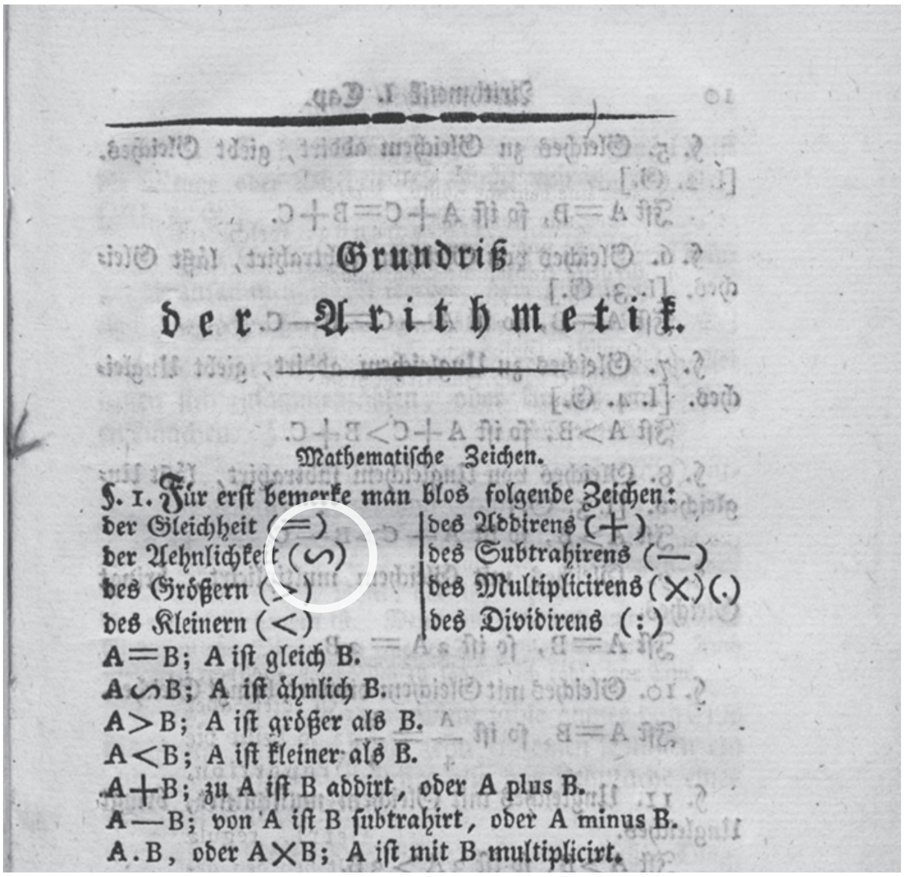
es mag die vorangesetzte a die größere oder die kleinere seyn. Dieses Zeichen scheint von Wallis zuerst gebraucht zu seyn. Es ist aber völlig unnöthig und unnütz, daher auch gar nicht in Gebrauch gekommen. Das Vorzeichen der Differenz liefert die nöthige Bestimmung von selbst.

Bei deutschen Schriftstellern ist ∽ das Zeichen der Aehnlichkeit, ein liegendes lateinisches S. Leibnitz und Wolf haben es zuerst angewandt. Oft gebraucht man das Wort ähnlich selbst. Vergl. Miscellan. Berolin. Part. III. p. 159.

Mit dem Begriffe hat auch Gauß das Zeichen ≡ als Zeichen der Zahlen-Congruenz eingeführt (s. den Art. Zahl. II. 1.). $\sqrt{-1}$ wird oft, auch in diesem Wörterbuche, durch i bezeichnet.

Hat eine Größe mehrere Werthe, so wird nach Cauchy der Inbegriff aller Werthe durch Einschließung in doppelte

∽ *variation sequence to 223D*
Klügel 1831.



Grundriß

der Arithmetik.

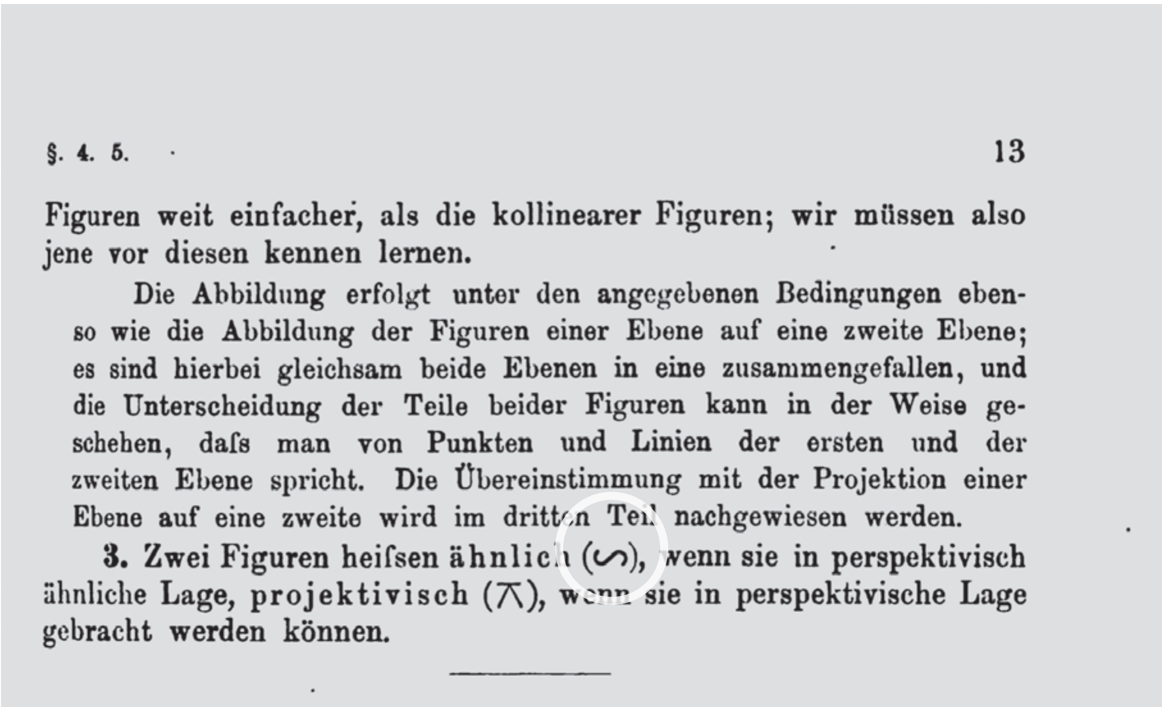
Mathematische Zeichen.

§. 1. Für erst bemerke man blos folgende Zeichen:

der Gleichheit (=)	des Addirens (+)
der Aehnlichkeit (∽)	des Subtrahirens (—)
des Größern (>)	des Multiplicirens (×)(.)
des Kleinern (<)	des Dividirens (:)

A=B; A ist gleich B.
A∽B; A ist ähnlich B.
A>B; A ist größer als B.
A<B; A ist kleiner als B.
A+B; zu A ist B addirt, oder A plus B.
A—B; von A ist B subtrahirt, oder A minus B.
A.B, oder A×B; A ist mit B multiplicirt.

∽ *variation sequence to 223D*
Lorenz 1798.



§. 4. 5. 13

Figuren weit einfacher, als die kollinearer Figuren; wir müssen also jene vor diesen kennen lernen.

Die Abbildung erfolgt unter den angegebenen Bedingungen ebenso wie die Abbildung der Figuren einer Ebene auf eine zweite Ebene; es sind hierbei gleichsam beide Ebenen in eine zusammengefallen, und die Unterscheidung der Teile beider Figuren kann in der Weise geschehen, daſs man von Punkten und Linien der ersten und der zweiten Ebene spricht. Die Übereinstimmung mit der Projektion einer Ebene auf eine zweite wird im dritten Teil nachgewiesen werden.

3. Zwei Figuren heiſsen ähnlich (∽), wenn sie in perspektivisch ähnliche Lage, projektivisch (⊼), wenn sie in perspektivische Lage gebracht werden können.

∽ *variation sequence to 223D*
Henrici/Treutlein 1881.

VERKLAARING

der Merkteekens in dit Werk gebruikt.

$=$ Beteekent gelyk.

$+$ ——— meer; dus $a+b$ is eeven zoo veel als a tot b vergaard.

$-$ ——— min; dat is $a-b$, wil zoo veel zeggen als a min b.

$\times$ of ( ), verbeeld vermeenigvuldigt: Dus is $a\times b$ zoo veel als a vermeenigvuldigt door b; eeven zoo is het met $(a+b)c$ of $(a+b)\times c$.

$>$ beteekend grooter: dat is $6 > 4$ of 6 grooter als 4.

$<$ ——— kleiner: dus $3 < 5$ of 3 kleiner als 5.

△ ——— driehoek.

□ ——— vierkant, vierhoek of *paralelogram.*

∽ ——— gelykvormig, wanneer a gelykvormig is aan b, schryft men het zelve a ∽ b.

⊥ beteekent loodrecht.

INLEI-

∽ *variation sequence to 223D*
Mauduit 1764, p. xxiiii (top), p. 109, 116.

in een en zelvde punt A *ontmoeten.*

BETOOGINGE.

Laat ons voor een oogenblik veronderstellen, dat, de rechte MN de lyn PQ in het stip A ontmoet, maar dat RS het die zelvde PQ in eenig ander stip als B doet, zoo is het klaar, dat 'er alleen betoogt moet worden dat de stippen A en B in elkander smelten en op een vallen; of 't geen op het zelvde uitkomt, dat AP$=$is aan BP. Dewyl de lynen PM en QN; PR en QS eevenwydig aan elkander zyn ieder aan ieder zoo zyn de △ⁿ. APM en AQN ∽, als meede de △ⁿ. BPR en BQS;
Door de eersten is AP: AQ$=$PM: QN (*n*). . . ,
en door de stelling . . , PM: QN$=$PR: QS,
de △ⁿ. BPQ en BQS geeven . . . PR: QS $=$BP:

(*n*) Eucl. Def. I. 6.

116 INLEIDING TOT DE

QN zoo wel eevenwydig zynde als LI en NS, (door de saamenstelling) zyn de △ⁿ. CIL en OSN ∽; dus is CI: OS$=$IL: NS (*w*); maar door de eigenschappen van de elips heeft men IL: NS$=$IF: MS, dus ook CI: OS$=$IF: MS (*a*); waar uit volgt, dat de rechthoekige △ⁿ. CIF en OSM ook ∽ zyn (*w*), en by gevolg de lynen CF en MP eevenwydig aan elkander (*c*).

V. GRONDLES.

§. 95. *Het vierkant* $\overline{PM}^2$ (Fig. 18.) *van eene ordinaat* PM *aan een diameeter* CE *in de elips, staat tot den regthoek* EP$\times$ eP *of* $\overline{CE}^2 - \overline{CP}^2$ *der abscissen* EP *en* eP, *gelyk het vierkant* $\overline{CF}^2$ *van den halven meede-diameeter* CF *staat tot het vierkant* $\overline{CE}^2$ *van den halven diameeter* CE *op welken*

| *divisé par*, entre deux nombres. Voir ×. Devant un nombre seul signifie *le réciproque de* (Int § 22 , II § 2 P21). Dans les parties I-IV il a aussi la signification du signe f.

√ *racine arithmétique*. Il se place devant un nombre positif (II § 6).

√* *racines algébriques*. Il se place devant un nombre réel ou imaginaire (II § 9 P11).

! *factorielle* (III § 1 P30, 31).

> *est plus grand que*. Il se place entre deux nombres réels finis (II § 5), entre un nombre fini et l'infini (V § 1 P6 , § 3 P7), ou deux transfinis (VI § 2 P13).

< *est plus petit que*. Voir >.

ɟ *fonction*. Voir f.

' ‘ Signes qui forment des fonctions (Intr § 21). Voir ⌣‘, ⌢‘, Nc‘, Ω‘, etc.

a⊢⊣b , a−b , a⊢b , a⊣b *intervalles de a à b*, avec, ou sans les extrêmes (Intr § 2 , V § 4 P41-45).

| Signe du produit intérieur de deux nombres complexes du même ordre (V § 4 P24).

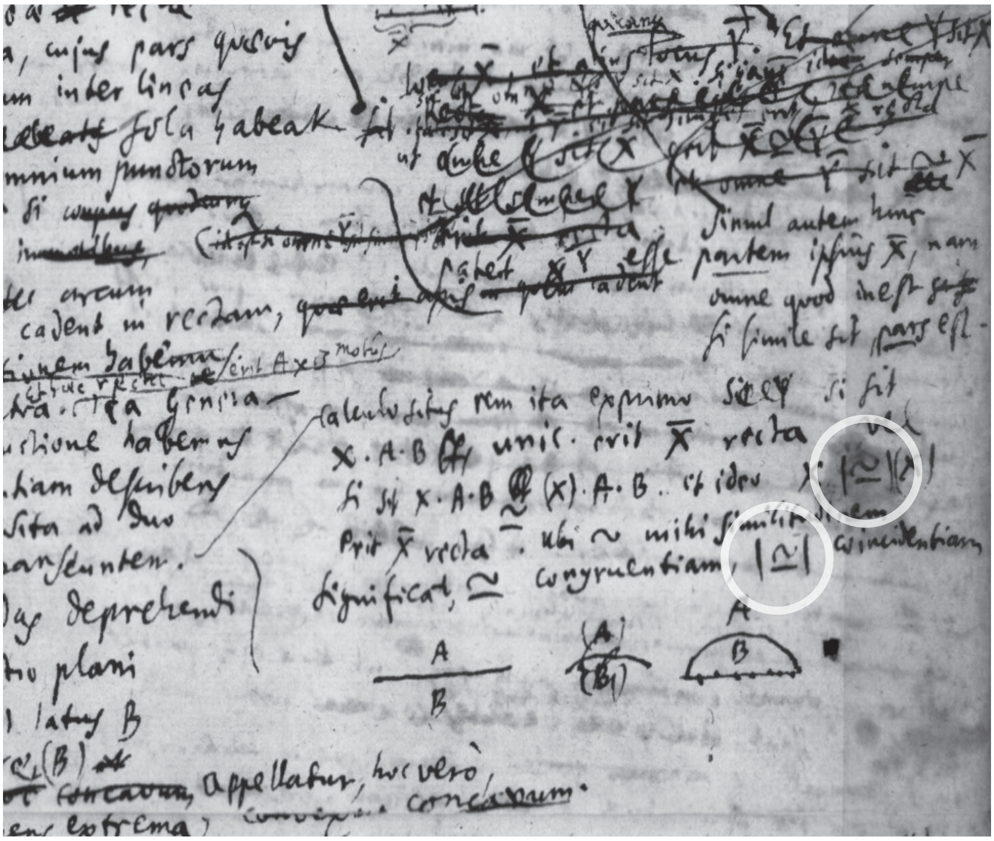
$\binom{b}{a}$ Signe de la substitution de *b* à *a* (Intr § 28).

∽ *est semblable*, ou *de la même puissance*; on l'écrit entre deux classes (VI § 1 P1).

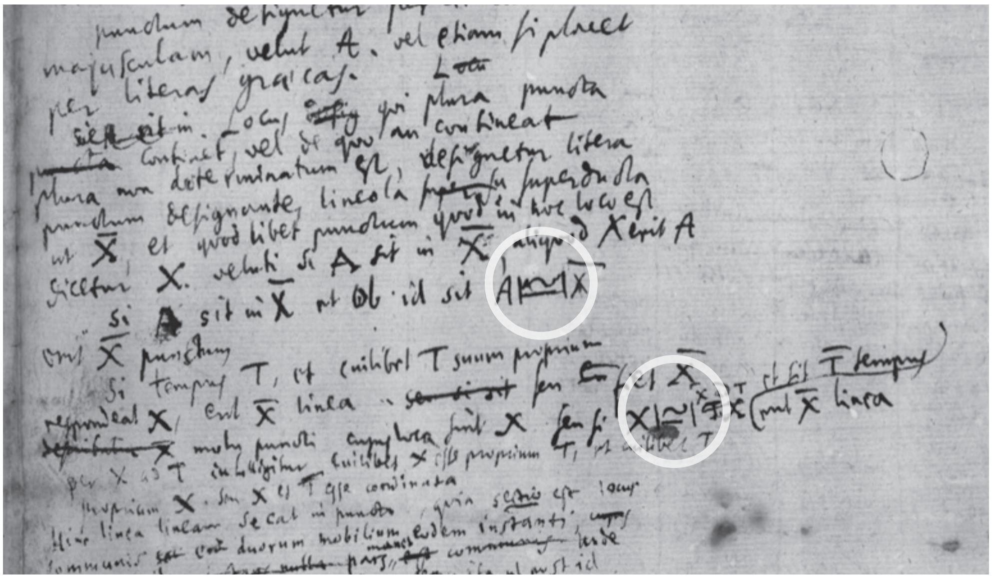
II *deuxième classe de nombres transfinis* (VI § 2 P27).

| , ↑, ↓ (Intr § 32-33). Ils ne figurent pas dans le Formulaire.

∽ *variation sequence to 223D*
Peano 1895.



|≃| LEIBNIZIAN COINCIDENCE
LH 35 I 1, f .1v



|≃| LEIBNIZIAN COINCIDENCE
LH 35 I 13, f. 12r

fuisset aggressus demonstrare, in triangulo duo quaecunque latera esse tertio majora; id enim ex tali definitione statim consequebatur.

(2) Ego varias lineae rectae definitiones habeo: veluti *Recta* est linea, cujus pars quaevis est similis toti, quanquam Recta non solum inter lineas, sed etiam inter magnitudines hoc sola habeat. Sit locus $\overline{X}$ (fig. 82), et locus alius quicunque $\overline{Y}$, qui insit priori, seu cujusque punctum quodvis Y sit X; si jam $\overline{Y}$ est simile ipsi $\overline{X}$, *erit* $\overline{X}$ recta. Simul autem hinc patet, Y esse *partem* ipsius $\overline{X}$, nam omne quod inest si simile sit, *pars* est.

(3) Definio etiam *rectam*, locum omnium punctorum ad duo puncta sui situs unicorum. Et hinc si quaecunque magnitudo moveatur duobus punctis immotis, mota quidem puncta arcum circuli describent, quiescentia autem omnia cadent in rectam, in quam cadent omnium illorum Circulorum centra. Et haec recta erit Axis Motus. Ita generationem rectae et circuli una eademque constructione habemus. At punctum extra rectam positum, circumferentiam describens, infinita percurrit puncta, eodem modo sita ad duo illa puncta immota et ad rectam per ea transeuntem. Calculo situs rem ita exprimo: Si sit X . A . B unic., erit $\overline{X}$ recta, vel si sit X . A . B ∽ (X) . A . B et ideo X ≌ (X), erit $\overline{X}$ recta, ubi ∽ mihi similitudinem significat, ≌ congruentiam, |≌| coincidentiam.

(4) Sed ad Euclideas demonstrationes perficiendas deprehendi hac opus esse definitione, ut *recta* sit sectio plani utrinque se habens eodem modo, ut latus A (fig. 83) et latus B, cum in *curva*

|≌| LEIBNIZIAN COINCIDENCE
Gerhard 1858, p. 185.

XXXV, I, 13
12 recto

Ad calculum situs constituendum utile est omnia à verbis reduci ad signa, remque eo usque produci donec habeatur Analysis, id est donec demonstrationes theorematum sine ope ingenii, certo ratiocinandi filo prodeant.

Punctum designetur per literam majusculam, velut A, vel enim si placet per literas graecas.

Locus qui plura puncta continet, vel de quo an contineat plura non determinatum est, designetur litera punctum designante, lineola superducta ut $\overline{X}$, et quodlibet punctum quod in hoc loco est dicetur X. Veluti si A sit in $\overline{X}$, aliquod X erit A.

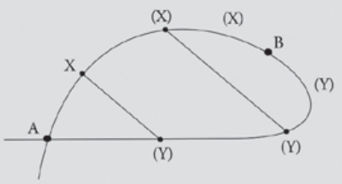
Si A sit in $\overline{X}$ et ob id sit $A|\simeq|\overline{X}$. erit $\overline{X}$ punctum.

Si tempus T, et cuilibet T suum proprium respondeat X, erit $\overline{X}$ linea. Seu fiet $\overline{X}$ motu puncti; complura sint X seu si $X|\simeq|X$ ad T et sit $\overline{T}$ tempus erit $\overline{X}$ linea per X ad T; intelligitur cuilibet X esse proprium T, et cuilibet T proprium X. Seu X et T esse coordinata.

Hinc linea lineam secat in puncto, quia **sectio** est locus communis duorum mobilium eodem instanti, ita ut post id instans nihil sit ipsis mobilibus commune. Sed haec definitio non potest applicari ad sectionem in motu superficierum et corporum, quia duae lineae generantes suam quaevis superficiem, et duae superficies generantes quaevis suum corpus rari tales sunt ut ex toto vel parte congruere possint. At punctum puncto semper congruit sectionis ergo definitioni nostrae generali standum, ut sit totum commune duabus magnitudinibus partem communem non habentibus.

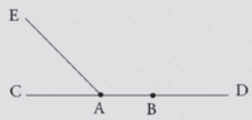
XXXV, I, 13
12 verso

Ex natura similitudinis consequitur rectas duas non nisi in uno puncto sibi occurrere posse. Habeant commune punctum A et inde egrediantur AX et AY et rursus concurrant in B. Moveantur puncta X et Y velocitatibus, quae sunt ut AXB ad AYB, ita concurrent in puncto B. Sit autem cujuscunque puncti motus uniformis. Cum sit $A(X)\simeq AX$ et $A(Y)\simeq AY$ et motus per A(X) vel A(Y) similis motui per AX, AY, atque adeò A(X)(Y) et AXY determinentur similiter, erit et $A(X)(Y)\simeq AXY$. Cum ergo non coincidant X et Y, neque etiam coincident (X) et (Y) adeoque nec poterit dari punctum B.



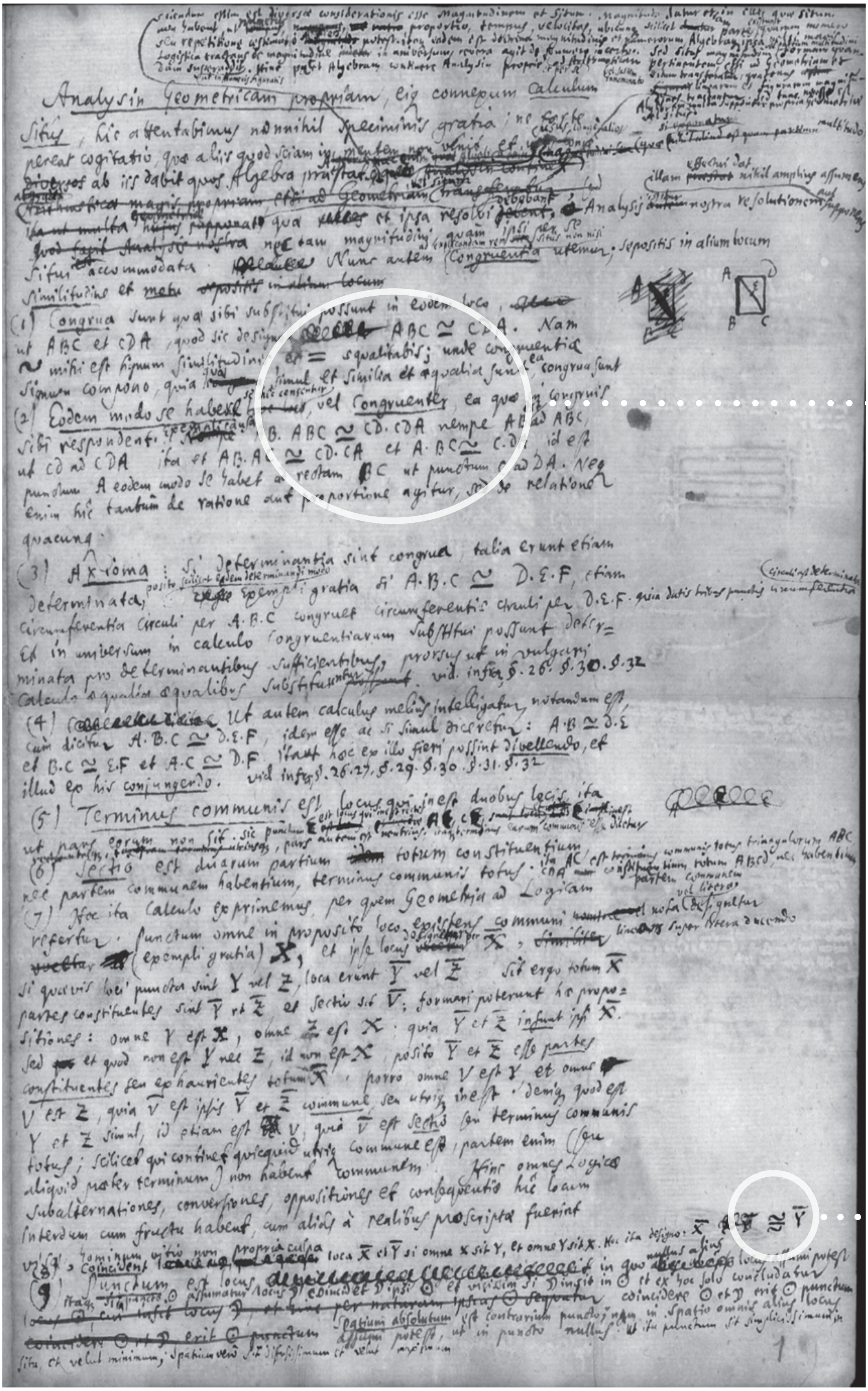
Hinc datis duobus punctis determinata est recta, quae puncta connectit; seu rectae extremis congruentes totae congruent.

Et datis duobus punctis determinata est recta infinita transiens per duo puncta. Nam determinata est recta AB, nec produci potest utrinque nisi uno modo, ut versus C vel D, nam si ex A versus C et E produci posset, rectae EAB et CAB darentur, plus quam punctum commune habentes.



Duae rectae AB et CD sunt similes inter se: nam AB, pariter ac CD, determinatur ex eo ut duo puncta connectuntur per rectam cujus pars sit similis

|≃| LEIBNIZIAN COINCIDENCE; ≃ *variation sequence to 2243*
de Risi 2007, p. 604, 605.

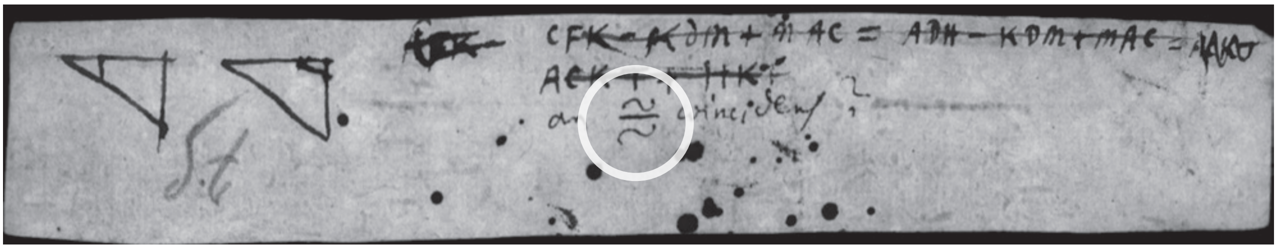

Analysin Geometricam propriam, eique connexum calculum
Situs, hic attentabimus nonnihil speciminis gratia; ne forte
pereat cogitatio, quae aliis quod sciam in mentem non venit
(1) Congrua sunt quae sibi substitui possunt in eodem loco, ABC ≃ CDA. Nam
ut ABC et CDA, quod sic designo = aequalitatis; unde congruentiae
∼ mihi est signum similitudinis simul et similia et aequalia sunt, ea congrua sunt
signum compono, quia
(2) Eodem modo se habent, vel Congruenter, ea quae in congruis
sibi respondent. B. ABC ≃ CD.CDA nempe AB ad ABC
ut CD ad CDA ita et AB.AC ≃ CD.CA et A.BC ≃ C.DA id est
punctum A eodem modo se habet ad rectam BC ut punctum C ad DA. Nec
enim hic tantum de ratione aut proportione agitur, sed de relatione
quacunque.
(3) Axioma: Si determinantia sint congrua, talia erunt etiam
determinata, posito scilicet eodem determinandi modo; exempli gratia si A.B.C ≃ D.E.F, etiam
circumferentia circuli per A.B.C congruet circumferentiae circuli per D.E.F.
Et in universum in calculo congruentiarum substitui possunt deter=
minata pro determinantibus sufficientibus, prorsus ut in vulgari
calculo aequalia aequalibus substituuntur. vid. infra §.26. §.30. §.32
(4) Ut autem calculus melius intelligatur, notandum est,
cum dicitur A.B.C ≃ D.E.F, idem esse ac si simul diceretur: A.B ≃ D.E
et B.C ≃ E.F et A.C ≃ D.F. Itaque hoc ex illo fieri possint divellendo, et
illud ex his conjungendo. vid. infra §.26.27. §.29. §.30. §.31. §.32
(5) Terminus communis est locus qui inest duobus locis, ita
ut nulla eorum pars non sit
(6) Sectio est duarum partium totum constituentium
nec partem communem habentium, terminus communis totus
(7) Hoc ita calculo exprimemus, per quem Geometria ad Logicam
refertur. Punctum omne in proposito loco existens communi
(exempli gratia) X, et ipse locus X̄,
Si quaevis loci puncta sint Y vel Z, loca erunt Ȳ vel Z̄. Sit ergo totum X̄
partes constituentes sint Ȳ et Z̄ et sectio sit V̄. formari poterunt hae propo=
sitiones: omne Y est X, omne Z est X: quia Ȳ et Z̄ insunt ipsi X̄.
Sed et quod non est Y nec Z, id non est X, posito Ȳ et Z̄ esse partes
constituentes seu exhaurientes totum X̄. porro omne V est Y et omne
V est Z, quia V̄ est ipsis Ȳ et Z̄ commune seu utrique inest; denique quod est
Y et Z simul, id etiam est V, quia V̄ est sectio seu terminus communis
totus; scilicet qui continet quicquid utrique commune est; partem enim (seu
aliquid praeter terminum) non habent communem. Hinc omnes Logicae
subalternationes, conversiones, oppositiones et consequentiae hic locum
interdum cum fructu habent, cum aliis à realibus proscriptae fuerint
vitio hominum non propria culpa.
X̄ ≊ Ȳ

≃

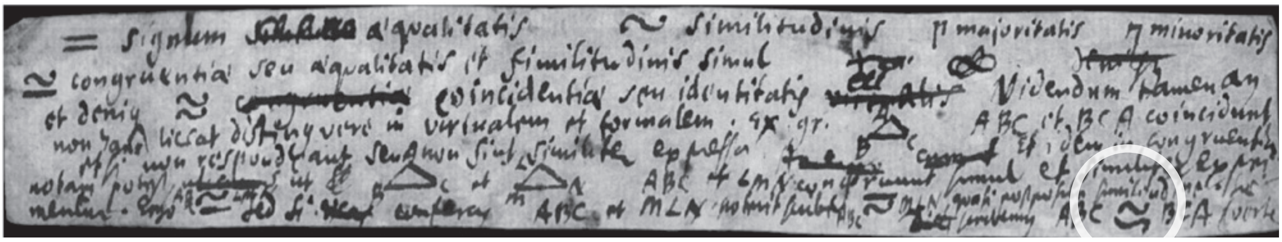
≊

≊ *variation sequence to 2A6C*

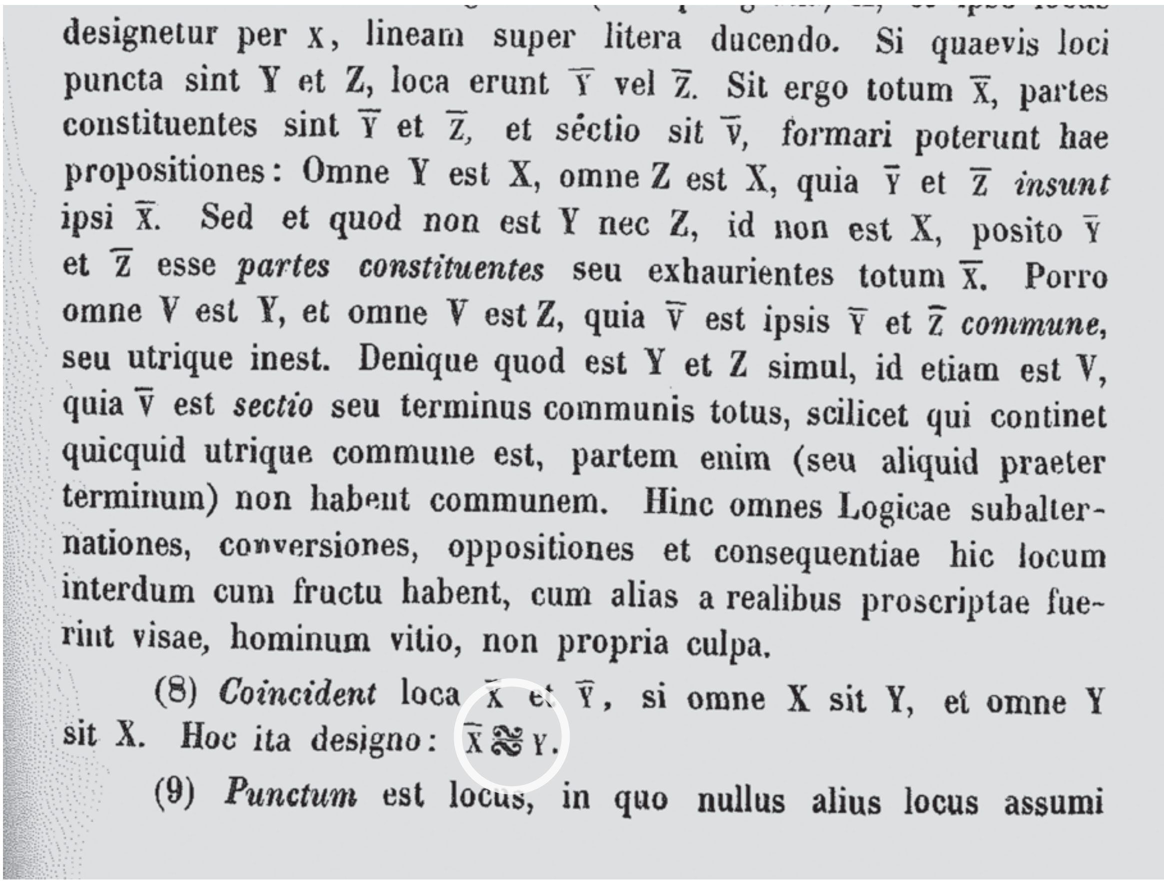
LH 35 I 13, f .1r



⩬ *variation sequence to 2A6C*
LH 35 I 14, f. 75r



INVERTED LAZY S OVER LAZY S
LH 35 I 14, f. 75v

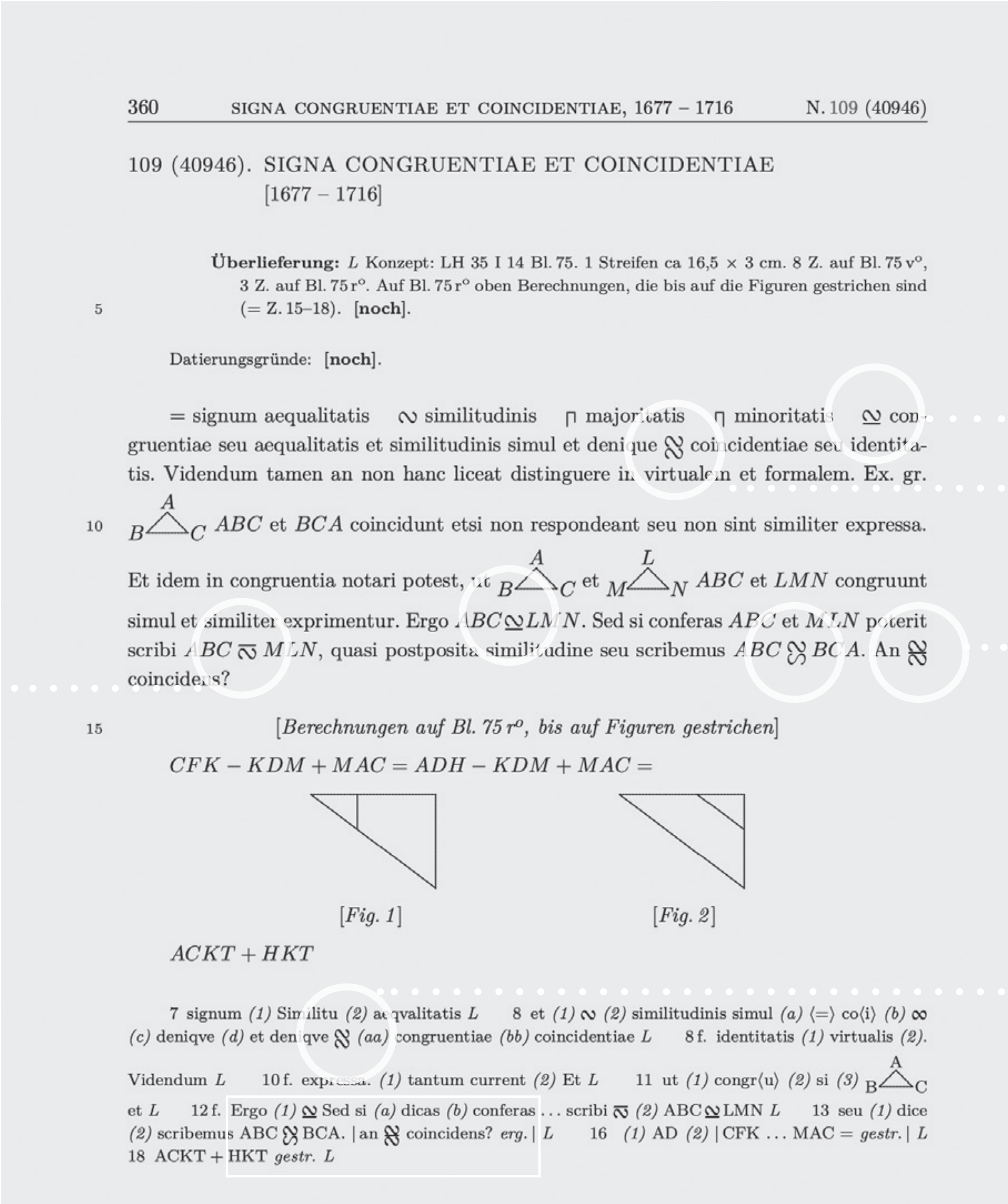


designetur per $\overline{X}$, lineam super litera ducendo. Si quaevis loci puncta sint Y et Z, loca erunt $\overline{Y}$ vel $\overline{Z}$. Sit ergo totum $\overline{X}$, partes constituentes sint $\overline{Y}$ et $\overline{Z}$, et sectio sit $\overline{V}$, formari poterunt hae propositiones: Omne Y est X, omne Z est X, quia $\overline{Y}$ et $\overline{Z}$ *insunt* ipsi $\overline{X}$. Sed et quod non est Y nec Z, id non est X, posito $\overline{Y}$ et $\overline{Z}$ esse *partes constituentes* seu exhaurientes totum $\overline{X}$. Porro omne V est Y, et omne V est Z, quia $\overline{V}$ est ipsis $\overline{Y}$ et $\overline{Z}$ *commune*, seu utrique inest. Denique quod est Y et Z simul, id etiam est V, quia $\overline{V}$ est *sectio* seu terminus communis totus, scilicet qui continet quicquid utrique commune est, partem enim (seu aliquid praeter terminum) non habent communem. Hinc omnes Logicae subalternationes, conversiones, oppositiones et consequentiae hic locum interdum cum fructu habent, cum alias a realibus proscriptae fuerint visae, hominum vitio, non propria culpa.

(8) *Coincident* loca $\overline{X}$ et $\overline{Y}$, si omne X sit Y, et omne Y sit X. Hoc ita designo: $\overline{X}$ ⩬ $\overline{Y}$.

(9) *Punctum* est locus, in quo nullus alius locus assumi

⩬ *variation sequence to 2A6C*
Gerhard 1858, p. 173.



360 SIGNA CONGRUENTIAE ET COINCIDENTIAE, 1677 – 1716 N. 109 (40946)

109 (40946). SIGNA CONGRUENTIAE ET COINCIDENTIAE
[1677 – 1716]

Überlieferung: *L* Konzept: LH 35 I 14 Bl. 75. 1 Streifen ca 16,5 × 3 cm. 8 Z. auf Bl. 75 v°, 3 Z. auf Bl. 75 r°. Auf Bl. 75 r° oben Berechnungen, die bis auf die Figuren gestrichen sind (= Z. 15–18). **[noch]**.

Datierungsgründe: **[noch]**.

= signum aequalitatis ∾ similitudinis ⊓ majoritatis ⊓ minoritatis ≃ congruentiae seu aequalitatis et similitudinis simul et denique ≈ coincidentiae seu identitatis. Videndum tamen an non hanc liceat distinguere in virtualem et formalem. Ex. gr. $\overset{A}{B\triangle C}$ ABC et BCA coincidunt etsi non respondeant seu non sint similiter expressa. Et idem in congruentia notari potest, ut $\overset{A}{B\triangle C}$ et $\overset{L}{M\triangle N}$ ABC et LMN congruunt simul et similiter exprimentur. Ergo ABC ≃ LMN. Sed si conferas ABC et MLN poterit scribi ABC ≂ MLN, quasi postposita similitudine seu scribemus ABC ∾ BCA. An ⩬ coincidens?

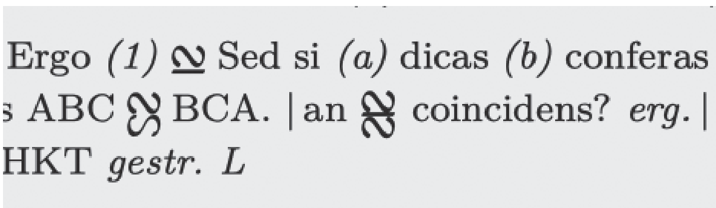
[*Berechnungen auf Bl. 75 r°, bis auf Figuren gestrichen*]

$CFK - KDM + MAC = ADH - KDM + MAC =$

[*Fig. 1*] [*Fig. 2*]

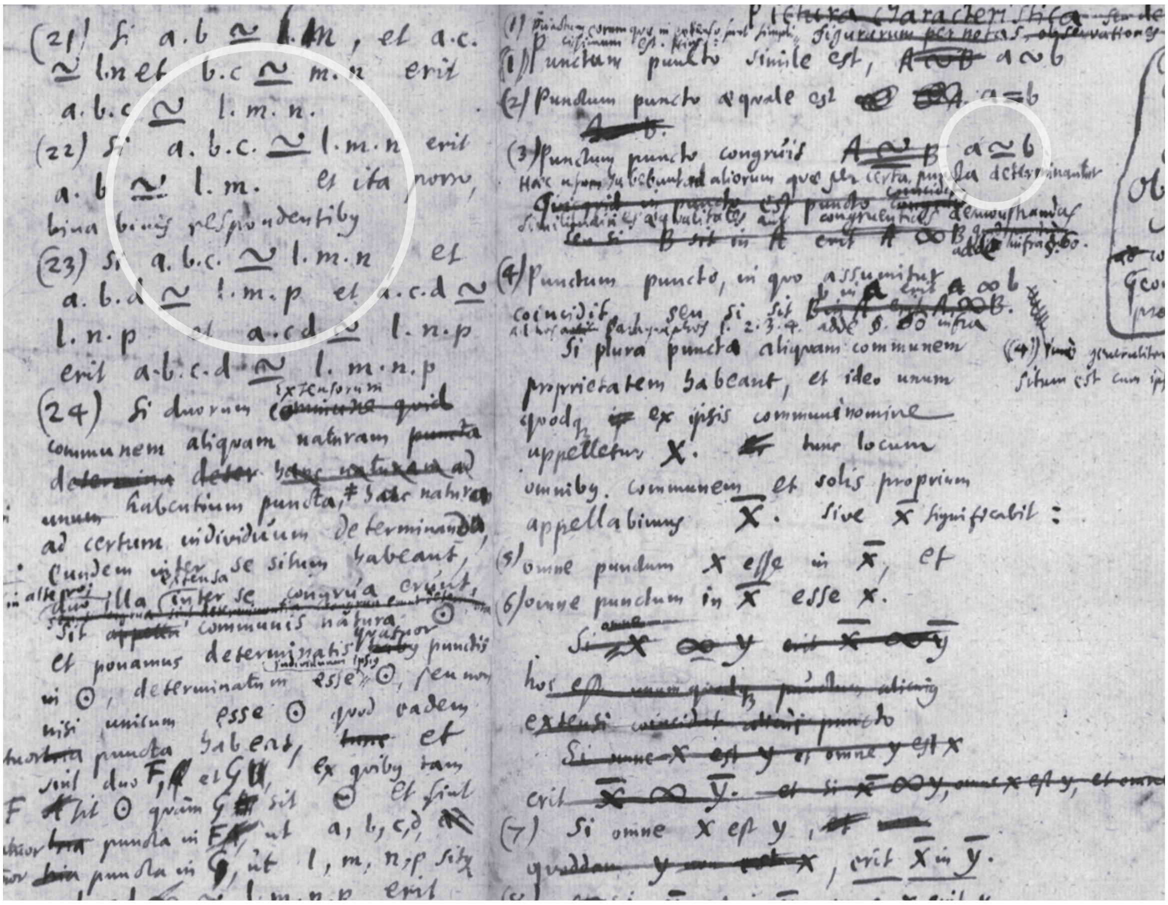
$ACKT + HKT$

7 signum *(1)* Similitu *(2)* aeqvalitatis *L* 8 et *(1)* ∾ *(2)* similitudinis simul *(a)* ⟨=⟩ co⟨i⟩ *(b)* ∞ *(c)* deniqve *(d)* et deniqve ≈ *(aa)* congruentiae *(bb)* coincidentiae *L* 8 f. identitatis *(1)* virtualis *(2)*. Videndum *L* 10 f. expressa. *(1)* tantum current *(2)* Et *L* 11 ut *(1)* congr⟨u⟩ *(2)* si *(3)* $\overset{A}{B\triangle C}$ et *L* 12 f. Ergo *(1)* ≃ Sed si *(a)* dicas *(b)* conferas ... scribi ≂ *(2)* ABC ≃ LMN *L* 13 seu *(1)* dice *(2)* scribemus ABC ∾ BCA. | an ⩬ coincidens? *erg.* | *L* 16 *(1)* AD *(2)* | CFK ... MAC = *gestr.* | *L* 18 ACKT + HKT *gestr.* *L*

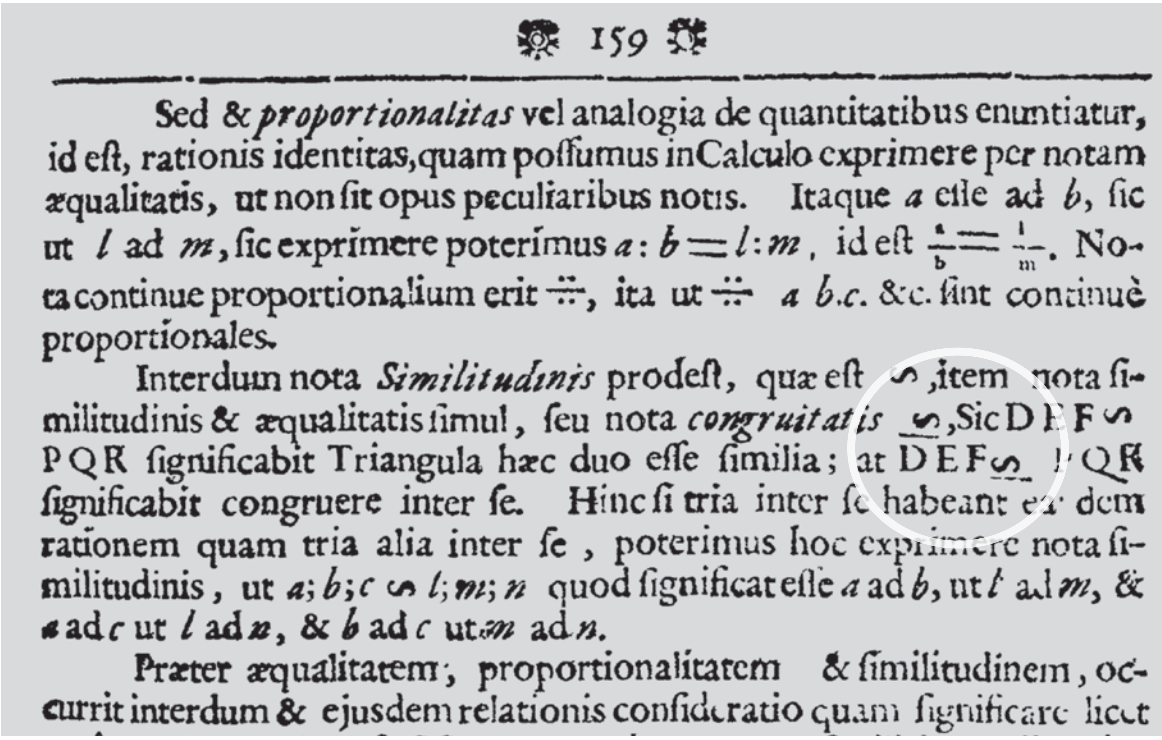


Ergo *(1)* ≃ Sed si *(a)* dicas *(b)* conferas
s ABC ∾ BCA. | an ⩬ coincidens? *erg.* |
HKT *gestr.* *L*

∾ INVERTED LAZY S OVER LAZY S;
⩬ *variation sequence to 2A6C,* ≃ *variation sequence to 2243,* ≈ *variation sequence to 2248,* ≂ *variation sequence to 2242*.
Mathesis vs. 2, Hannover 2025 (PDF), p. 360



≃ *variation sequence to 2243*
LH 35 I 14, fol. 1r



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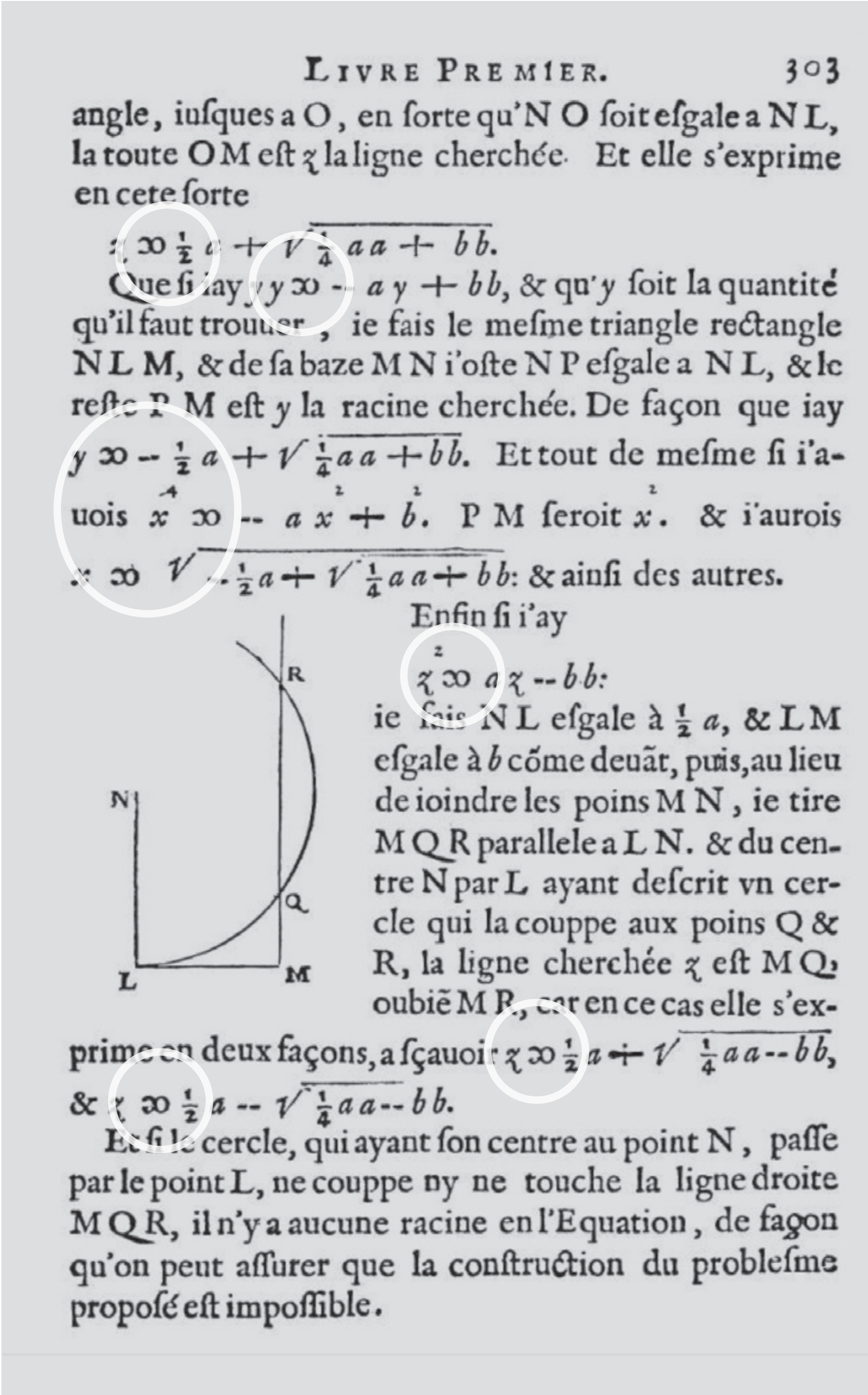
Sed & *proportionalitas* vel analogia de quantitatibus enuntiatur, id eſt, rationis identitas, quam poſſumus in Calculo exprimere per notam æqualitatis, ut non ſit opus peculiaribus notis. Itaque *a* eſſe ad *b*, ſic ut *l* ad *m*, ſic exprimere poterimus $a : b = l : m$, id eſt $\frac{a}{b} = \frac{l}{m}$. Nota continue proportionalium erit ∺, ita ut ∺ *a b.c.* &c. ſint continuè proportionales.

Interdum nota *Similitudinis* prodeſt, quæ eſt ∽, item nota ſimilitudinis & æqualitatis ſimul, ſeu nota *congruitatis* ≌, Sic DEF ∽ PQR ſignificabit Triangula hæc duo eſſe ſimilia; at DEF ≌ PQR ſignificabit congruere inter ſe. Hinc ſi tria inter ſe habeant eandem rationem quam tria alia inter ſe, poterimus hoc exprimere nota ſimilitudinis, ut *a*; *b*; *c* ∽ *l*; *m*; *n* quod ſignificat eſſe *a* ad *b*, ut *l* ad *m*, & *a* ad *c* ut *l* ad *n*, & *b* ad *c* ut *m* ad *n*.

Præter æqualitatem, proportionalitatem & ſimilitudinem, occurrit interdum & ejuſdem relationis conſideratio quam ſignificare licet

∽ *variation sequence to 22CD*
Monitum de Characteribus Algebraica, Miscellanea Berolinensia, 1710, p. 159



angle, iuſques a O, en ſorte qu'N O ſoit eſgale a N L, la toute O M eſt z la ligne cherchée. Et elle s'exprime en cete ſorte

$$z \infty \tfrac{1}{2} a + \sqrt{\tfrac{1}{4} aa + bb}.$$

Que ſi i'ay $yy \infty - ay + bb$, & qu'y ſoit la quantité qu'il faut trouuer, ie fais le meſme triangle rectangle N L M, & de ſa baze M N i'oſte N P eſgale a N L, & le reſte P M eſt y la racine cherchée. De façon que i'ay $y \infty - \tfrac{1}{2} a + \sqrt{\tfrac{1}{4} aa + bb}$. Et tout de meſme ſi i'auois $x^4 \infty - a x^2 + b^2$. P M ſeroit x^2. & i'aurois $x \infty \sqrt{-\tfrac{1}{2} a + \sqrt{\tfrac{1}{4} aa + bb}}$: & ainſi des autres.

Enfin ſi i'ay

$$z^2 \infty az - bb:$$

ie fais N L eſgale à $\tfrac{1}{2} a$, & L M eſgale à b cõme deuãt, puis, au lieu de ioindre les poins M N, ie tire M Q R parallele a L N. & du centre N par L ayant deſcrit vn cercle qui la couppe aux poins Q & R, la ligne cherchée z eſt M Q, oubiẽ M R, car en ce cas elle s'exprime en deux façons, a ſçauoir $z \infty \tfrac{1}{2} a + \sqrt{\tfrac{1}{4} aa - bb}$, & $z \infty \tfrac{1}{2} a - \sqrt{\tfrac{1}{4} aa - bb}$.

Et ſi le cercle, qui ayant ſon centre au point N, paſſe par le point L, ne couppe ny ne touche la ligne droite M Q R, il n'y a aucune racine en l'Equation, de façon qu'on peut aſſurer que la conſtruction du probleſme propoſé eſt impoſſible.

∞ CARTESIAN EQUAL

Descartes, La Géométrie, 1637, p. 303

Here the type composer utilized a turned **œ** letter from which he carved off the horizontal bar of the **e**, as a makeshift for ∞. Rather than sticking to that desperate solution, we see ∞ being graphically a rotated variant of 221D ∝ PROPORTIONAL TO.

2°.

— in — facit +

Esto $+d - e$ ducendum in $+c - b$, quod erit per praecedentem

$+dc - db - \ldots$

Sumpto $+v \infty +d - e$, et $+x \infty +c - b$ erit etiam

$+v + e \infty +d$, et $+x + b \infty +c$ et ad invicem ductis

$+vx \infty +dc - db - ec \cdot eb$ et

$+vx + vb + ex + eb \infty +dc$

Et locis $+v$ et $+x$ in hac ultima aequatione, ubi simul non conjunguntur, substitutis ipsarum valoribus, erit

$+vx + bd - be + ec - eb + eb \infty +dc$ quae reducitur ad

∞ CARTESIAN EQUAL
LAA III-2 p. 698. – Equal sign introduced and mainly used by René Descartes.

43. JOHANN JAKOB FERGUSON FÜR LEIBNIZ

[Hannover, Frühjahr 1680]. [42. 44.]

Überlieferung:

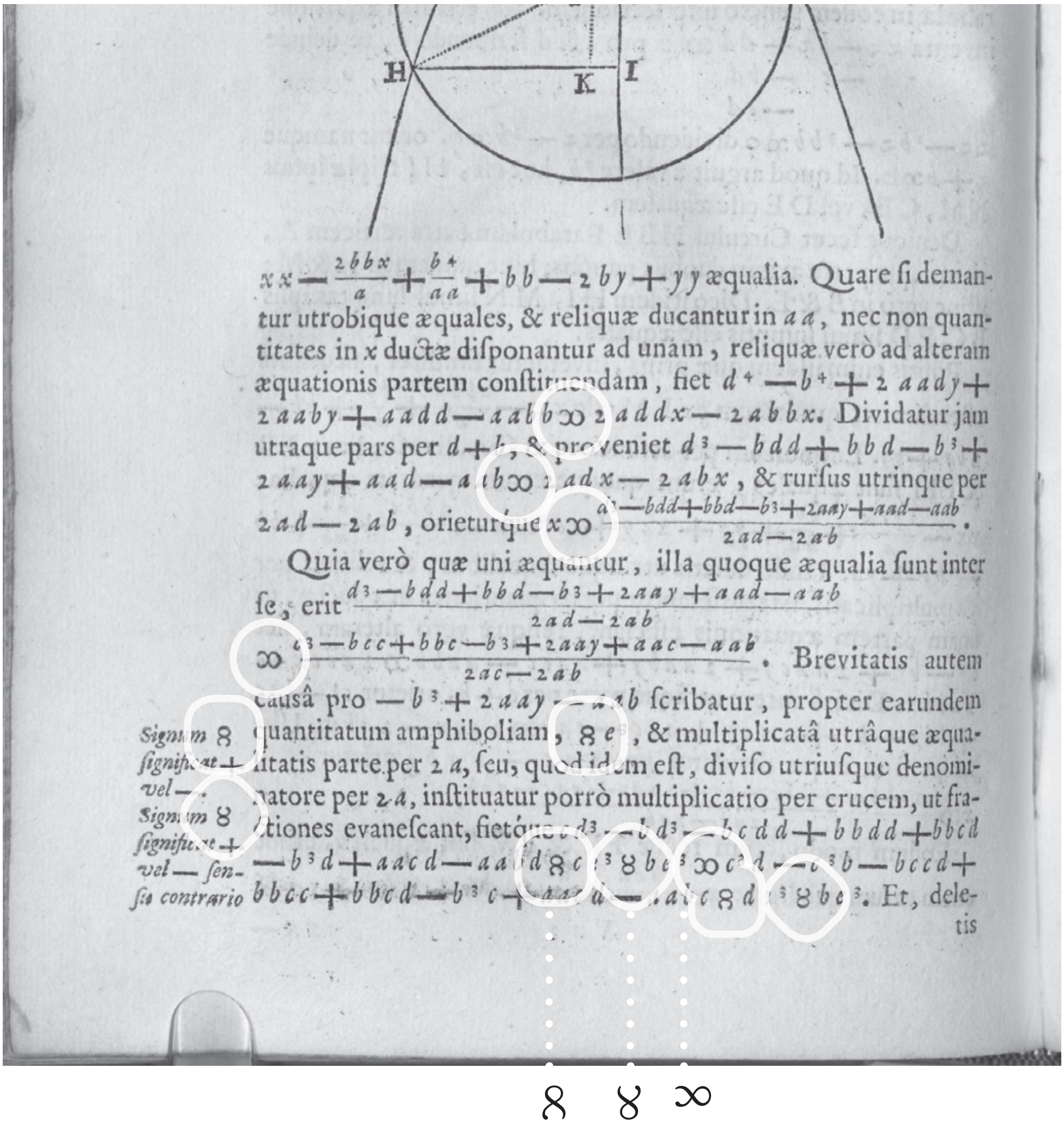
K Abfertigung: *LH* XXXV 12,2 Bl. 32—33. 1 Bog. 2°. 1 S. (Bl. 33 v°). Bemerkung von Leibniz' Hand. Auf Bl. 32 r° Aufzeichnung von Leibniz zur gleichen Thematik; auf Bl. 33 r° und Bl. 32 v° Aufzeichnung von Leibniz zum Alhazenschen Problem. — (Unsere Druckvorlage)

Ponatur latus quadrati $\infty\ ax + b$ eritque quadratum $aaxx + 2abx + bb$ addatur c et Cubus $aaxx + 2abx + bb + c$, aequale cubo cujus latus $dx + f$ ergo $\infty\ d^3x^3 + 3ddfxx + 3dffx + f^3$, sit jam $bb + c \infty f^3$ habebitur $d^3x^3 + 3ddfxx + 3dffx \infty aaxx + 2abx$ sive $d^3xx + 3ddfx + 3dff \infty aax + 2ab$ sit iterum $3dff \infty 2ab$, et erit $d^3xx + 3ddfx \infty aax$ sive $d^3x + 3ddf \infty aa$ vel $x \infty \frac{aa - 3ddf}{d^3}$ unde $dx + f \infty \frac{aa - 2ddf}{dd}$ sive $\infty \frac{aa}{dd} - 2f$ latus Cubi.

∞ CARTESIAN EQUAL. LAA III-3 p. 102.

			neum $\delta\gamma\pi\rho$	tur spatium $\delta\gamma\pi\rho$.
ibid.	l. 24.		$\frac{trdy}{z} = rdx$	v. pag. 284. l. 14. ubi $dy \infty \frac{zdx}{t}$.
pag.	286.	l. 9.	- - - - -	Nam $\frac{zdz}{\sqrt{rr - zz}} \infty FE$, omnia autem $FE \infty CA$ seu $\sqrt{rr - zz}$, z indefinite accipitur pro quavis DG.

∞ CARTESIAN EQUAL
LAA III-7 p. 137



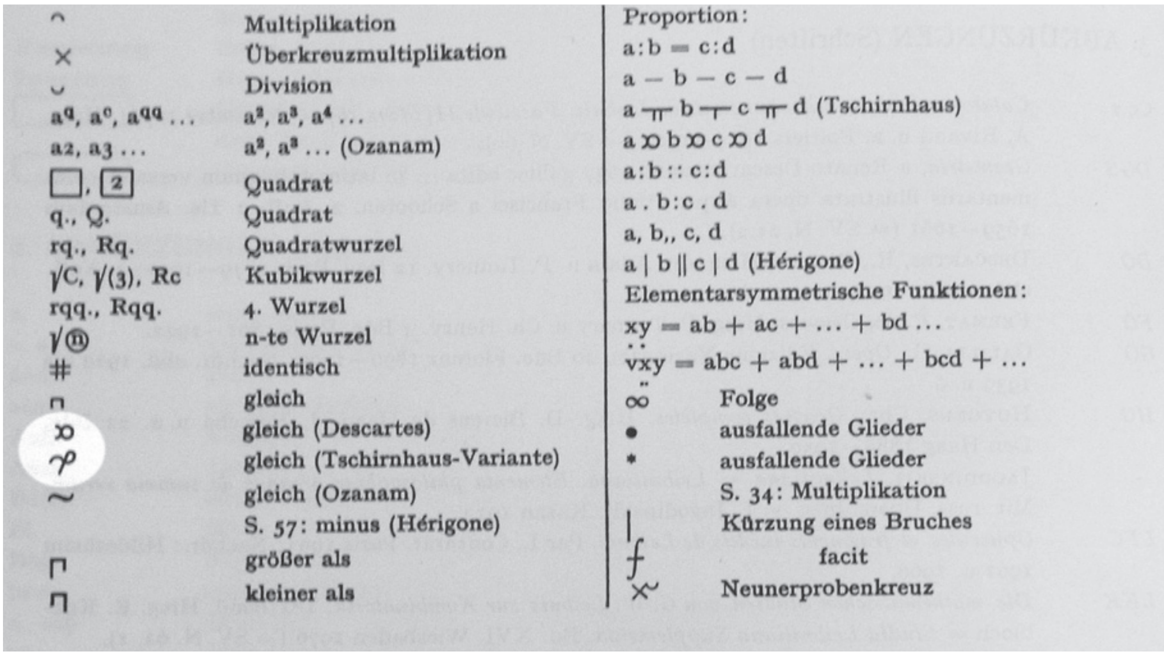
Signum 8 significat + vel —.
Signum 8 significat + vel — sensu contrario

$xx - \frac{2bbx}{a} + \frac{b^4}{aa} + bb - 2by + yy$ æqualia. Quare si demantur utrobique æquales, & reliquæ ducantur in aa, nec non quantitates in x ductæ disponantur ad unam, reliquæ verò ad alteram æquationis partem constituendam, fiet $d^4 - b^4 + 2aady + 2aaby + aadd - aabb \infty 2addx - 2abbx$. Dividatur jam utraque pars per $d + b$, & proveniet $d^3 - bdd + bbd - b^3 + 2aay + aad - aab \infty 2adx - 2abx$, & rursus utrinque per $2ad - 2ab$, orieturque $x \infty \frac{d^3 - bdd + bbd - b^3 + 2aay + aad - aab}{2ad - 2ab}$.

Quia verò quæ uni æquantur, illa quoque æqualia sunt inter se, erit $\frac{d^3 - bdd + bbd - b^3 + 2aay + aad - aab}{2ad - 2ab} \infty \frac{c^3 - bcc + bbc - b^3 + 2aay + aac - aab}{2ac - 2ab}$. Brevitatis autem causâ pro $- b^3 + 2aay - aab$ scribatur, propter earundem quantitatum amphiboliam, 8 e, & multiplicatâ utrâque æquationis parte per $2a$, seu, quod idem est, diviso utriusque denominatore per $2a$, instituatur porrò multiplicatio per crucem, ut fractiones evanescant, fietque $cd^3 - bd^3 - bcdd + bbdd + bbcd - b^3d + aacd - aa$ [illegible] d 8 c^3 8 $bc^3 \infty c^3d - c^3b - bccd + bbcc + bbcd - b^3c$ [illegible] c 8 d 8 bc^3. Et, deletis

8 8 ∞

∞ CARTESIAN EQUAL, ȣ LEIBNIZIAN CONGRUENCE-2,
8 LEIBNIZIAN CONGRUENCE-2 INVERTED
Francisci à Schooten In Geometriam Renati Des Cartes Commentarii, p. 340. Amsterdam 1659.
The image is from an anthology of Descartes' Geometria, this copy was in possession of G. W. Leibniz. It shows that van Schooten created the characters ȣ and 8 on the model of Descartes' sign for *equal* ∞; here he uses them for the meanings "plusminus" and "minusplus". Leibniz eventually adopted these characters to denote "congruence".
Source: GWLB Hannover, Leibn. Marg. 178, 1



⁀	Multiplikation	Proportion:	
×	Überkreuzmultiplikation	a:b = c:d	
‿	Division	a — b — c — d	
aq, ac, aqq ...	a^2, a^3, a^4 ...	a ╥ b ┬ c ╥ d (Tschirnhaus)	
a2, a3 ...	a^2, a^3 ... (Ozanam)	a ∞ b ∞ c ∞ d	
□, [2]	Quadrat	a:b :: c:d	
q., Q.	Quadrat	a . b:c . d	
rq., Rq.	Quadratwurzel	a, b,, c, d	
√C, √(3), Rc	Kubikwurzel	a | b || c | d (Hérigone)	
rqq., Rqq.	4. Wurzel	Elementarsymmetrische Funktionen:	
√ⓝ	n-te Wurzel	xy = ab + ac + ... + bd ...	
#	identisch	vxy = abc + abd + ... + bcd + ...	
⊓	gleich	∞	Folge
∞	gleich (Descartes)	•	ausfallende Glieder
∞	gleich (Tschirnhaus-Variante)	*	ausfallende Glieder
~	gleich (Ozanam)		S. 34: Multiplikation
	S. 57: minus (Hérigone)	|	Kürzung eines Bruches
⊓	größer als	ƒ	facit
⊓	kleiner als	✕	Neunerprobenkreuz

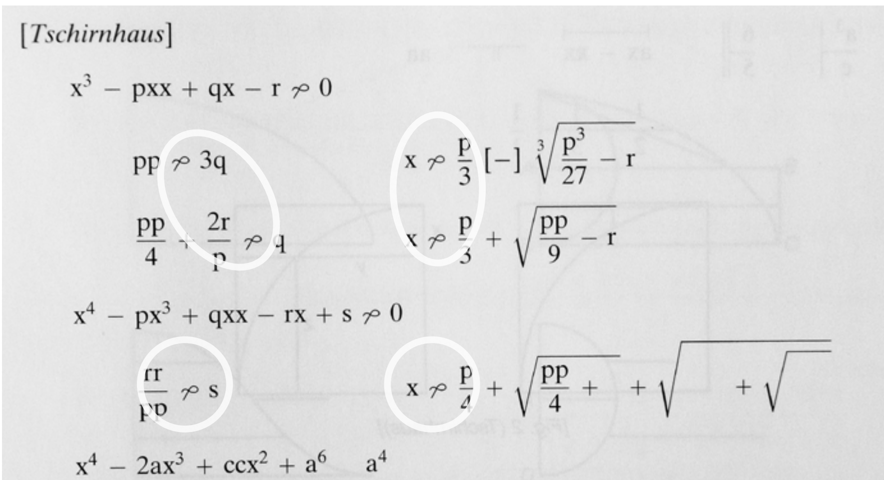
∞ CARTESIAN EQUAL – key to symbols, LAA VII-1

German natural scientist Ehrenfried Walther von Tschirnhaus (1651–1708) adopted Descartes' symbol ∞ for *equal,* but wrote it in a more sloppy version with a straight downwards going line. This led the editors of the Leibniz Akademie-Ausgabe (LAA) to decide to distinguish the two variants, and so these two came into use for many decades. Initially we proposed a second character:

∞ TSCHIRNHAUS EQUAL

which reflects this typographic convention. In certain situations it is desirable to maintain the distinction for historiographical reasons, to trace different authors and writing habits. On the other hand, ∞ and ∞ actually bear the same meaning: *equal*. Therefore we propose to encode ∞ as a new character but to encode the Tschirnhaus variant as a variation sequence:

```
xb17;CARTESIAN EQUAL;Sm;0;ON;;;;;N;;;;;
xb17 FE00; with descender; # CARTESIAN EQUAL
```



[*Tschirnhaus*]

x³ − pxx + qx − r ∞ 0

pp ∞ 3q   x ∞ p/3 [−] ∛(p³/27 − r)

pp/4 + 2r/p ∞ q   x ∞ p/3 + √(pp/9 − r)

x⁴ − px³ + qxx − rx + s ∞ 0

rr/pp ∞ s   x ∞ p/4 + √(pp/4 + + √(+ √(

x⁴ − 2ax³ + ccx² + a⁶   a⁴

∞ *variation sequence to CARTESIAN EQUAL* (Tschirnhaus variant)
LAA VII-2 p. 715

N. 26 ARITHMETISCHE KREISQUADRATUR 1673–1676 301

kan sien daer, AB is $\frac{1}{8}$ van AC dat het differ. ontrent is $\frac{1}{2}$ sec: soude dan diff: van de geheele AB. ontrent 3 secunden.

Maer soo men de $\angle ACB$, 2 mahl, in 2 gelijcke deelen deelt, dan is AB, een weijnig kleijnder als $\frac{1}{5}$ deel van AC (wen AB is $\propto AC$) en de $\angle$en differ. als men kan sien in de wercking bouen, daer AB is $\frac{1}{5}$ deel van AC, dat de differentie is ontrent 12 sec.

Daerom wen de sijde AB is $\propto AC$ ofte een wenig kleijnder, het is genoeg om de $\angle ACB$, te deelen in 2 mahl, in 2 gelijcke deel, de $\angle$ sal ontrent $\frac{4}{5}$ deel, van 1 minut differen (als men met de 2 eerste termen, als $\frac{b}{1} - \frac{b^3}{3} \propto$ de arcus ADE werckt) van de Tab. sinus; ende hoe naeder het kombt tot $\frac{1}{3}$ deel van AC, hoeweeniger het verschiet.

Soo AB is $\frac{1}{3}$ deel van AC ofte een wenig groter soo heeft men van nooden de $\angle ACB$

∝ *variation sequence to CARTESIAN EQUAL* (Tschirnhaus variant)
LAA VII-6 p. 301

66 EHRENFRIED WALTHER VON TSCHIRNHAUS AN LEIBNIZ, 17. IV. 1677 N. 30

sive cubica $x^3 - pxx + qx - r \propto o$ etc.; si jam saltem unicus terminus debeat auferri supponatur $x \propto a + y$ et transmutatur aequatio in qua unicus terminus debet auferri; ope $x \propto a + y$ in aliam; ubi y incognita radix, in qua ponatur ille terminus auferendus $\propto o$ atque sic inveniemus qua ratione a sit assumenda ad terminum illum auferendum. Sit eg. in hac aequatione $xx - px + q \propto o$ auferendus secundus terminus fiat $x \propto y + a$ jam vero

$$\begin{array}{l} xx \propto yy + 2ay + aa \propto o \\ -px \propto \quad - \quad py - pa \\ +q \propto \qquad\qquad q \end{array}$$ adeoque ponendo $2ay - py \propto o$ erit $2a - p \propto o$

et $a \propto \frac{p}{2}$, hinc patet debere fieri $x \propto y + \frac{p}{2}$ ad secundum terminum in aequatione quadratica

∝ *variation sequence to CARTESIAN EQUAL* (Tschirnhaus variant)
LAA III-2 p. 66; III-2 p. 285 (below)

incognitae potestates ordine per divisionem inserendo ac assumendo semper quotientes aequaliter compositas, quarum omnium possibilium modorum determinatus semper numerus facile exhibetur; hanc vero Methodum in praesentia abunde declaravi et specimina exhibui; sed non ita pridem ad majorem perfectionem deduxi. 2^{da} est supponendo formulas omnes possibiles radicalium $x \propto \sqrt{a} + \sqrt{b}$, $x \propto \sqrt[3]{a} + b$, $x \propto \sqrt{a + \sqrt{b + \sqrt{c}}}$ quae facile omnes quot esse possunt numero determinantur et tunc liberandae sunt ab signis radicalibus atque comparatio instituenda. Specimen Tibi exhibebo ad formulas Cardanicas obtinendas sit $x \propto \sqrt[3]{a} + \sqrt[3]{b}$ supponatur jam $\sqrt[3]{a} \propto c$ et $\sqrt[3]{b} \propto d$ et habebimus has tres aequationes $x \propto c + d$, $a \propto c^3$ et $b \propto d^3$ quibus reductis inveniemus aequationem absque signo radicali (ut Tibi jam notum erit juxta Methodum D. de Beaune radicalia signa auferendi, quaeque

N. 46 NOTAE AD TRIANGULA NUMERORUM ET AD ALGEBRAM, Erste Hälfte Mai 1676 287

[*Vierter Teil*]

$$\cancel{a} + \cancel{b} \propto \cancel{cc} + \cancel{2cd} + dd$$

$$a \propto cc \qquad b \propto 2cd$$

$$\cancel{a^2} + \cancel{2ab} + b^2 \propto \cancel{c^3} + \cancel{3cd^3} + 3c^3d + d^3$$

$$a^2 \propto c^3 \qquad 2ab \propto 3c^2d \qquad b^2 \propto 3cd^2 + d^3$$

$$a \propto \sqrt{c^3} \qquad b \propto \frac{3c^2d}{2a} \qquad \frac{9c^4dd}{4\cancel{c^3}} \propto 3c^3d + d^3$$

$$\frac{9cdd}{4}$$

$$9cdd \propto 12c^3d + d^3$$

$$9cd \propto 12c^3 + dd$$

$$dd \propto 9cd - 12c^3$$

$$d \propto 3c + \sqrt{9cc - 12c^3}$$

$$d \propto 3c + c\sqrt{9 - 12c}$$

$\propto$ *variation sequence to CARTESIAN EQUAL* (Tschirnhaus variant)
LAA VII-8 p. 287; III-2 p. 380 (below)

380 EHRENFRIED WALTHER VON TSCHIRNHAUS AN LEIBNIZ, 10. IV. 1678 N. 154

ratione determinentur. Atque sic haec porro sese ita in infinitum habere; sed prolixioribus non opus, cum operanti juxta ea quae diximus haec sese statim manifestabunt. Attamen ut omni ex parte satisfaciam, Demonstratio possibilitatis poterat universalius et facilius sic absolvi; aequationes seu quaestiones ex aequaliter compositis primis et simplicissimis quantitatibus $x + y \propto a$ et $xy \propto b$ reducuntur ad quadraticam $yy - ay + b \propto o$; $x + y + z \propto a$, $xy + xz + yz \propto b$, $xyz \propto c$ ad Cubicam $y^3 - ayy + by - c \propto o$; $x + y + z + t \propto a$, $xy + xz + xt + yz + yt + zt \propto b$, $xyz + xyt + xzt + yzt \propto c$, $xyzt \propto d$ ad quadrato-quadraticam $y^4 - ay^3 + byy - cy + d \propto o$ atque sic porro ubi jam notum et facillime demonstratur.

Jam vero 2[do] aequationes

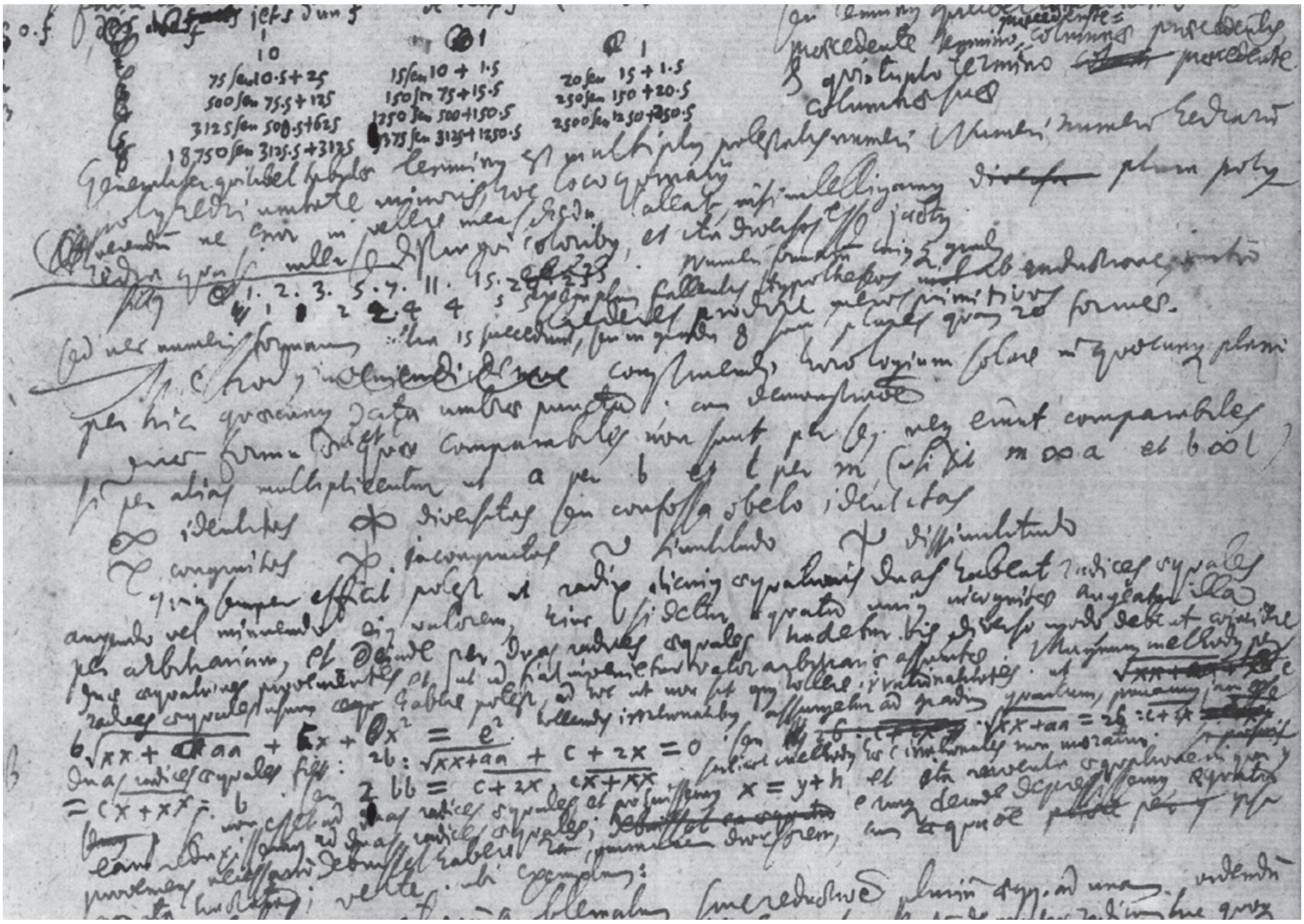
$xx + yy \propto a$, $xy \propto b$ possunt reduci ad $xx + yy \propto a$ et $xxyy \propto bb$ etc.
$x^3 + y^3 \propto a$ $\qquad$ $x^3 + y^3 \propto a$ $\quad$ $x^3y^3 \propto b^3$
$x^4 + y^4 \propto a$ $\qquad$ $x^4 + y^4 \propto a$ $\quad$ $x^4y^4 \propto b^4$

item per superiora Theoremata aequationes

$xx + yy + zz \propto a$, $\quad xy + xz + yz \propto b$, $\quad xyz \propto c$
$x^3 + y^3 + z^3 \propto a$
$x^4 + y^4 + z^4 \propto a$

reducuntur ad aequationes

$xx + yy + zz \propto a$, $\quad xxyy + yyzz + xxzz \propto$ cognitae $\quad xxyyzz \propto cc$
$x^3 + y^3 + z^3 \propto a$ $\quad x^3y^3 + y^3z^3 + x^3z^3$ quantitati $\quad x^3y^3z^3 \propto c^3$
$x^4 + y^4 + z^4 \propto a$ $\quad x^4y^4 + y^4z^4 + x^4z^4$ $\quad x^4y^4z^4 \propto c^4$



The marked lines read:

Duae formulae et quae comparabiles non sunt per se, neque erunt comparabiles si per alias multiplicentur ut *a* per *b* et *l* per *m,* (nisi sit $m \infty a$ et $b \infty l$)

∞ identitas ⧞ diversitas seu confossa obelo identitas

[LEIBNIZIAN CONGRUENCE] congruitas [LEIBNIZIAN CONGRUENCE WITH VERTICAL BAR] incongruitas ∾ similitudo [LEIBNIZIAN DISSIMILARITY] dissimilitudo

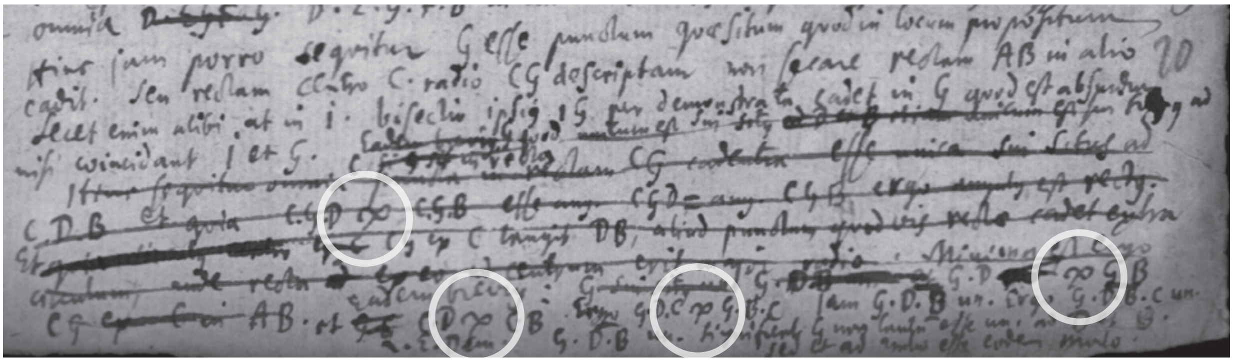
This detail of Leibniz's manuscript LH 35 VIII 30, f. 119v, shows ∞ (221E), ⧞ (29DE) and ∾ (variant to 223E) alongside the characters:

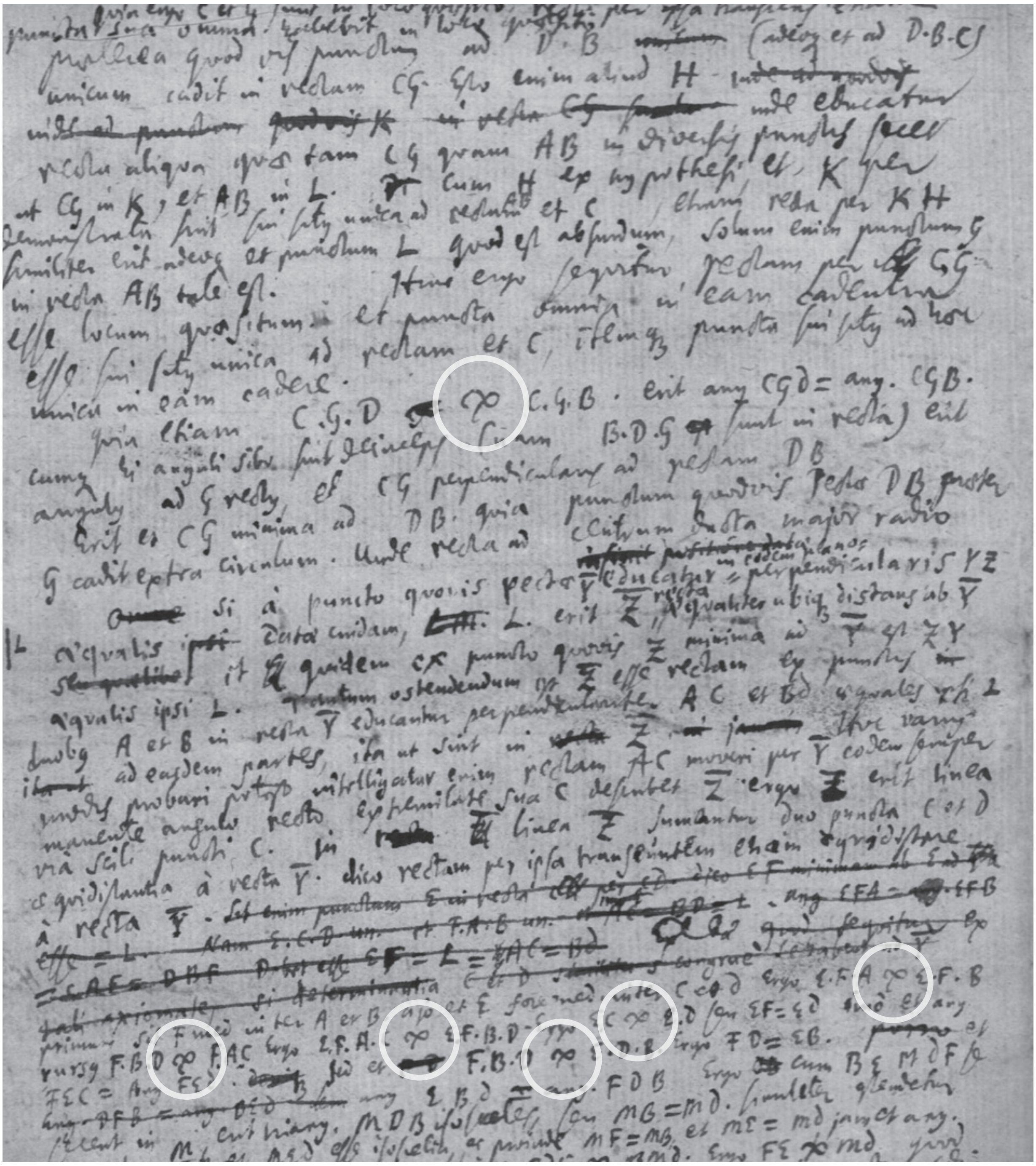
[LEIBNIZIAN CONGRUENCE] LEIBNIZIAN CONGRUENCE

[LEIBNIZIAN CONGRUENCE WITH VERTICAL BAR] LEIBNIZIAN CONGRUENCE WITH VERTICAL BAR

[LEIBNIZIAN DISSIMILARITY] LEIBNIZIAN DISSIMILARITY

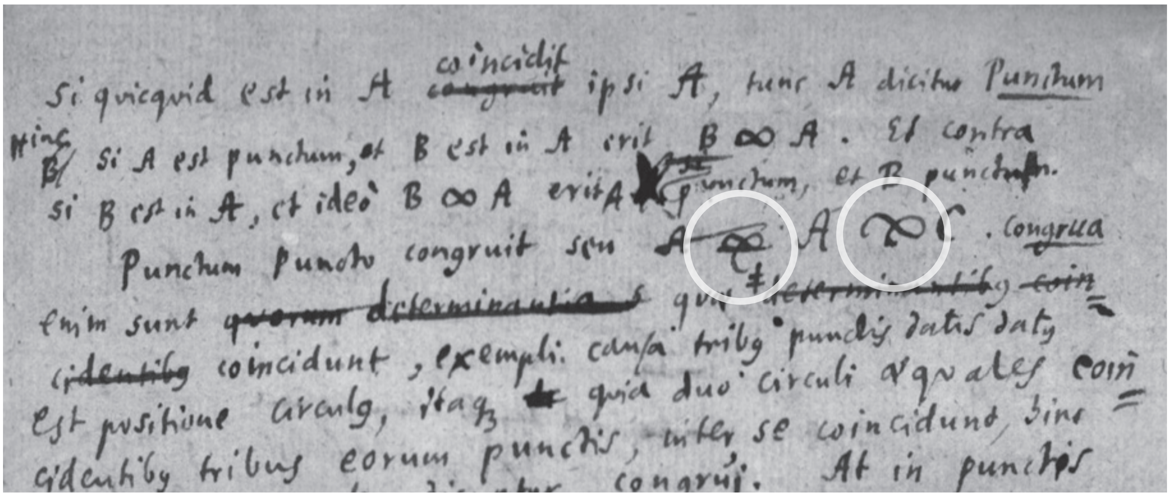
We prefer the latter character [LEIBNIZIAN DISSIMILARITY] not to be seen as a mere variant of 2241 ≁ NOT TILDE and to give it its own codepoint. The obliqueness of the dash in 2241, together with the distinct lazy-S shape, does not let a unification under one codepoint seem appropriate.



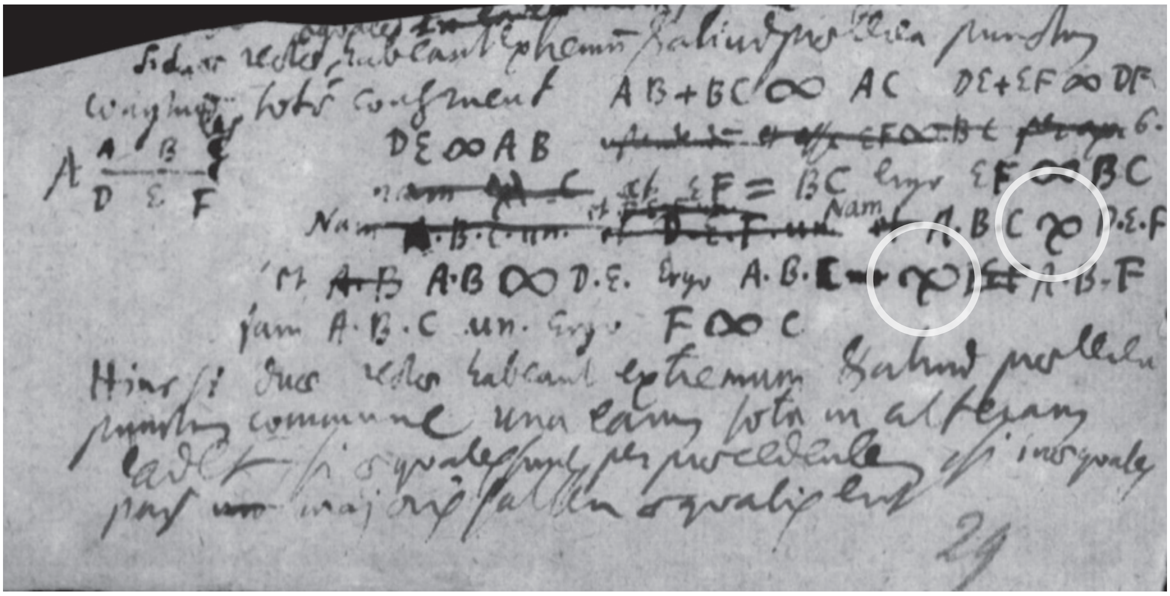


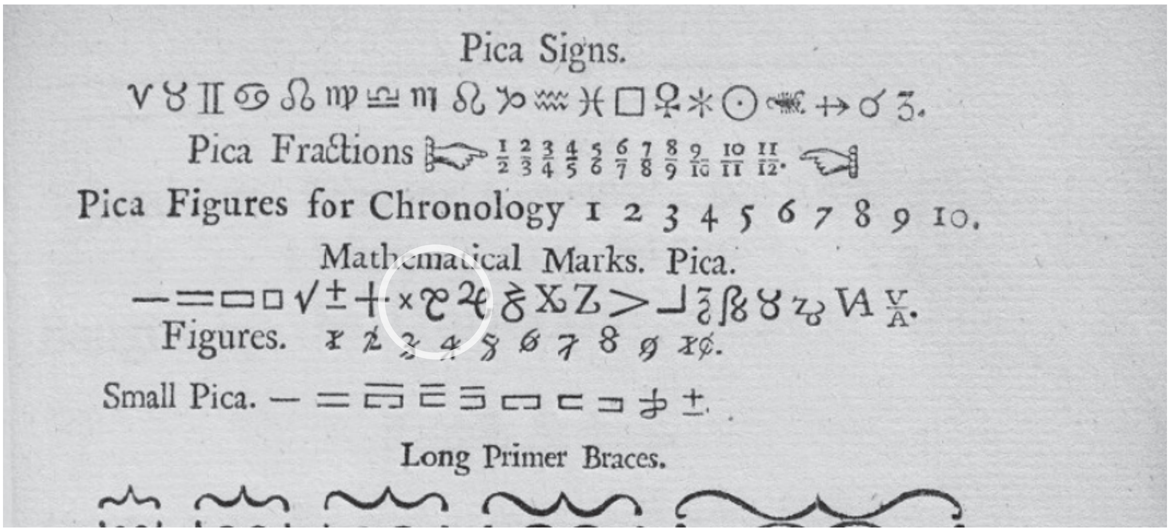
ꝏ LEIBNIZIAN CONGRUENCE

Ms. LH 35 I 14, fol. 20r, 20v. *This manuscript is under preparation for edition*.



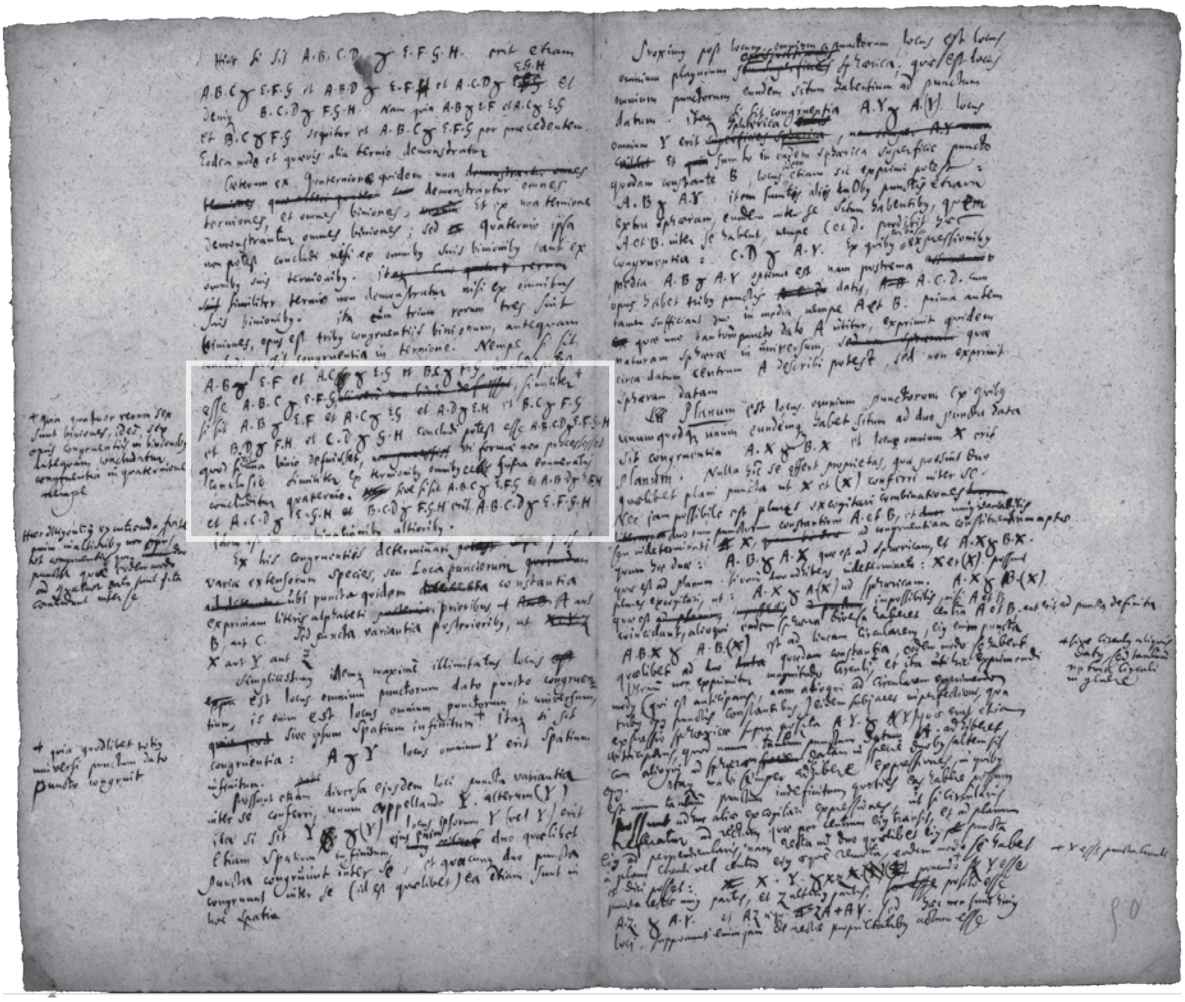
LEIBNIZIAN CONGRUENCE
Ms. LH 35 I 14, fol. 27r (top), fol. 29r

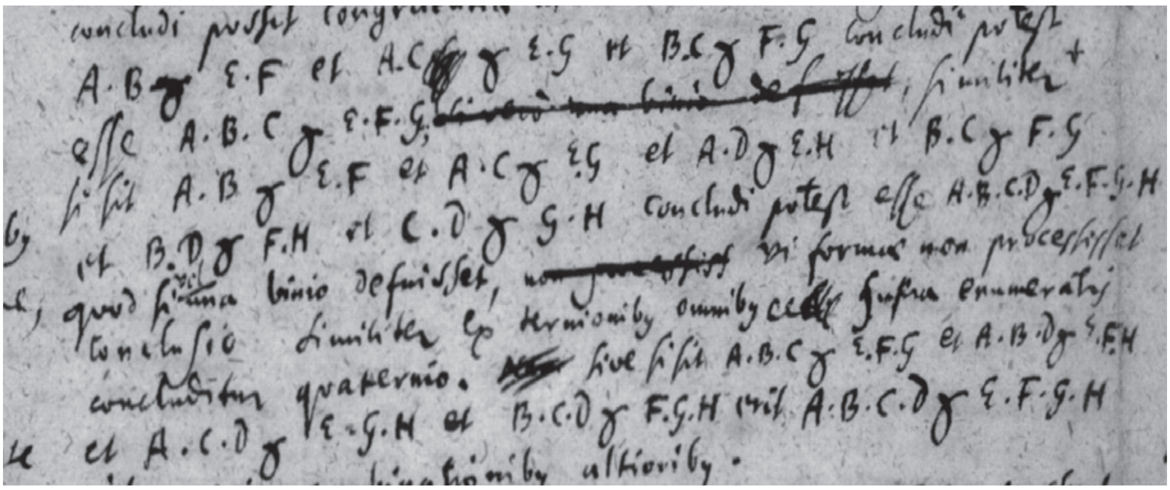




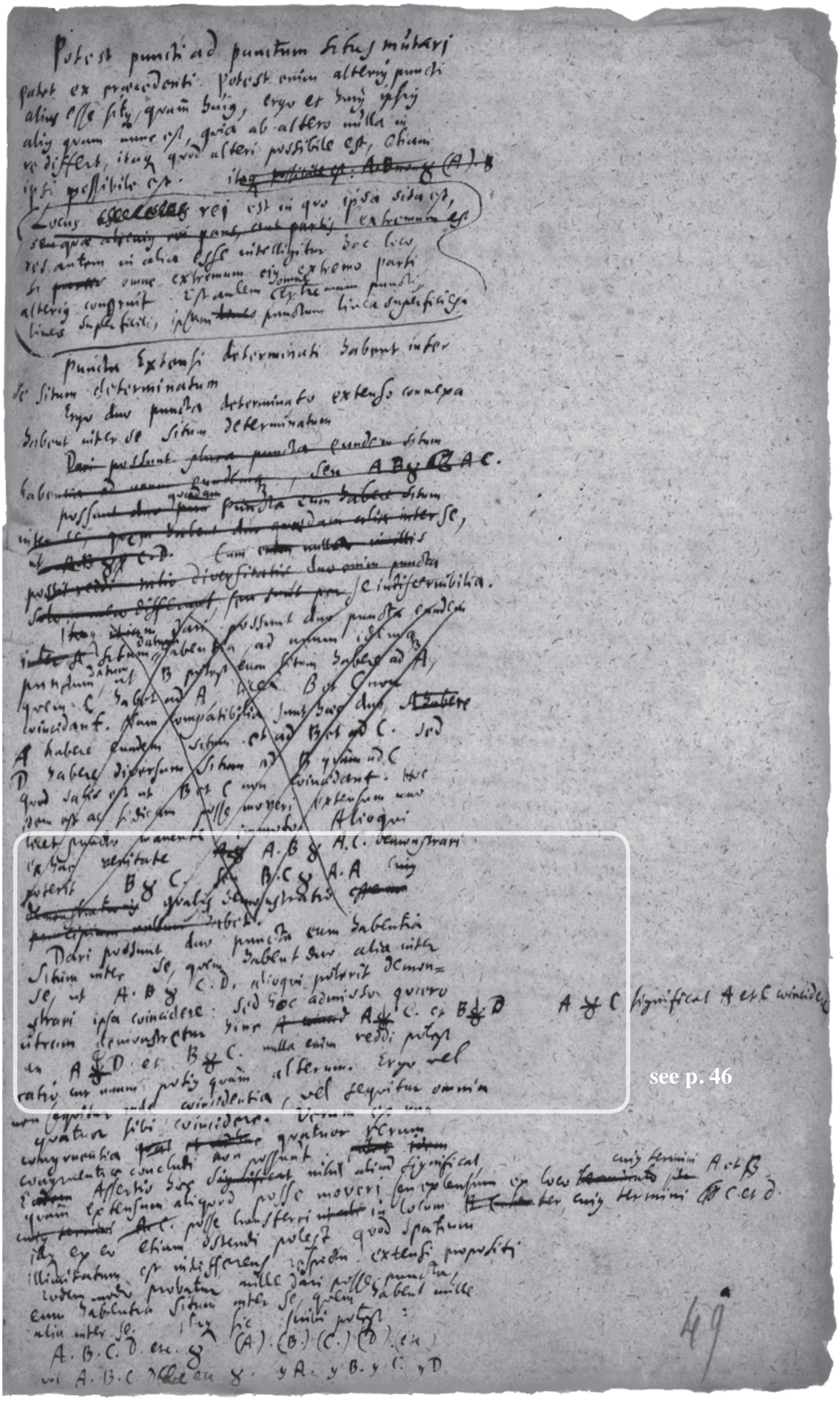
LEIBNIZIAN CONGRUENCE
From a type specimen by Dr. John Fell, Oxford 1695. Source: Bodleian Library Oxford

Leibniz used an even greater and rather complex variety of symbols for *congruence*: ≃, ⌓, ꝏ, ჯ, ჯ̶ and ჯ̶̶. In this set, ꝏ is a glyph derived from the letter *c*, but its shape also reflects the intention to show a relation to the *infinity* symbol ∞. In a similar way he developed the symbols ჯ, ჯ̶ and ჯ̶̶ on the basis of the shape of ∝ CARTESIAN EQUAL and are used very frequently. They form a group in which the base character (ჯ) gets differentiated in terms of the aspect of *coincidence* (with or without). – First, a few examples from Leibniz's manuscripts.

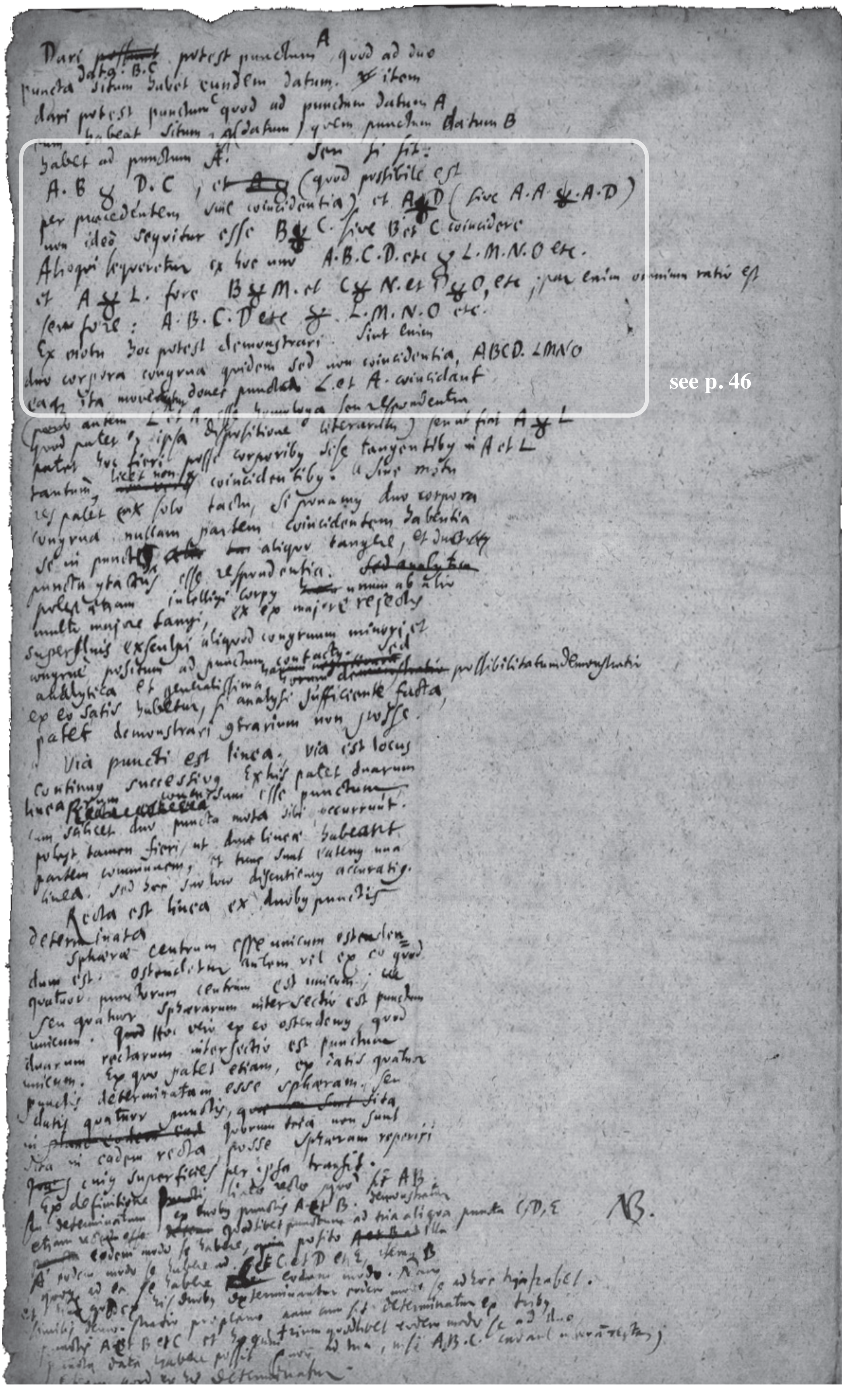




ჯ LEIBNIZIAN CONGRUENCE-2
LH 35 I 11, fol. 47v-46r (top); detail of fol. 47v


see p. 46

LH 35 I 11, fol. 49r



Dari ~~possunt~~ potest punctum A quod ad duo puncta data B.C situm habet eundem datum. Et item dari potest punctum C quod ad punctum datum A habeat situm, ad datum, quem punctum datum B habet ad punctum A. Sint si fit: A.B ∞ D.C, et ~~A.E~~ (quod possibile est per praecedentem sine coincidentia) et A.D (sive A.A ∞ A.D) non ideo sequitur esse B ∞ C. sive B et C coincidere. Alioqui sequeretur ex hoc uno A.B.C.D. etc ∞ L.M.N.O etc. et A ∞ L. fore B ∞ M. et C ∞ N. et D ∞ O, etc, [illegible] semper fore: A.B.C.D etc ∞ L.M.N.O etc. Ex motu hoc potest demonstrari. Sint enim duo corpora congrua quidem sed non coincidentia, ABCD. LMNO eaque ita moveantur donec puncta L et A coincidant [illegible]

[illegible]

Via puncti est linea. [illegible]

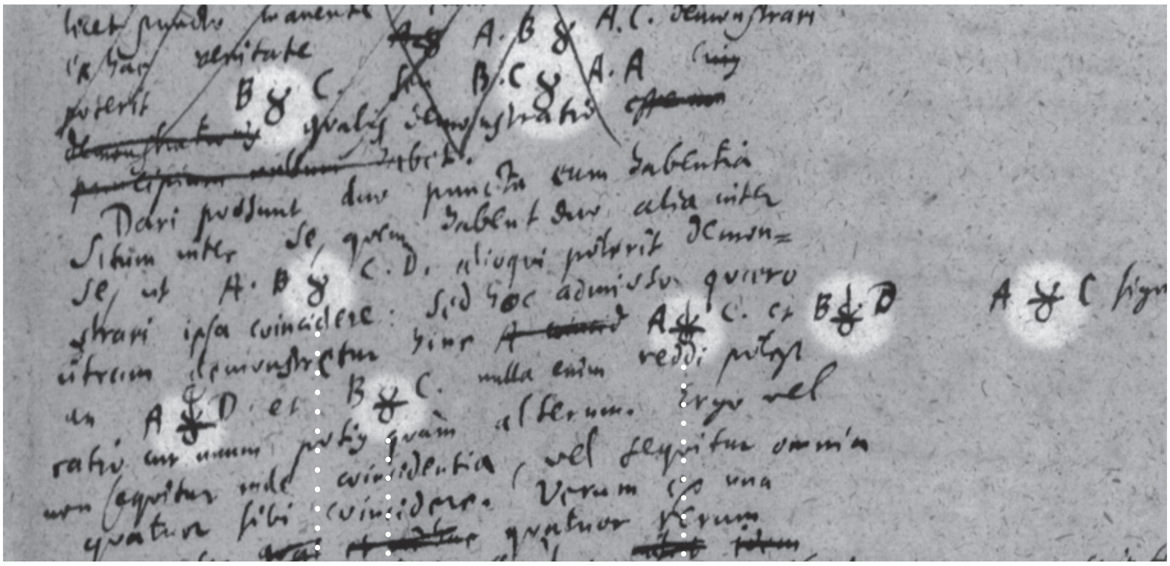
[illegible]

see p. 46

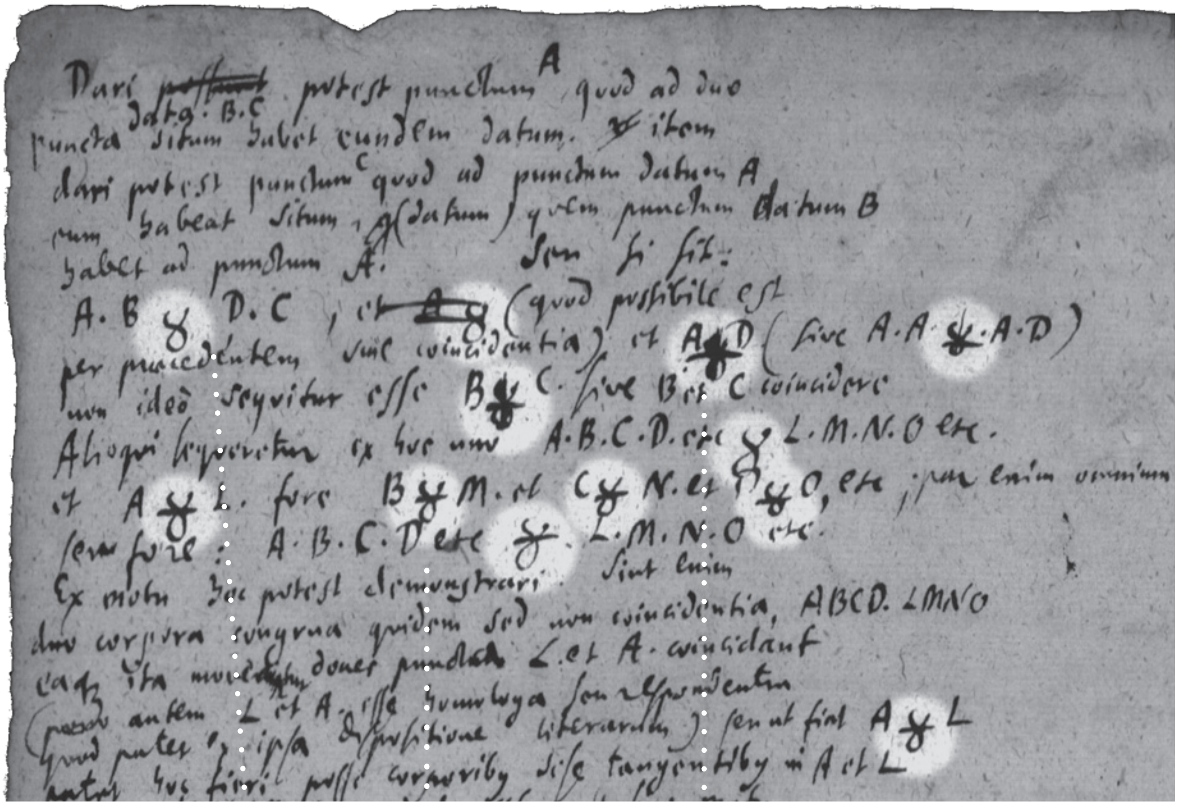
LH 35 I 11, fol. 49v

The shape of LEIBNIZIAN CONGRUENCE-2 is in a way similar to ȣ GREEK SMALL LETTER OMICRON UPSILON, which we propose in doc. **L-2535** (N5335R). They have different meanings and function in the mathematical context:. is a relation symbol whereas ȣ is a Greek letter used as a variable. Visually and ȣ are clearly different in their position on the baseline. – See L-2535 p. 10 for further explanation.

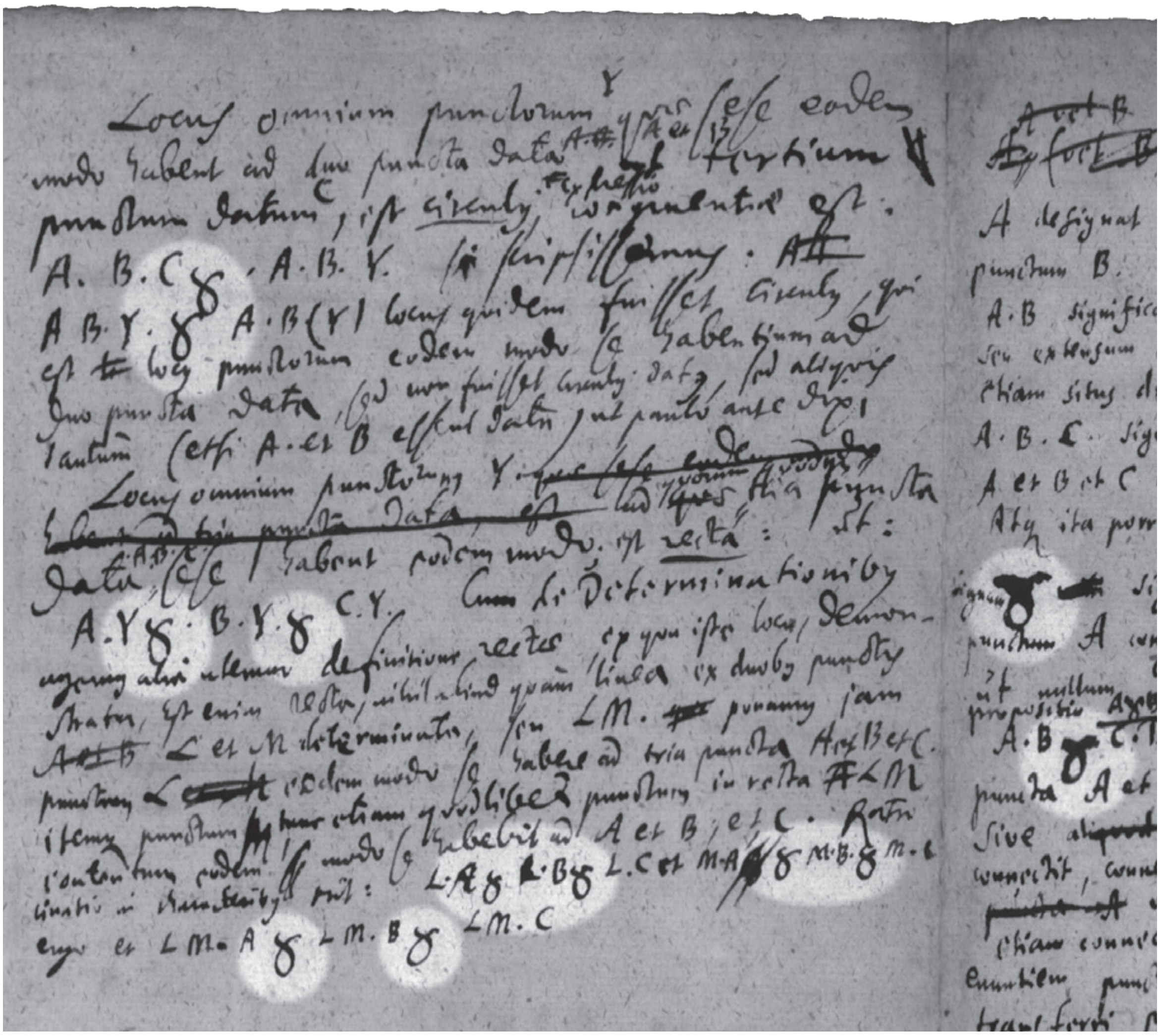
LEIBNIZIAN CONGRUENCE-2, LEIBNIZIAN CONGRUENCE-2 WITH HORIZONTAL BAR, LEIBNIZIAN CONGRUENCE-2 WITH HORIZONTAL AND VERTICAL BAR



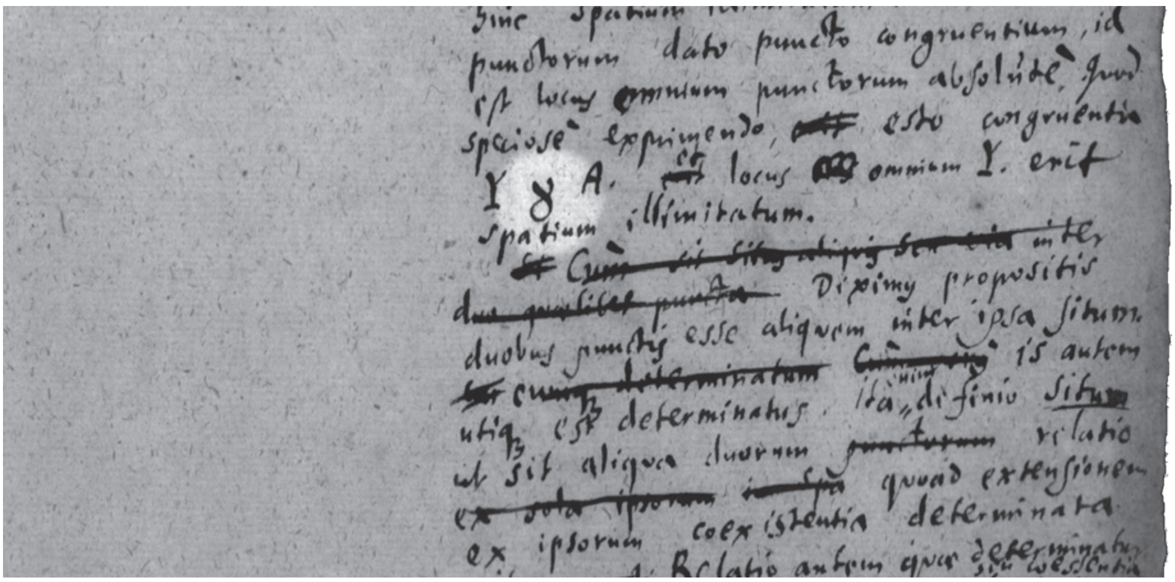
LH 35 I 11, part of fol. 49r



LH 35 I 11, part of fol. 49v



♉ LEIBNIZIAN CONGRUENCE-2
LH 35 I 11, part of fol. 47r



♉ LEIBNIZIAN CONGRUENCE-2
LH 35 I 11, part of fol. 49r

N. 10 (40848) DE PERFECTIONE CHARACTERISTICAE NOVAE, 1679 (?) 83

inter se, seu omnia puncta esse unum et idem. Nam quod unum punctum A alteri alicui C non coincidat, non potest aliter demonstrari, quam quod aliud quoddam punctum datur, B, cujus respectu diversum habent situm, ita ut $A.B.$ non ♉ $C.B.$

Potest puncti ad punctum situs mutari patet ex praecedenti. Potest enim alterius puncti alius esse situs, quam hujus, ergo et hujus ipsius alius quam nunc est, quia ab altero nulla in re differt, itaque quod alteri possibile est, etiam ipsi possibile est.

Locus rei est in quo ipsa sita est, res autem in alia esse intelligitur hoc loco, si omne extremum ejus extremo parti alterius congruit. Est autem omne extremum puncti, lineae superficiei, ipsum punctum linea superficies.

Puncta Extensi determinati habent inter se situm determinatum. Ergo duo puncta determinato extenso connexa habent inter se situm determinatum.

Dari possunt duo puncta eum habentia situm inter se, quem habent duo alia inter se, ut $A.B$ ♉ $C.D$. Alioqui poterit demonstrari ipsa coincidere: sed hoc admisso quaero utrum demonstretur hinc A ♉ C et B ♉ D an A ♉ D et B ♉ C. Nulla enim reddi potest ratio cur unum potius quam alterum. Ergo vel non sequitur inde coincidentia, vel sequitur omnia quatuor sibi coincidere. Verum ex una congruentia quatuor rerum congruentiae concludi non possunt. Assertio haec nihil aliud significat, quam extensum aliquod posse moveri seu extensum ex loco cujus termini A et B posse transferri in locum cujus termini C et D.

84 DE PERFECTIONE CHARACTERISTICAE NOVAE, 1679 (?) N. 10 (40848)

quem habent milla alia inter se. Itaque sic scribi potest: $A.B.C.D.$ etc. ♉ $(A).(B).(C).(D).$ (etc) vel $A.B.C.D$ etc. ♉ $yA.yB.yC.yD.$

Dari potest punctum A, quod ad duo puncta data $B.C$ situm habet eundem datum. Item dari potest punctum C quod ad punctum datum A eum habeat situm (datum), quem punctum datum B habet ad punctum A. Seu si sit: $A.B$ ♉ $D.C$ (quod possibile est per praecedentem sine coincidentia) et A ♉ D (sive $A.A$ ♉ $A.D$) non ideo sequitur esse B ♉ C. sive B et C coincidere. Alioqui sequeretur ex hoc uno $A.B.C.D.$ etc. ♉ $L.M.N.O.$ etc. et A ♉ L. fore B ♉ M. et C ♉ N. et D ♉ O, etc.; par enim omnium ratio est seu fore $A.B.C.D$ etc. ♉ $L.M.N.O$ etc.

Ex motu hoc potest demonstrari. Sint enim duo corpora congrua quidem sed non coincidentia, $ABCD$. $LMNO$. eaque ita moveantur donec puncta L. et A. coincidant (porro autem L. et A. esse homologa seu respondentia quod patet ex ipsa dispositione literarum) seu ut fiat A ♉ L. Patet hoc fieri posse corporibus sese tangentibus in A et L tantum, licet non coincidentibus. Sine motu res patet ex solo tactu, si ponamus duo corpora congrua nullam partem coincidentem habentia se in puncto aliquo tangere, et duo puncta contactus esse respondentia. Potest etiam intelligi corpus unum ab alio multo majore tangi, et ex majore rejectis superfluis exsculpi aliquod congruum minori et congrue positum ad punctum contactus. Sed analytica et generalissima harum possibilitatum demonstratio ex eo satis habetur, si analysi sufficiente facta, patet demonstrari contrarium non posse.

♉ LEIBNIZIAN CONGRUENCE-2, ♉̶ LEIBNIZIAN CONGRUENCE-2 WITH HORIZONTAL BAR

Philiumm vs. 2 (2023), p. 83, 84

by De Witt.[1] Wallis[2] wrote ȣ for + or −, and ȣ for the contrary. The sign ȣ was used in a restricted way, by James Bernoulli;[3] he says, "ȣ significat + in pr. e − in post. hypoth.," i.e., the symbol stood for + according to the first hypothesis, and for −, according to the second hypothesis. He used this same symbol in his *Ars conjectandi* (1713), page 264. Van Schooten wrote also ȣ for ∓. It should be added that ȣ appears also in the older printed Greek books as a ligature or combination of two Greek letters, the omicron *o* and the upsilon *υ*. The ȣ appears also as an astronomical symbol for the constellation Taurus.

Da Cunha[4] introduced ±' and ±', or ±' and ∓', to mean that the upper signs shall be taken simultaneously in both or the lower signs shall be taken simultaneously in both. Oliver, Wait, and Jones[5] denoted positive or negative *N* by $^{\pm}N$.

211. The symbol [*a*] was introduced by Kronecker[6] to represent

ȣ LEIBNIZIAN CONGRUENCE-2, ȣ LEIBNIZIAN CONGRUENCE-2 INVERTED
Cajori I p. 246. In this paragraph Cajori explains the different usage of this two symbols for "+ or –" and "- or +" by van Schooten , Bernoulli and Wallis. A variety of symbols was used during the 17th century for denoting plus-minus. Leibniz used the same symbols in a different context in order to denote *congruence*, hence the proposed character name in this proposal.
Despite of what Cajori writes here about the similar looking characters *omicron-upsilon* (ȣ, see top of p. 46 and doc. L-2535 on letterlike symbols p. 10) and the astrological *Taurus* symbol ♉ (2649), ȣ must not be mixed up with neither of them.

binomium *a* ȣ √*bc*,

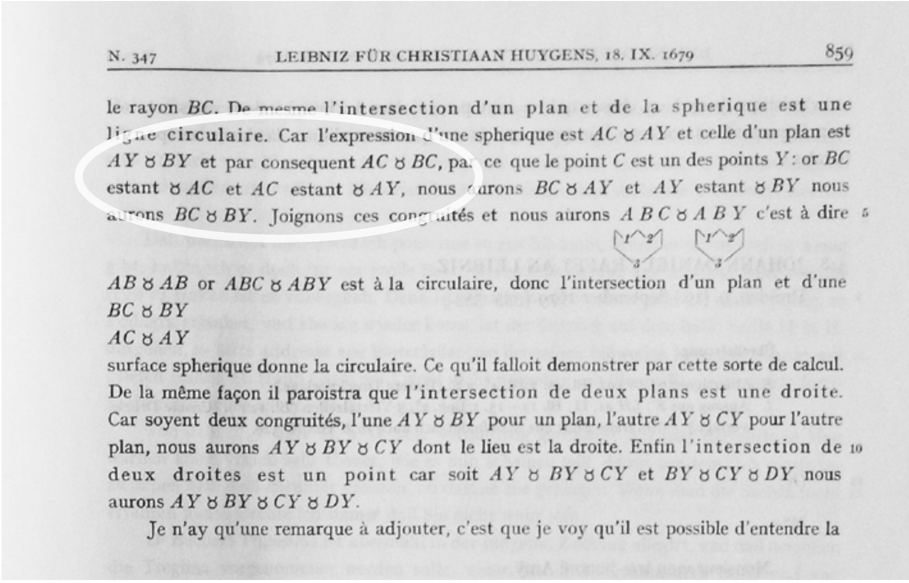
ȣ LEIBNIZIAN CONGRUENCE-2 INVERTED; Descartes, Geometria, p. 330

Where the First Term hath the Sign + (becauſe made by Multiplying + into +;) The Second Term is wanting (becauſe − *ya*³ and + *ya*³ deſtroy each other:) In the Third Term, *yy* hath − (becauſe made of + *y* into − *y*;) and *b*, *d*, have the ſame Terms as in the Quadraticks, (which Sign, be it + or −, we here deſign by ȣ, and its contrary by ȣ:) In the Fourth Term, *b* hath the ſame Sign as before (becauſe Multiplied into + *y*;) but *d* the contrary to what it had (becauſe Multiplyed into − *y*.) And thus far it holds conſtantly, whatever be the Signs of *p*, *q*, *r*.

ȣ LEIBNIZIAN CONGRUENCE-2, ȣ LEIBNIZIAN CONGRUENCE-2 INVERTED
Wallis, Algebra, p. 210

(ȣ *ſignificat* + *in pr.* & − *in poſt. hypoth.*

ȣ LEIBNIZIAN CONGRUENCE-2 INVERTED
Acta eruditorum 1701, p. 214



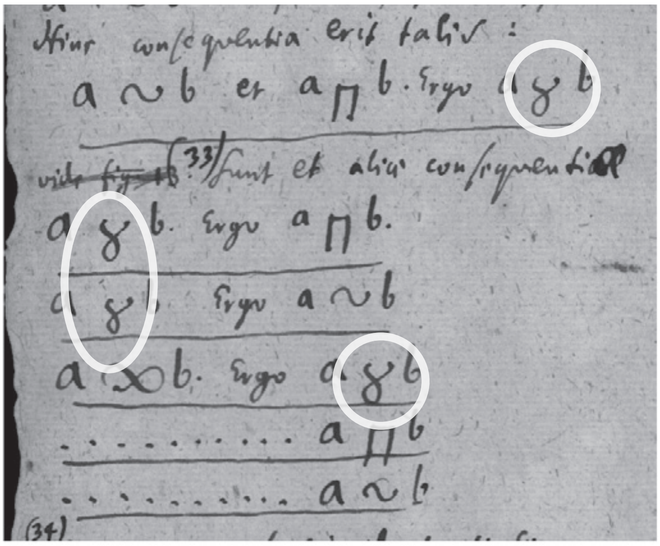
N. 347 LEIBNIZ FÜR CHRISTIAAN HUYGENS, 18. IX. 1679 859

le rayon BC. De mesme l'intersection d'un plan et de la spherique est une ligne circulaire. Car l'expression d'une spherique est AC ♉ AY et celle d'un plan est AY ♉ BY et par consequent AC ♉ BC, par ce que le point C est un des points Y: or BC estant ♉ AC et AC estant ♉ AY, nous aurons BC ♉ AY et AY estant ♉ BY nous aurons BC ♉ BY. Joignons ces congruités et nous aurons ABC ♉ ABY c'est à dire

AB ♉ AB or ABC ♉ ABY est à la circulaire, donc l'intersection d'un plan et d'une
BC ♉ BY
AC ♉ AY
surface spherique donne la circulaire. Ce qu'il falloit demonstrer par cette sorte de calcul. De la même façon il paroistra que l'intersection de deux plans est une droite. Car soyent deux congruités, l'une AY ♉ BY pour un plan, l'autre AY ♉ CY pour l'autre plan, nous aurons AY ♉ BY ♉ CY dont le lieu est la droite. Enfin l'intersection de deux droites est un point car soit AY ♉ BY ♉ CY et BY ♉ CY ♉ DY nous aurons AY ♉ BY ♉ CY ♉ DY.

Je n'ay qu'une remarque à adjouter, c'est que je voy qu'il est possible d'entendre la

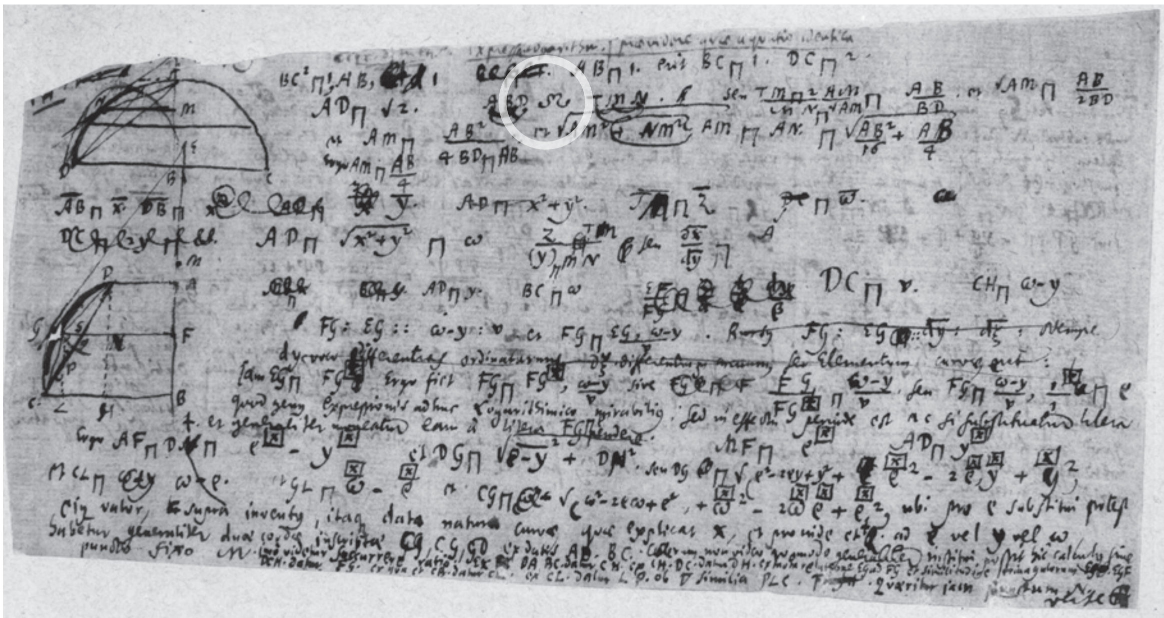
♉ LEIBNIZIAN CONGRUENCE-2
LAA III-2 p. 859.

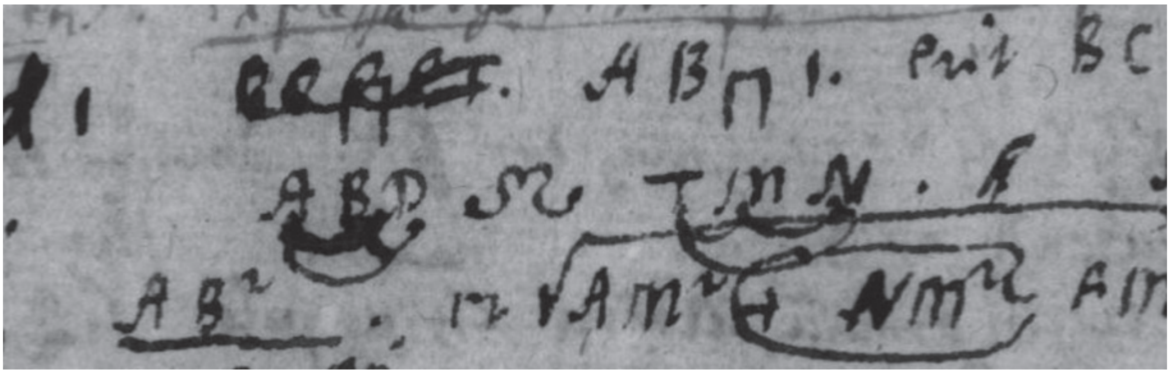


Hinc consequentia erit talis:
a ∾ b et a ⊓ b. Ergo a ♉ b
(33) Sunt et aliae consequentiae
a ♉ b. Ergo a ⊓ b.
a ♉ b Ergo a ∾ b
a ∞ b. Ergo a ♉ b
. . . . a ⊓ b
. . . . a ∾ b
(34)

♉ LEIBNIZIAN CONGRUENCE-2
LH 35 I 11 fol. 9r

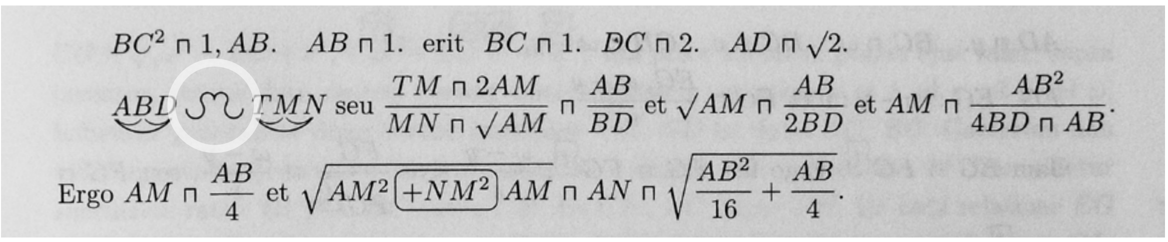
Leibniz used a variety of symbols to denote *similarity:* ∽, ʃ∽ and ⁓⁓. Of these, we propose ∽ as a variation sequence to 223D ∽ REVERSED TILDE. This variant is already referenced in the annotations to 223D, however, it does not show up in the *Standardized Variation Sequences* chapter of the 2200 block so far.
Two other, considerably different *similarity* signs remain for new encoding: ʃ∽ and ⁓⁓.





ʃ∽ LEIBNIZIAN SIMILARITY
LH 35 XII 1, fol. 343v;

– this is the same text in the LAA edition:



$BC^2 \sqcap 1, AB.\quad AB \sqcap 1.$ erit $BC \sqcap 1.\quad DC \sqcap 2.\quad AD \sqcap \sqrt{2}.$

$\underbrace{ABD}$ ʃ∽ $\underbrace{TMN}$ seu $\dfrac{TM \sqcap 2AM}{MN \sqcap \sqrt{AM}} \sqcap \dfrac{AB}{BD}$ et $\sqrt{AM} \sqcap \dfrac{AB}{2BD}$ et $AM \sqcap \dfrac{AB^2}{4BD \sqcap AB}$.

Ergo $AM \sqcap \dfrac{AB}{4}$ et $\sqrt{AM^2 \boxed{+NM^2}}\, AM \sqcap AN \sqcap \sqrt{\dfrac{AB^2}{16} + \dfrac{AB}{4}}$.

ʃ∽ LEIBNIZIAN SIMILARITY
LAA VII-7 p. 595

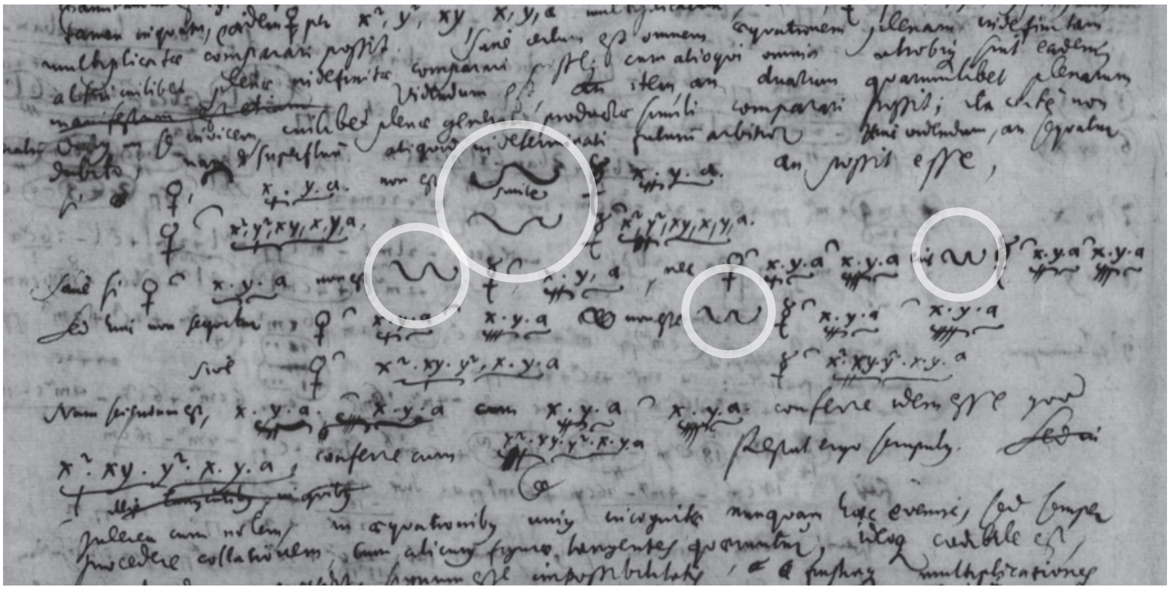
(10) Weitere neue Notationen

Wohl im April 1676 verwendet Leibniz mit ∽ ein neues Symbol für die Ähnlichkeit von Dreiecken. Ob er es auch andernorts einsetzt, ist bislang nicht bekannt. Das Beispiel:

$\underbrace{ABD}$ ∽ $\underbrace{TMN}$ (N. 66)

Im gleichen Stück entwickelt er schrittweise eine neue Notation für die eindeutige Zuordnung bestimmter geometrischer Größen zueinander. Er geht von einer Kurve aus,

∽ LEIBNIZIAN SIMILARITY
LAA VII-7 p. LIII



∿ LEIBNIZIAN SIMILARITY-2
LH 35 V 1 fol. 4v;
the same part in the edition:

Hinc videndum, an sequatur si

♀ ⌒ $\underbrace{x.y.a.}_{/}$ non est ∿ ☿ ⌒ $\underbrace{x.y.a.}_{//}$ an possit esse,

♀ ⌒ $\underbrace{x^2, y^2, xy, x, y, a.}_{/}$ ∿ ☿ ⌒ $\underbrace{x^2, y^2, xy, x, y, a.}_{//}$

Sane si ♀ ⌒ $\underbrace{x.y.a.}_{/}$ non est ∿ ☿ ⌒ $\underbrace{x.y.a}_{//}$, nec ♀ ⌒ $\underbrace{x.y.a}_{/}$ ⌒ $\underbrace{x.y.a}_{///}$ erit ∿ ☿ ⌒ $\underbrace{x.y.a}_{//}$ ⌒ $\underbrace{x.y.a.}_{///}$ Sed hinc non sequitur

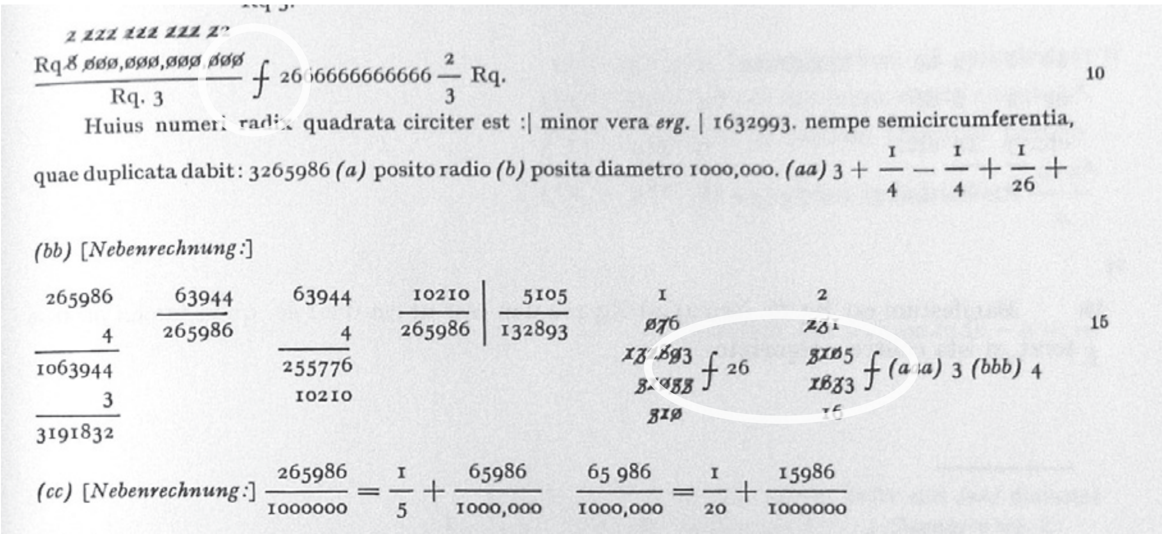
♀ ⌒ $\underbrace{x.y.a.}_{/}$ ⌒ $\underbrace{x.y.a}_{///}$ non esse ∿ ☿ ⌒ $\underbrace{x.y.a.}_{//}$ ⌒ $\underbrace{x.y.a.}_{////}$

sive ♀ ⌒ $\underbrace{x^2.xy.y^2.x.y.a}_{/}$ ☿ ⌒ $\underbrace{x^2.xy.y^2.x.y.a}_{//}$

---

19 *Zu* ∿: simile

∿ LEIBNIZIAN SIMILARITY-2
LAA VII-3 p. 75



$\frac{\text{Rq} 8\,000{,}000{,}000\,000}{\text{Rq. } 3}$ ⨍ 2666666666666 $\frac{2}{3}$ Rq.

Huius numeri radix quadrata circiter est :| minor vera *erg.* | 1632993. nempe semicircumferentia, quae duplicata dabit: 3265986 *(a)* posito radio *(b)* posita diametro 1000,000. *(aa)* $3 + \frac{1}{4} - \frac{1}{4} + \frac{1}{26} +$

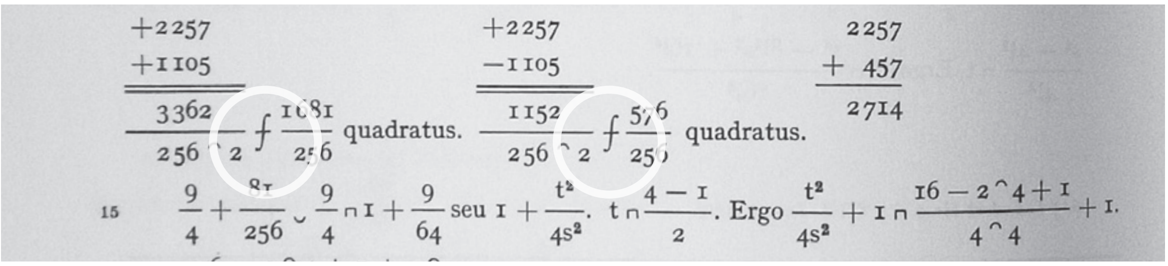
(bb) [*Nebenrechnung*:]

(cc) [*Nebenrechnung*:] $\frac{265986}{1000000} = \frac{1}{5} + \frac{65986}{1000{,}000}$ $\frac{65\,986}{1000{,}000} = \frac{1}{20} + \frac{15986}{1000000}$

⨍ FACIT SYMBOL – LAA VII-1 p. 65

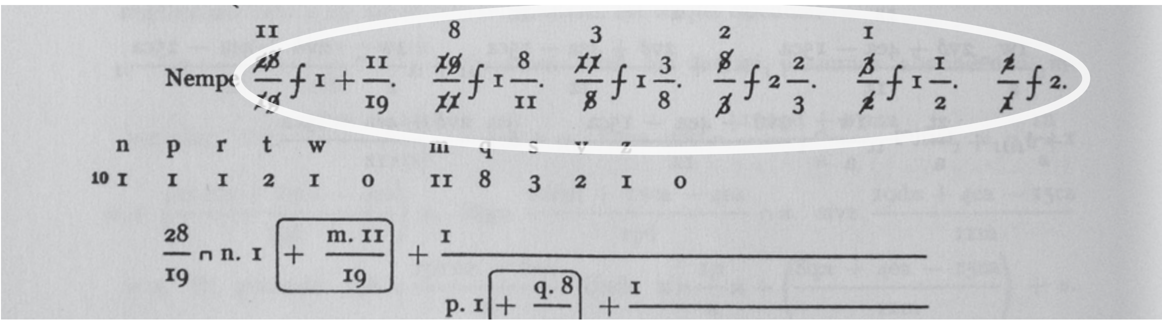
Leibniz used various script-style forms of the lowercase f for *facit* in his writings. In order to suitably represent them by one unambiguous symbol which make it distinguishable from both the ordinary (upright) f as well as the italic *f;* it is an established practice in the LAA edition for many decades to represent this expression by a specially shaped, “upright cursive” f with a descender and a reversed stress pattern (which not in any case was executed properly).

There is another similar looking character, LATIN SMALL LETTER F WITH HOOK (0192) which is defined as a currency character for *Florin* but which also gets used as an alphabetic character in the Ewe language. Since this unification is rather problematic already, we advise that 0192 not getting further loaded with other meanings. Regardless of a certain optical likeness the reason for including this character is mainly its distinctive purpose and function as an element of mathematical notation. The meaning is also different from that of the modern “function symbol” as which 0192 is annotated, additionally.



⨍ FACIT SYMBOL

LAA VII-1 p. 352



⨍ FACIT SYMBOL

LAA VII-1 p. 508

N. 38₁₈ DIFFERENZEN, FOLGEN, REIHEN 1672–1676 551

sive (30) $c \sqcap \dfrac{\begin{array}{l}+9,\ 25fa^2\\ ∓\quad 81\ldots\end{array}}{\begin{array}{l}∓3 ⌢ 125\beta^2\\ \quad 27\,..\\ +3 ⌢ 45\,..\\ \quad 75\,..\end{array}} \sqcap \dfrac{\begin{array}{l}+3 ⌢ 25fa^2\\ ∓3 ⌢\ 9\ldots\end{array}}{\begin{array}{l}∓125\beta^2\\ ..\ 27\,..\\ +\ 45\,..\\ ..\ 75\,..\end{array}} \sqcap \dfrac{\begin{array}{l}+3 ⌢ 25fa^2\\ ∓\,..\quad 9\ldots\end{array}}{\begin{array}{l}∓[152]\beta^2\\ +\ 120\,..\end{array}}.$

Ac denique erit (31) $b \sqcap \dfrac{\begin{array}{ll}-4,125a^3f & -6,3,25a^3f\\ \ldots\ 27\ldots & ±\ ..\ ..\ 9\ldots\\ ±\,..\ 45\ldots & \\ ..\ ..\ 75\ldots & \end{array}}{\begin{array}{l}∓9,125\beta^3\\ ..\ ..\ 27\,..\\ +\ ..\ 45\,..\\ ..\ ..\ 75\,..\end{array}}$, seu $b \sqcap \dfrac{\begin{array}{l}±\ 642fa^3\\ -\ 802\ldots\end{array}}{\begin{array}{l}∓1368\beta^3\\ +1080\,..\end{array}}.$

550,15–551,5 *Nebenrechnungen:*

zu Z. 15: 4 15 ⌢ 25 / 225 ʄ 25 / 9 9 ⌢ 9

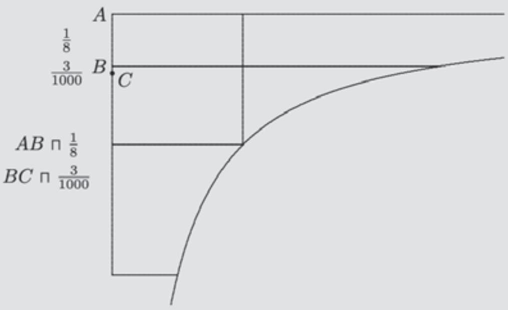
zu Z. 1–5: +9, 25 ∓99 ∓3 ⌢ 125 9 ⌢ 15 / ±18 ∓3 ⌢ 27 9 ⌢ 25 / ∓81 3 ⌢ ∓152 3 ⌢ 45 / 3 ⌢ 75

ʄ FACIT SYMBOL
LAA VII-3 p. 553 (top),
LAA VII-6 p. 449 (right)

These samples demonstrate the intentional use of a specific character for “facit” in order to distinguish it from the the ordinary italic *f*.

N. 43 ARITHMETISCHE KREISQUADRATUR 1673–1676 449

Quaeritur log. a 10. Inveniamus a 250 id est a 25 in 10. Habebimus et a 10 ex dato a 2. Est enim 5^3 in 2. Inveniemus a 250. si habeamus a $\frac{1}{250}$. Est autem notus log. ab $\frac{1}{256}$. Quaeratur differentia inter $\frac{1}{250}$ et $\frac{1}{256}$. Ea est $\frac{256-250}{250,256} \Big| \frac{6}{64000} \Big| \frac{3}{32000}$ eritque $\frac{1}{250} \sqcap \frac{1}{256} + \frac{3}{32000}$ vel $\sqcap \frac{1}{8} + \frac{3}{1000} \sqcap \frac{1024}{8000} \sqcap \frac{16}{125}$. Nam si hoc dividas per 32. habebis $\frac{1}{250}$ nam fit $\frac{1024}{8000}$ in $\frac{1}{32}$ dat $\frac{1024}{256000}$. Ergo quaerenda quantitas $\frac{d}{f} - \frac{d^2}{2f^2} + \frac{d^3}{3f^3}$ etc. ita ut d sit $\frac{3}{1000}$. et f. $\frac{1}{8}$.



[*Fig. 2*]

1–5 *Nebenbetrachtung:* $\frac{1}{250} - \frac{1}{256} \sqcap \frac{6}{64000} \Big| \frac{3}{32000}$. Ergo $\frac{1}{250} \sqcap \frac{1}{256} + \frac{3}{32000}$ cujus quaeritur logarithmus.

256 / 250 / 12800 / 512 / 64000

0 / 1 / 22 / 256000 ʄ 250 / 10244 / 1022 / 10

5. Unicode Character Properties

```
xb01;LEIBNIZIAN EQUAL;Sm;0;ON;;;;;N;;;;;
xb02;LEIBNIZIAN EQUAL WITH DOUBLE VERTICALS;Sm;0;ON;;;;;N;;;;;
xb03;LEIBNIZIAN EQUAL WITH SMALL S;Sm;0;ON;;;;;N;;;;;
xb04;LEIBNIZIAN GREATER;Sm;0;ON;;;;;N;;;;;
xb05;LEIBNIZIAN LESS;Sm;0;ON;;;;;N;;;;;
xb06;LEIBNIZIAN GREATER WITH SMALL P;Sm;0;ON;;;;;N;;;;;
xb07;LEIBNIZIAN LESS WITH SMALL P;Sm;0;ON;;;;;N;;;;;
xb08;LEIBNIZIAN GREATER-LESS;Sm;0;ON;;;;;N;;;;;
xb09;INVERTED SQUARE LEFT OPEN BOX OPERATOR;Sm;0;ON;;;;;N;;;;;
xb10;INVERTED SQUARE RIGHT OPEN BOX OPERATOR;Sm;0;ON;;;;;N;;;;;
xb11;TWO-LINE GREATER;Sm;0;ON;;;;;N;;;;;
xb12;TWO-LINE LESS;Sm;0;ON;;;;;N;;;;;
xb13;COMMENSURABILITY;Sm;0;ON;;;;;N;;;;;
xb14;INCOMMENSURABILITY;Sm;0;ON;;;;;N;;;;;
xb15;COMMENSURABILITY IN SQUARE;Sm;0;ON;;;;;N;;;;;
xb16;INCOMMENSURABILITY IN SQUARE;Sm;0;ON;;;;;N;;;;;
xb17;CARTESIAN EQUAL;Sm;0;ON;;;;;N;;;;;
xb18;LEIBNIZIAN CONGRUENCE;Sm;0;ON;;;;;N;;;;;
xb19;LEIBNIZIAN CONGRUENCE WITH VERTICAL BAR;Sm;0;ON;;;;;N;;;;;
xb20;LEIBNIZIAN CONGRUENCE-2;Sm;0;ON;;;;;N;;;;;
xb21;LEIBNIZIAN CONGRUENCE-2 INVERTED;Sm;0;ON;;;;;N;;;;;
xb22;LEIBNIZIAN CONGRUENCE-2 WITH HORIZONTAL BAR;Sm;0;ON;;;;;N;;;;;
xb23;LEIBNIZIAN CONGRUENCE-2 WITH HORIZONTAL AND VERTICAL BAR;Sm;0;ON;;;;;N;;;;;
xb24;LEIBNIZIAN COINCIDENCE;Sm;0;ON;;;;;N;;;;;
xb25;INVERTED LAZY S OVER LAZY S;Sm;0;ON;;;;;N;;;;;
xb26;LEIBNIZIAN SIMILARITY;Sm;0;ON;;;;;N;;;;;
xb27;LEIBNIZIAN SIMILARITY-2;Sm;0;ON;;;;;N;;;;;
xb28;LEIBNIZIAN DISSIMILARITY;Sm;0;ON;;;;;N;;;;;
xb29;FACIT SYMBOL;Sm;0;ON;;;;;N;;;;;
```

```
xb17 FE00; with descender; # CARTESIAN EQUAL
223D FE00; lazy S variant; # REVERSED TILDE
2243 FE00; lazy S variant; # ASYMPTOTICALLY EQUAL TO
22CD FE00; lazy S variant; # REVERSED TILDE EQUALS
2242 FE00; lazy S variant; # MINUS TILDE
2248 FE00; lazy S variant; # ALMOST EQUAL TO
2A6C FE00; lazy S variant; # SIMILAR MINUS SIMILAR
22DC FE00; parallelised form; # EQUAL TO OR LESS-THAN
22DD FE00; parallelised form; # EQUAL TO OR GREATER-THAN
```

6. Bibliography

LAA – refers to: Leibniz, Gottfried Wilhelm: Sämtliche Schriften und Briefe. (‘Leibniz-Akademie-Ausgabe’, many volumes)
LH – refers to: Leibniz’s original manuscripts, GWLB Hanover

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online

Leibniz-Akademie-Ausgabe (LAA, general edition of Leibniz’s writings)

LAA series VII (mathematical manuscripts, volumes 3 to 7 available online)

ISO/IEC JTC 1/SC 2/WG 2
PROPOSAL SUMMARY FORM TO ACCOMPANY SUBMISSIONS
FOR ADDITIONS TO THE REPERTOIRE OF ISO/IEC 10646[1]
Please fill all the sections A, B and C below.
Please read Principles and Procedures Document (P & P) from http://std.dkuug.dk/JTC1/SC2/WG2/docs/principles.html for guidelines and details before filling this form.
Please ensure you are using the latest Form from http://std.dkuug.dk/JTC1/SC2/WG2/docs/summaryform.html.
See also http://std.dkuug.dk/JTC1/SC2/WG2/docs/roadmaps.html for latest *Roadmaps.*

A. Administrative

1. **Title:** **Proposal to encode historical mathematical relations**
2. Requester's name: Uwe Mayer, Siegmund Probst, David Rabouin, Elisabeth Rinner, Andreas Stötzner, Achim Trunk, Charlotte Wahl
3. Requester type (Member body/Liaison/Individual contribution): Individual (work group)
4. Submission date: 2025-11.24.
5. Requester's reference (if applicable): LUCP L-2530
6. Choose one of the following:
   This is a complete proposal: Yes
   (or) More information will be provided later:

B. Technical – General

1. Choose one of the following:
   a. This proposal is for a new script (set of characters): No
      Proposed name of script:
   b. The proposal is for addition of character(s) to an existing block: No
      Name of the existing block:
2. Number of characters in proposal: 38
3. Proposed category (select one from below - see section 2.2 of P&P document):

A-Contemporary		B.1-Specialized (small collection)	Yes	B.2-Specialized (large collection)	
C-Major extinct		D-Attested extinct		E-Minor extinct	
F-Archaic Hieroglyphic or Ideographic				G-Obscure or questionable usage symbols	

4. Is a repertoire including character names provided? Yes
   a. If YES, are the names in accordance with the "character naming guidelines" in Annex L of P&P document? Yes
   b. Are the character shapes attached in a legible form suitable for review? Yes
5. Fonts related:
   a. Who will provide the appropriate computerized font to the Project Editor of 10646 for publishing the standard?
      Andreas Stötzner
   b. Identify the party granting a license for use of the font by the editors (include address, e-mail, ftp-site, etc.):
      Andreas Stötzner Gestaltung, Klauflügelweg 21, 88400 Biberach/R., Germany, as@signographie.de
6. References:
   a. Are references (to other character sets, dictionaries, descriptive texts etc.) provided? Yes
   b. Are published examples of use (such as samples from newspapers, magazines, or other sources) of proposed characters attached? Yes
7. Special encoding issues:
   Does the proposal address other aspects of character data processing (if applicable) such as input, presentation, sorting, searching, indexing, transliteration etc. (if yes please enclose information)? No

8. Additional Information:
Submitters are invited to provide any additional information about Properties of the proposed Character(s) or Script that will assist in correct understanding of and correct linguistic processing of the proposed character(s) or script. Examples of such properties are: Casing information, Numeric information, Currency information, Display behaviour information such as line breaks, widths etc., Combining behaviour, Spacing behaviour, Directional behaviour, Default Collation behaviour, relevance in Mark Up contexts, Compatibility equivalence and other Unicode normalization related information. See the Unicode standard at http://www.unicode.org for such information on other scripts. Also see Unicode Character Database (http://www.unicode.org/reports/tr44/) and associated Unicode Technical Reports for information needed for consideration by the Unicode Technical Committee for inclusion in the Unicode Standard.

[1] Form number: N4502-F (Original 1994-10-14; Revised 1995-01, 1995-04, 1996-04, 1996-08, 1999-03, 2001-05, 2001-09, 2003-11, 2005-01, 2005-09, 2005-10, 2007-03, 2008-05, 2009-11, 2011-03, 2012-01)

C. Technical - Justification

1. Has this proposal for addition of character(s) been submitted before? — Yes
   If YES explain: *see L-2519 (N5334), N5277 (L-2402n)*
2. Has contact been made to members of the user community (for example: National Body, user groups of the script or characters, other experts, etc.)? — Yes
   If YES, with whom? Leibniz-Archiv, Forschungsstelle der Leibniz-Edition, Niedersächsische Landesbibliothek (GWLB), Hanover, Göttingen Academy of Science and Humanities in Lower Saxony (DE), Philiumm research group of CNRS (UMR 7219, laboratoire SPHERE) / Université de Paris VII; general: scholars, researchers, authors and editors working in the field of science history and upon editions of historic text corpora (e.g. of G. W. Leibniz, but also many others)
   If YES, available relevant documents: L-2409, L-2410
3. Information on the user community for the proposed characters (for example: size, demographics, information technology use, or publishing use) is included? — Yes
   Reference:
4. The context of use for the proposed characters (type of use; common or rare) — Common
   Reference: mainly specialist usage, scholarly, worldwide
5. Are the proposed characters in current use by the user community? — Yes
   If YES, where? Reference: mainly Europe, Americas; other countries
6. After giving due considerations to the principles in the P&P document must the proposed characters be entirely in the BMP? — No
   If YES, is a rationale provided?
   If YES, reference:
7. Should the proposed characters be kept together in a contiguous range (rather than being scattered)? — No
8. Can any of the proposed characters be considered a presentation form of an existing character or character sequence? — Yes
   If YES, is a rationale for its inclusion provided? — *Yes*
   If YES, reference: *see explanations in chapter 4.*
9. Can any of the proposed characters be encoded using a composed character sequence of either existing characters or other proposed characters? — Yes
   If YES, is a rationale for its inclusion provided? — *Yes*
   If YES, reference: *see explanations in chapter 4.*
10. Can any of the proposed character(s) be considered to be similar (in appearance or function) to, or could be confused with, an existing character? — No
    If YES, is a rationale for its inclusion provided?
    If YES, reference:
11. Does the proposal include use of combining characters and/or use of composite sequences? — No
    If YES, is a rationale for such use provided?
    If YES, reference:
    Is a list of composite sequences and their corresponding glyph images (graphic symbols) provided? — No
    If YES, reference:
12. Does the proposal contain characters with any special properties such as control function or similar semantics? — No
    If YES, describe in detail (include attachment if necessary)
13. Does the proposal contain any Ideographic compatibility characters? — No
    If YES, are the equivalent corresponding unified ideographic characters identified?
    If YES, reference: