ISO/IEC JTC1/SC2/WG2 **N5336R**
LUCP L-2536

Universal Multiple-Octet Coded Character Set
International Organization for Standardization
Internationale Standardisierungs-Organisation
Organisation Internationale de Normalisation
Διεθνής Οργανισμός Τυποποίησης
Международная организация по стандартизации

Doc Type: Working Group Document

Title: Proposal to encode miscellaneous scientific symbols

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Action: for expert review, intended for Unicode 18.0 pipeline
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Requester's reference: LUCP L-2536

1. Background – Miscellaneous symbols in historic sources

This proposal is part of the research program upon historical mathematical sources, conducted by the CNRS Philiumm project (headed by Prof. David Rabouin, University of Paris) and supported by researchers from the Landesbibliothek Hanover (Germany). The aim of this project work is to achieve a standardized encoding for special mathematical characters in historic texts, which is required for accurate facsilime editions of those sources.
For more background information about the Philiumm project and the related research work, please visit the Philiumm website or see doc. no. N5277.
In this proposal we introduce a number of various symbols mainly from the field of historical mathematics. Some of them are combining symbols (or marks) which in the sources occur in combination with digits or other symbols.

2. Revision

After discussion with SEWG and UTC experts and following the subsequent recommendations, we have dropped a few candidates (for various reasons) and propose now a selection of 13 new characters, of which 3 are combining characters.

3. Characters

If this proposal gets accepted, the following characters will exist:

 CASTING-OUT-NINES

 LUNATE ENCIRCLED DIGIT ONE

 PROPORTION WITH ONE STROKE

 PROPORTION WITH TWO STROKES

 INFINITY WITH TWO DOTS

 INVOLVED

 LEIBNIZIAN ENCIRCLED V

 LEIBNIZIAN ENCIRCLED V IN BOX

 SUPERSCRIPT WAVE SYMBOL

 SUPERSCRIPT WAVE WITH OVERLINE SYMBOL

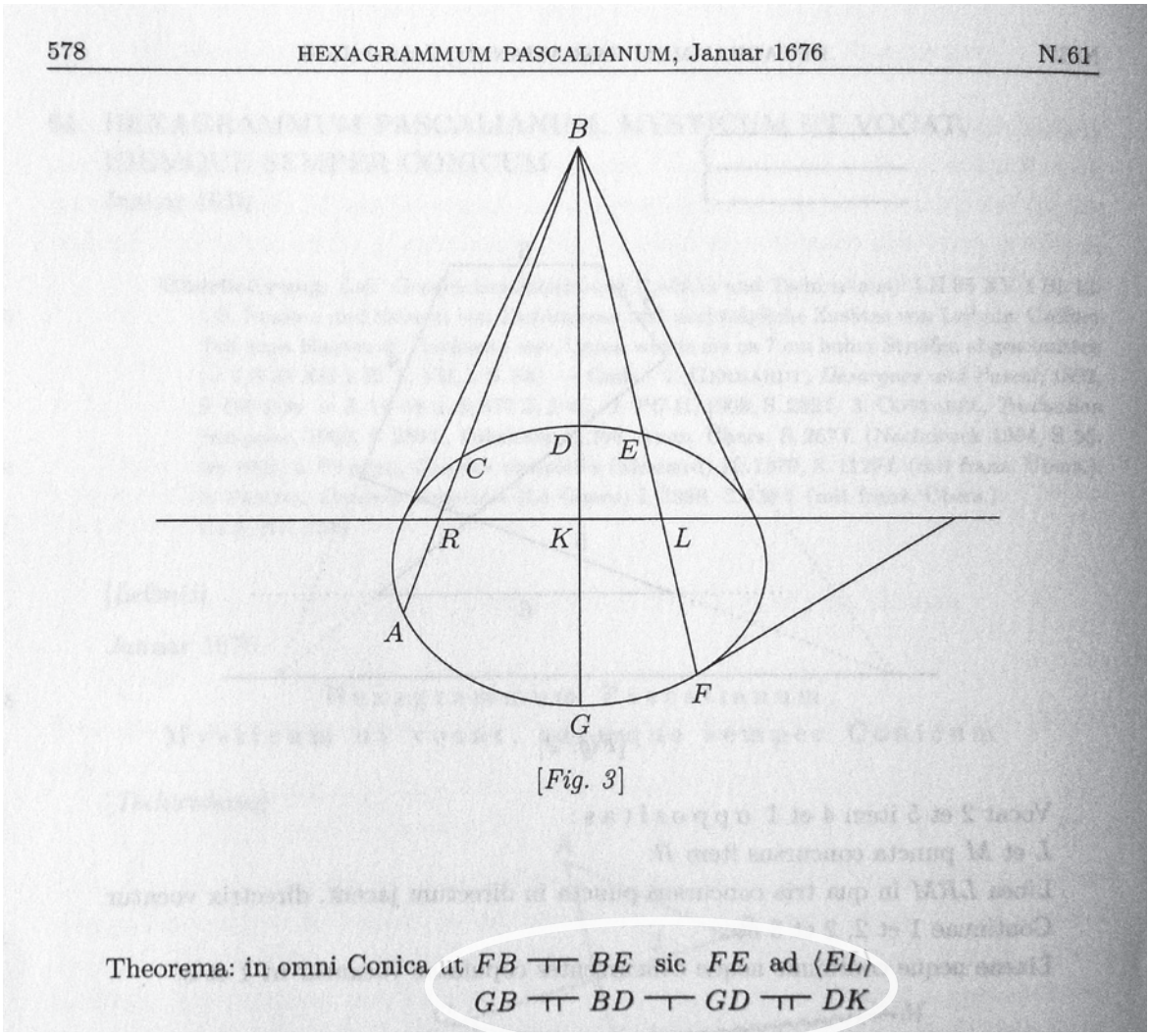
 COMBINING BOMBELLI POWER MARK

 COMBINING HALF CIRCLE BELOW

 COMBINING FACTOR MARK

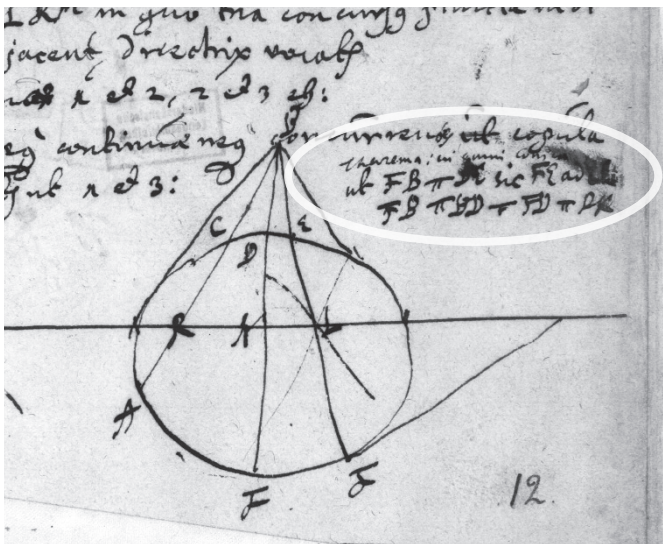
The three combining characters will occur in combination with single digits.
COMBINING FACTOR MARK should get the “overlay” (1) property.

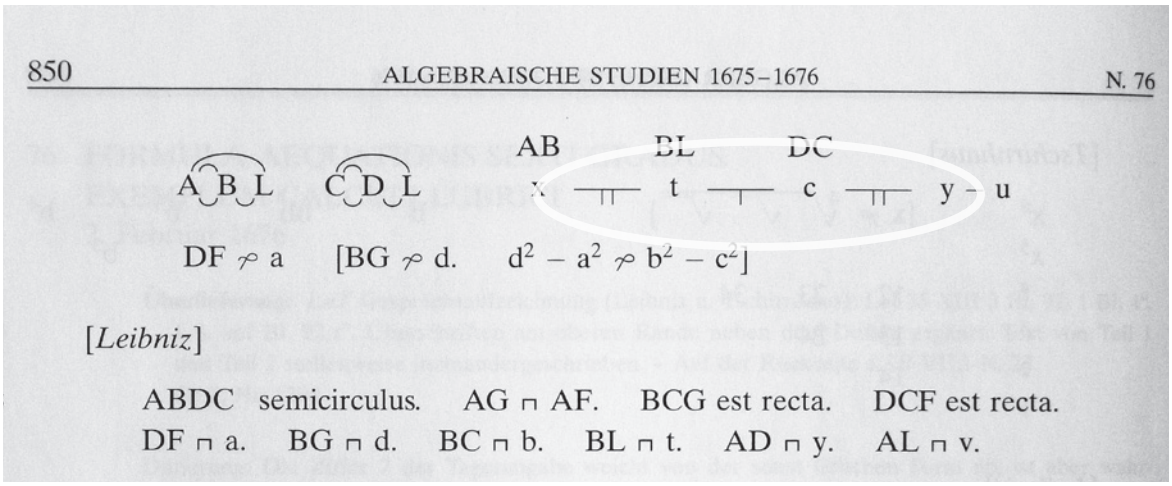
4. Figures and explanations



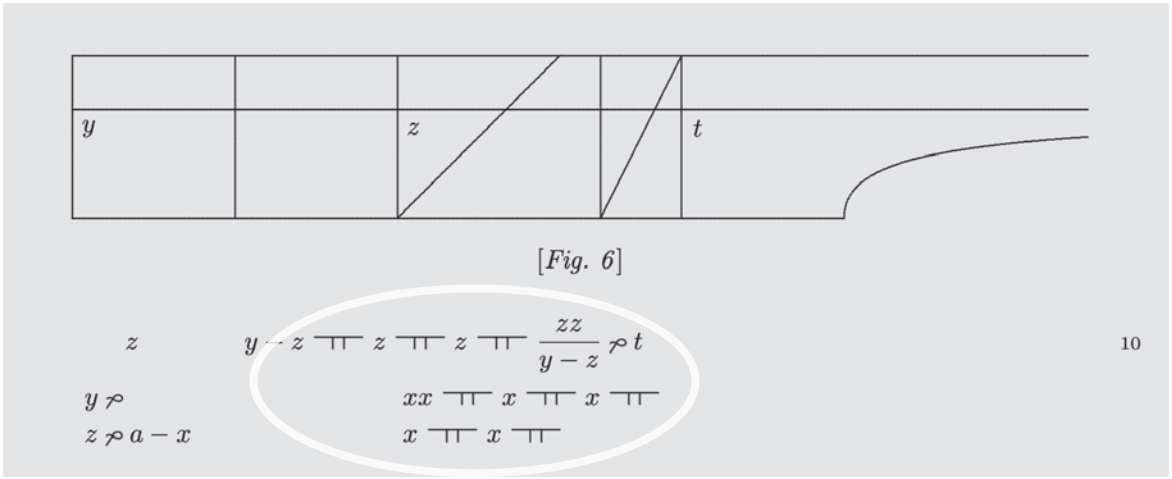
⊤ PROPORTION WITH ONE STROKE, ⫟ PROPORTION WITH TWO STROKES
LAA VII-7 p. 578

⊤ PROPORTION WITH ONE STROKE, ⫟ PROPORTION WITH TWO STROKES
LH 35 XV 1, fol. 12r

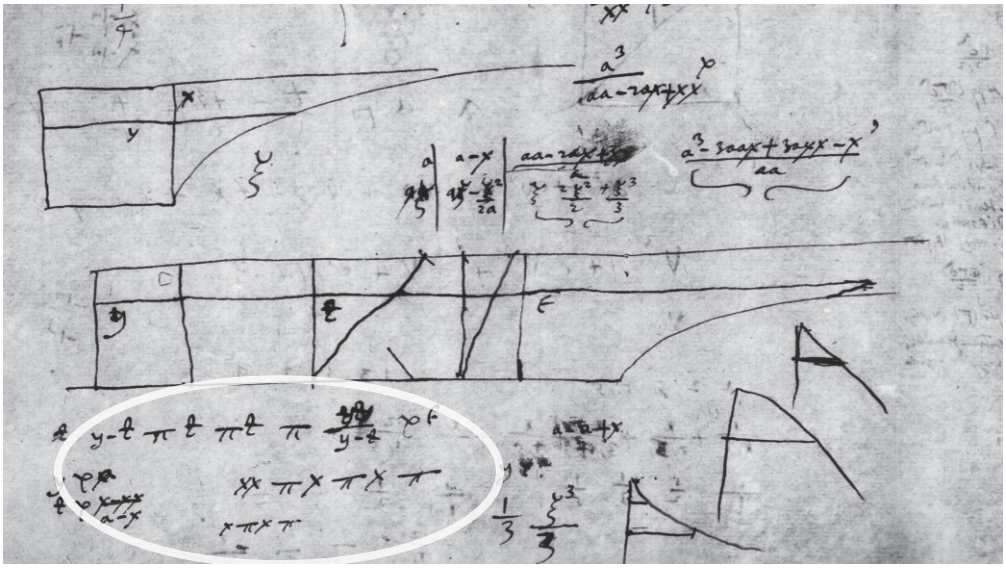




⫟ PROPORTION WITH ONE STROKE, ⫪ PROPORTION WITH TWO STROKES
LAA VII-2 p. 850



⫪ PROPORTION WITH TWO STROKES
LAA VII-6 p. 271



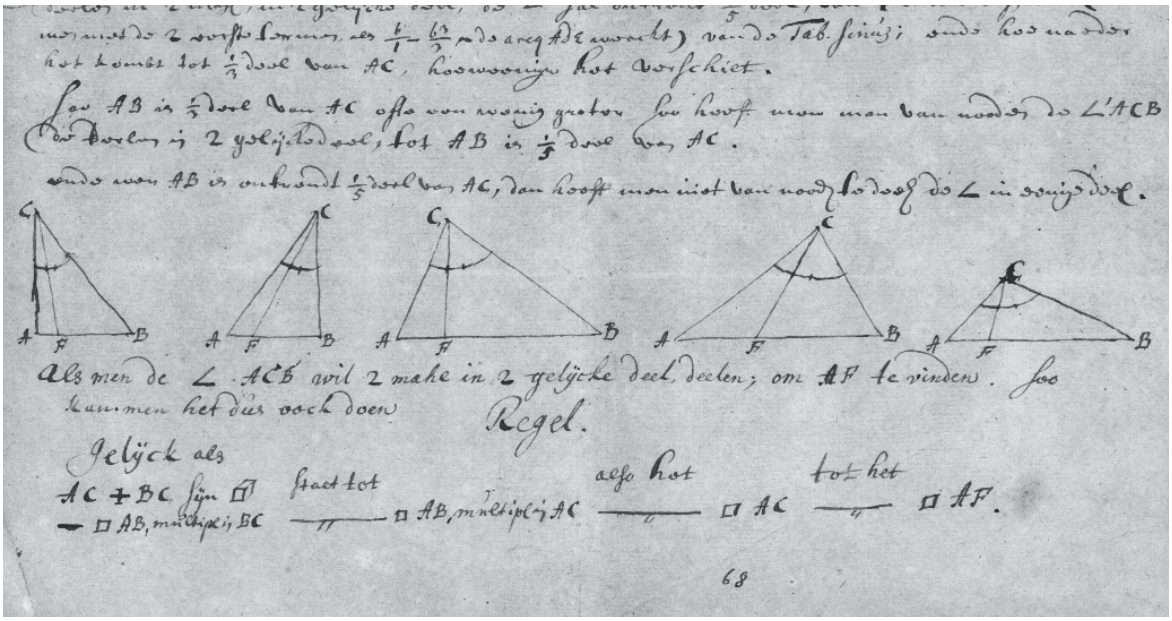
⫪ PROPORTION WITH TWO STROKES
LH 35 V 6, fol. 10v

Als men de $\angle ACB$ wil 2 mahl in 2 gelijcke deel, deelen; om AF te vinden, soo kan men het dus oock doen[:]

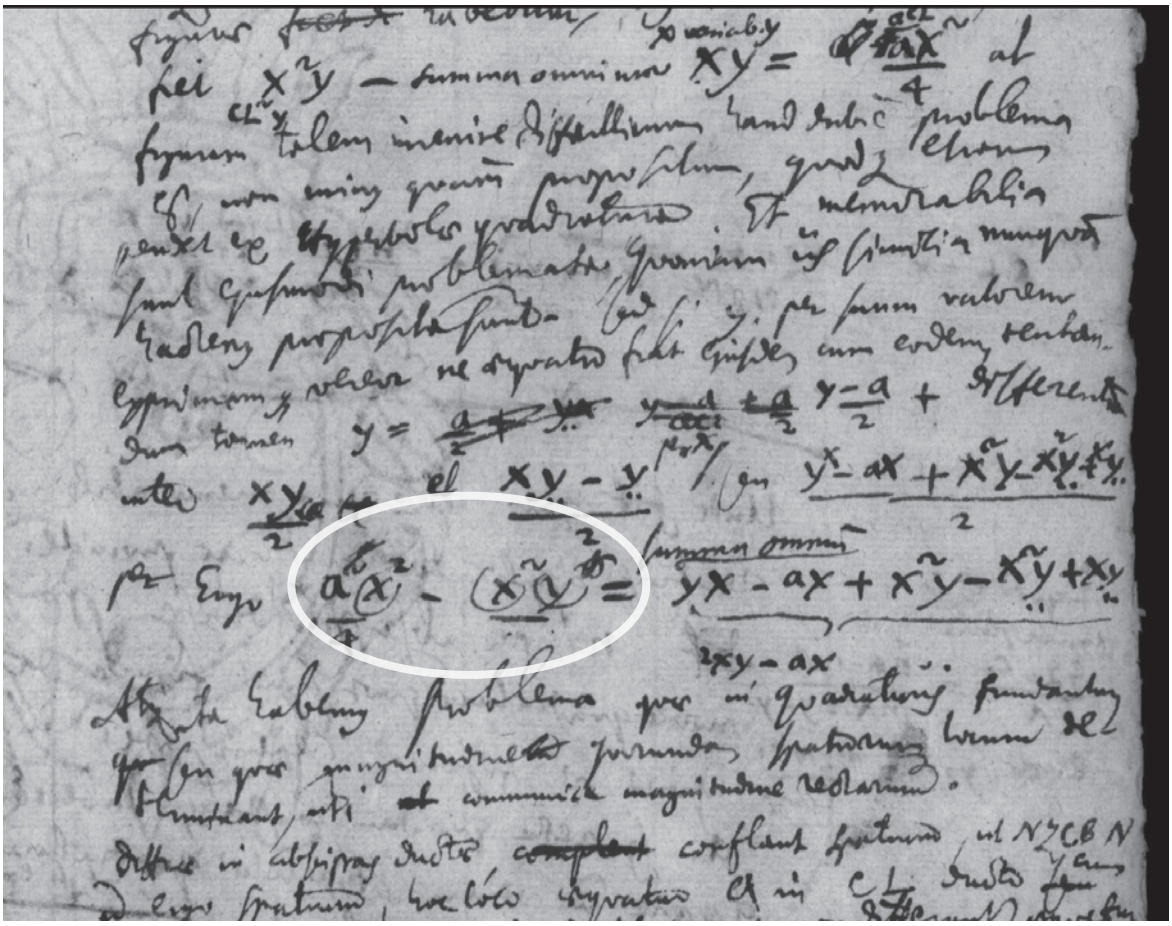
Regel.

Gelijck als
$AC + BC$, sijn $\square$ staet tot also het tot het
$-\square AB$, multipl. in BC $\sqcap$ $\square AB$, multipl. in AC $\sqcap$ $\square AC$ $\sqcap$ $\square AF$.

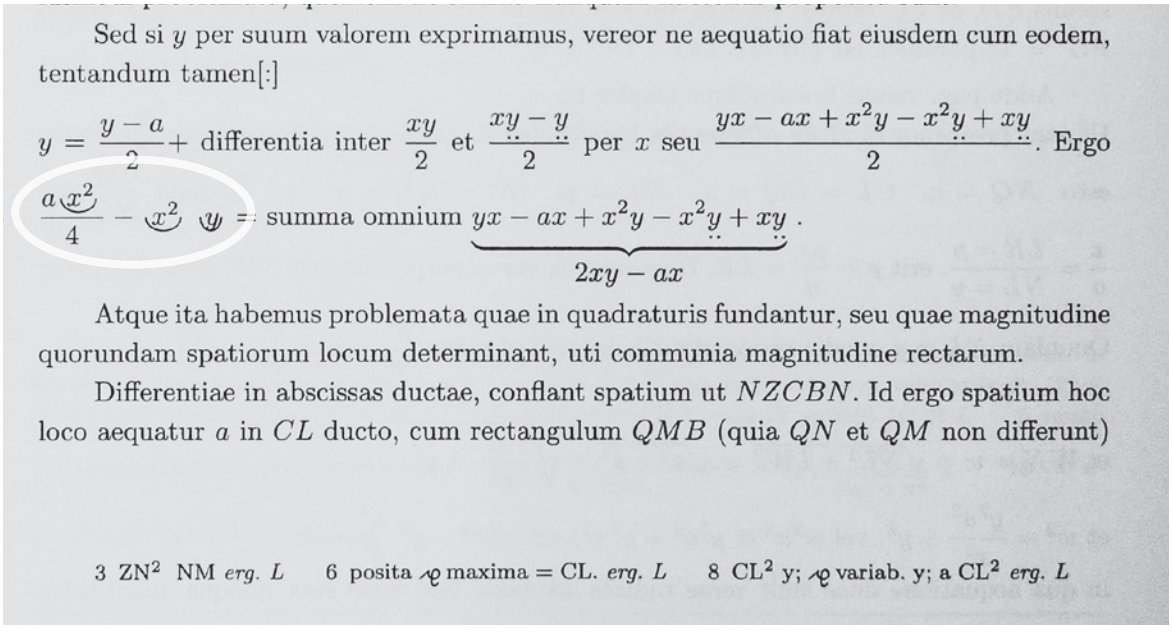
⫟ PROPORTION WITH TWO STROKES
This figure also shows the use of $\square$ WHITE CUBE (proposed in doc. L-2514 for u1F7F7).
LAA VII-6 P. 302



⫟ PROPORTION WITH TWO STROKES
LH 35 VIII 30, fol. 68r



◌͜ COMBINING HALF CIRCLE BELOW
The shape of the character is typically at least a half circle, often it approximates 3/5 of a circle. Hence it is considerably different from COMBINING DOUBLE BREVE BELOW (035C).
LH 35 XIII 3, fol. 250v



Sed si y per suum valorem exprimamus, vereor ne aequatio fiat eiusdem cum eodem, tentandum tamen[:]

$y = \frac{y-a}{2}+$ differentia inter $\frac{xy}{2}$ et $\frac{xy-y}{2}$ per x seu $\frac{yx-ax+x^2y-x^2y+xy}{2}$. Ergo

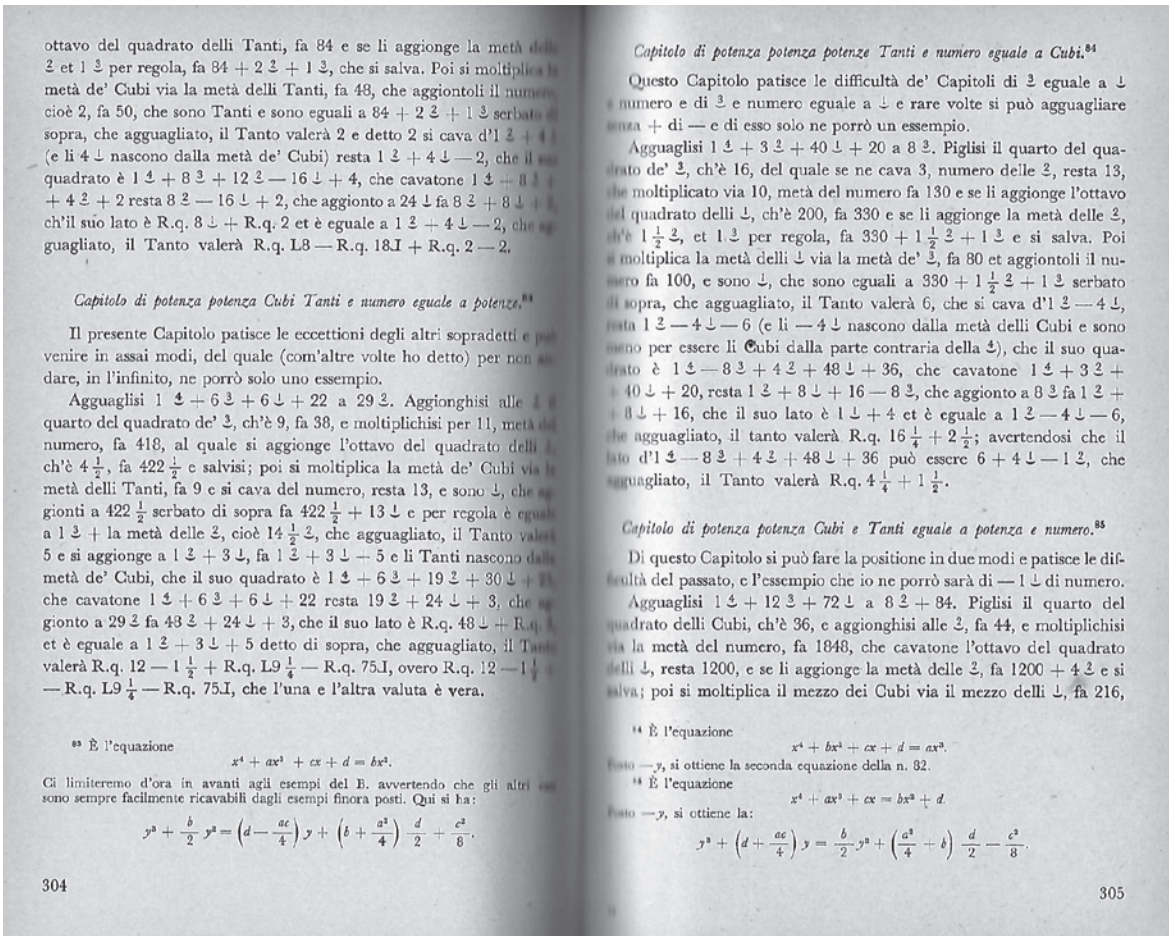
$\frac{a\,x^2}{4} - x^2\ y =$ summa omnium $\underbrace{yx-ax+x^2y-x^2y+xy}_{2xy-ax}$.

Atque ita habemus problemata quae in quadraturis fundantur, seu quae magnitudine quorundam spatiorum locum determinant, uti communia magnitudine rectarum.

Differentiae in abscissas ductae, conflant spatium ut $NZCBN$. Id ergo spatium hoc loco aequatur a in CL ducto, cum rectangulum QMB (quia QN et QM non differunt)

3 ZN^2 NM *erg. L* 6 posita ϱ maxima = CL. *erg. L* 8 CL^2 y; ϱ variab. y; a CL^2 *erg. L*

◌͜ COMBINING HALF CIRCLE BELOW
LAA VII-4 p. 824



ottavo del quadrato delli Tanti, fa 84 e se li aggionge la metà delli 2 et 1 3 per regola, fa 84 + 2 2 + 1 3, che si salva. Poi si moltiplica la metà de' Cubi via la metà delli Tanti, fa 48, che aggiontoli il numero, cioè 2, fa 50, che sono Tanti e sono eguali a 84 + 2 2 + 1 3 serbato di sopra, che agguagliato, il Tanto valerà 2 e detto 2 si cava d'1 2 + 4 1 (e li 4 1 nascono dalla metà de' Cubi) resta 1 2 + 4 1 — 2, che il suo quadrato è 1 4 + 8 3 + 12 2 — 16 1 + 4, che cavatone 1 4 + 8 3 + + 4 2 + 2 resta 8 2 — 16 1 + 2, che aggionto a 24 1 fa 8 2 + 8 1 + 8, ch'il suo lato è R.q. 8 1 + R.q. 2 et è eguale a 1 2 + 4 1 — 2, che agguagliato, il Tanto valerà R.q. L8 — R.q. 18I + R.q. 2 — 2.

Capitolo di potenza potenza Cubi Tanti e numero eguale a potenze.[83]

Il presente Capitolo patisce le eccettioni degli altri sopradetti e può venire in assai modi, del quale (com'altre volte ho detto) per non andare, in l'infinito, ne porrò solo uno essempio.

Agguaglisi 1 4 + 6 3 + 6 1 + 22 a 29 2. Aggionghisi alle 1 il quarto del quadrato de' 3, ch'è 9, fa 38, e moltiplichisi per 11, metà del numero, fa 418, al quale si aggionge l'ottavo del quadrato delli 1, ch'è $4\frac{1}{2}$, fa $422\frac{1}{2}$ e salvisi; poi si moltiplica la metà de' Cubi via la metà delli Tanti, fa 9 e si cava del numero, resta 13, e sono 1, che aggionti a $422\frac{1}{2}$ serbato di sopra fa $422\frac{1}{2} + 13\,1$ e per regola è eguale a 1 3 + la metà delle 2, cioè $14\frac{1}{2}$ 2, che agguagliato, il Tanto valerà 5 e si aggionge a 1 2 + 3 1, fa 1 2 + 3 1 + 5 e li Tanti nascono dalla metà de' Cubi, che il suo quadrato è 1 4 + 6 3 + 19 2 + 30 1 + 25, che cavatone 1 4 + 6 3 + 6 1 + 22 resta 19 2 + 24 1 + 3, che aggionto a 29 2 fa 48 2 + 24 1 + 3, che il suo lato è R.q. 48 1 + R.q. 3, et è eguale a 1 2 + 3 1 + 5 detto di sopra, che agguagliato, il Tanto valerà R.q. 12 — 1 $\frac{1}{2}$ + R.q. L9 $\frac{1}{4}$ — R.q. 75I, overo R.q. 12 — 1 $\frac{1}{2}$ + — R.q. L9 $\frac{1}{4}$ — R.q. 75I, che l'una e l'altra valuta è vera.

[83] È l'equazione

$$x^4 + ax^3 + cx + d = bx^2.$$

Ci limiteremo d'ora in avanti agli esempi del B. avvertendo che gli altri casi sono sempre facilmente ricavabili dagli esempi finora posti. Qui si ha:

$$y^3 + \frac{b}{2}y^2 = \left(d - \frac{ac}{4}\right)y + \left(b + \frac{a^2}{4}\right)\frac{d}{2} + \frac{c^2}{8}.$$

304

Capitolo di potenza potenza potenze Tanti e numero eguale a Cubi.[84]

Questo Capitolo patisce le difficultà de' Capitoli di 3 eguale a 1 e numero e di 3 e numero eguale a 1 e rare volte si può agguagliare senza + di — e di esso solo ne porrò un essempio.

Agguaglisi 1 4 + 3 3 + 40 1 + 20 a 8 3. Piglisi il quarto del quadrato de' 3, ch'è 16, del quale se ne cava 3, numero delle 2, resta 13, che moltiplicato via 10, metà del numero fa 130 e se li aggionge l'ottavo del quadrato delli 1, ch'è 200, fa 330 e se li aggionge la metà delle 2, ch'è $1\frac{1}{2}$ 2, et 1 3 per regola, fa 330 + $1\frac{1}{2}$ 2 + 1 3 e si salva. Poi si moltiplica la metà delli 1 via la metà de' 3, fa 80 et aggiontoli il numero fa 100, e sono 1, che sono eguali a 330 + $1\frac{1}{2}$ 2 + 1 3 serbato di sopra, che agguagliato, il Tanto valerà 6, che si cava d'1 2 — 4 1, resta 1 2 — 4 1 — 6 (e li — 4 1 nascono dalla metà delli Cubi e sono meno per essere li Cubi dalla parte contraria della 4), che il suo quadrato è 1 4 — 8 3 + 4 2 + 48 1 + 36, che cavatone 1 4 + 3 2 + + 40 1 + 20, resta 1 2 + 8 1 + 16 — 8 3, che aggionto a 8 3 fa 1 2 + + 8 1 + 16, che il suo lato è 1 1 + 4 et è eguale a 1 2 — 4 1 — 6, che agguagliato, il tanto valerà R.q. $16\frac{1}{4} + 2\frac{1}{2}$; avertendosi che il lato d'1 4 — 8 3 + 4 2 + 48 1 + 36 può essere 6 + 4 1 — 1 2, che agguagliato, il Tanto valerà R.q. $4\frac{1}{4} + 1\frac{1}{2}$.

Capitolo di potenza potenza Cubi e Tanti eguale a potenza e numero.[85]

Di questo Capitolo si può fare la positione in due modi e patisce le difficultà del passato, e l'essempio che io ne porrò sarà di — 1 1 di numero.

Agguaglisi 1 4 + 12 3 + 72 1 a 8 2 + 84. Piglisi il quarto del quadrato delli Cubi, ch'è 36, e aggionghisi alle 2, fa 44, e moltiplichisi via la metà del numero, fa 1848, che cavatone l'ottavo del quadrato delli 1, resta 1200, e se li aggionge la metà delle 2, fa 1200 + 4 2 e si salva; poi si moltiplica il mezzo dei Cubi via il mezzo delli 1, fa 216,

[84] È l'equazione

$$x^4 + bx^2 + cx + d = ax^3.$$

Posto $-y$, si ottiene la seconda equazione della n. 82.

[85] È l'equazione

$$x^4 + ax^3 + cx = bx^2 + d.$$

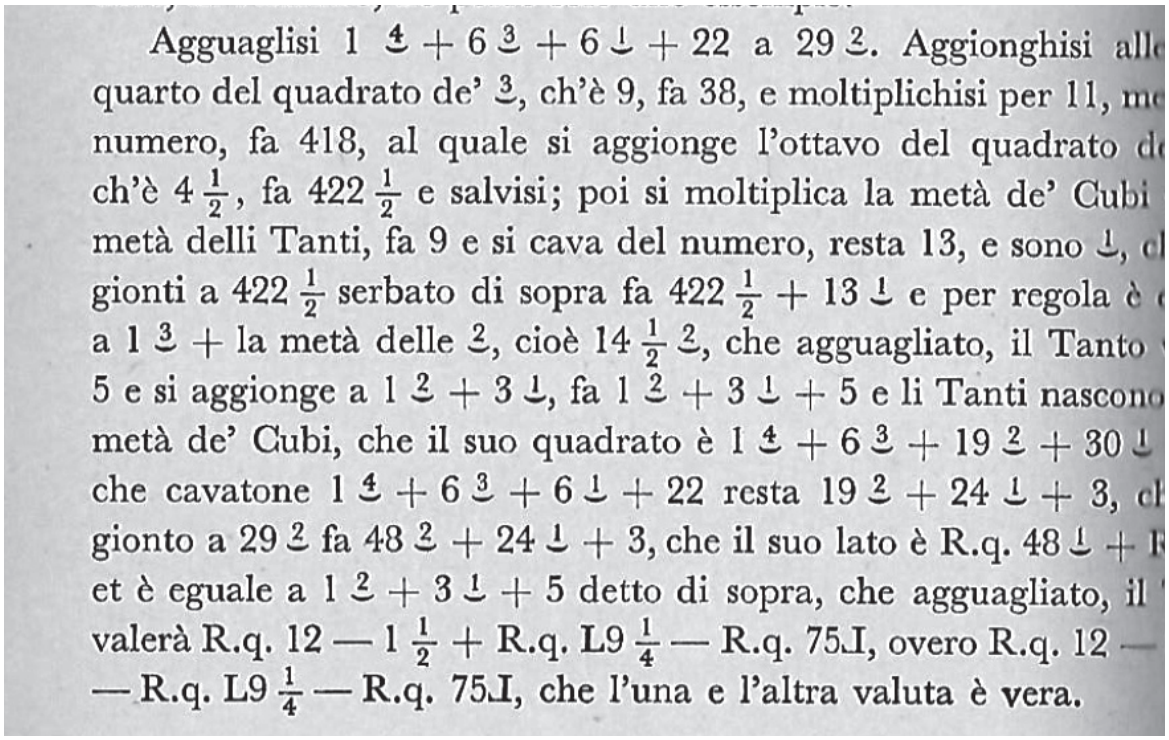
Posto $-y$, si ottiene la:

$$y^3 + \left(d + \frac{ac}{4}\right)y = \frac{b}{2}y^2 + \left(\frac{a^2}{4} + b\right)\frac{d}{2} - \frac{c^2}{8}.$$

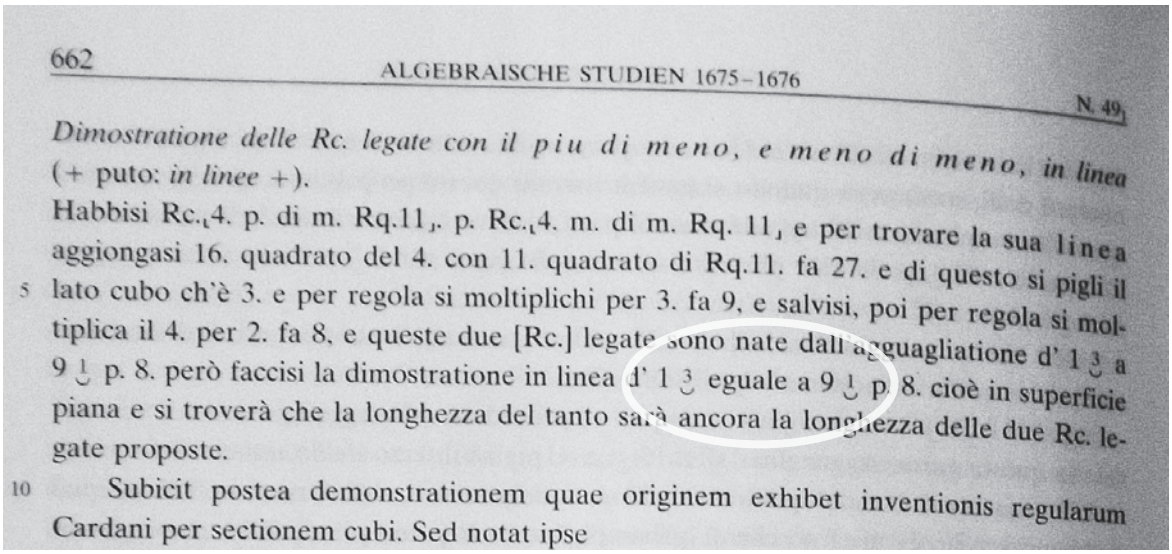
305

◌̳ COMBINING BOMBELLI POWER MARK

A digit combined with a small bow below as introduced by R. Bombelli, denotes the n-th power of a quantity. Example from Bombelli (1966).



Agguaglisi 1 4 + 6 3 + 6 1 + 22 a 29 2. Aggionghisi alle
quarto del quadrato de' 3, ch'è 9, fa 38, e moltiplichisi per 11, me
numero, fa 418, al quale si aggionge l'ottavo del quadrato de
ch'è $4\frac{1}{2}$, fa $422\frac{1}{2}$ e salvisi; poi si moltiplica la metà de' Cubi
metà delli Tanti, fa 9 e si cava del numero, resta 13, e sono 1, ch
gionti a $422\frac{1}{2}$ serbato di sopra fa $422\frac{1}{2} + 13\,1$ e per regola è
a 1 3 + la metà delle 2, cioè $14\frac{1}{2}$ 2, che agguagliato, il Tanto
5 e si aggionge a 1 2 + 3 1, fa 1 2 + 3 1 + 5 e li Tanti nascono
metà de' Cubi, che il suo quadrato è 1 4 + 6 3 + 19 2 + 30 1
che cavatone 1 4 + 6 3 + 6 1 + 22 resta 19 2 + 24 1 + 3, ch
gionto a 29 2 fa 48 2 + 24 1 + 3, che il suo lato è R.q. 48 1 + I
et è eguale a 1 2 + 3 1 + 5 detto di sopra, che agguagliato, il
valerà R.q. 12 — 1 $\frac{1}{2}$ + R.q. L9 $\frac{1}{4}$ — R.q. 75I, overo R.q. 12 —
— R.q. L9 $\frac{1}{4}$ — R.q. 75I, che l'una e l'altra valuta è vera.

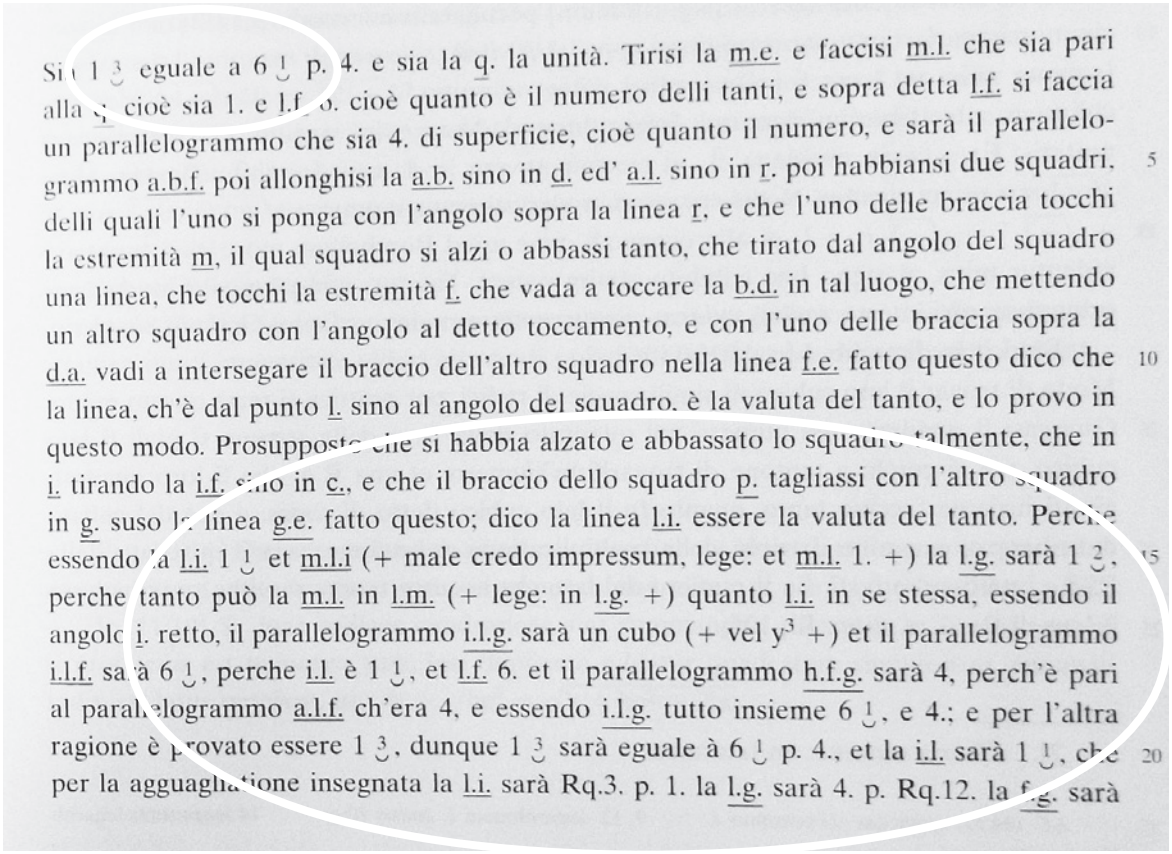


662 ALGEBRAISCHE STUDIEN 1675–1676 N. 49

Dimostratione delle Rc. legate con il piu di meno, e meno di meno, in linea (+ puto: *in linee* +).

Habbisi Rc.$_{\iota}$4. p. di m. Rq.11$_{\lrcorner}$. p. Rc.$_{\iota}$4. m. di m. Rq. 11$_{\lrcorner}$ e per trovare la sua linea aggiongasi 16. quadrato del 4. con 11. quadrato di Rq.11. fa 27. e di questo si pigli il lato cubo ch'è 3. e per regola si moltiplichi per 3. fa 9, e salvisi, poi per regola si moltiplica il 4. per 2. fa 8, e queste due [Rc.] legate sono nate dall'agguagliatione d' 1 $\overset{3}{\smile}$ a 9 $\overset{1}{\smile}$ p. 8. però faccisi la dimostratione in linea d' 1 $\overset{3}{\smile}$ eguale a 9 $\overset{1}{\smile}$ p. 8. cioè in superficie piana e si troverà che la longhezza del tanto sarà ancora la longhezza delle due Rc. legate proposte.

Subicit postea demonstrationem quae originem exhibet inventionis regularum Cardani per sectionem cubi. Sed notat ipse

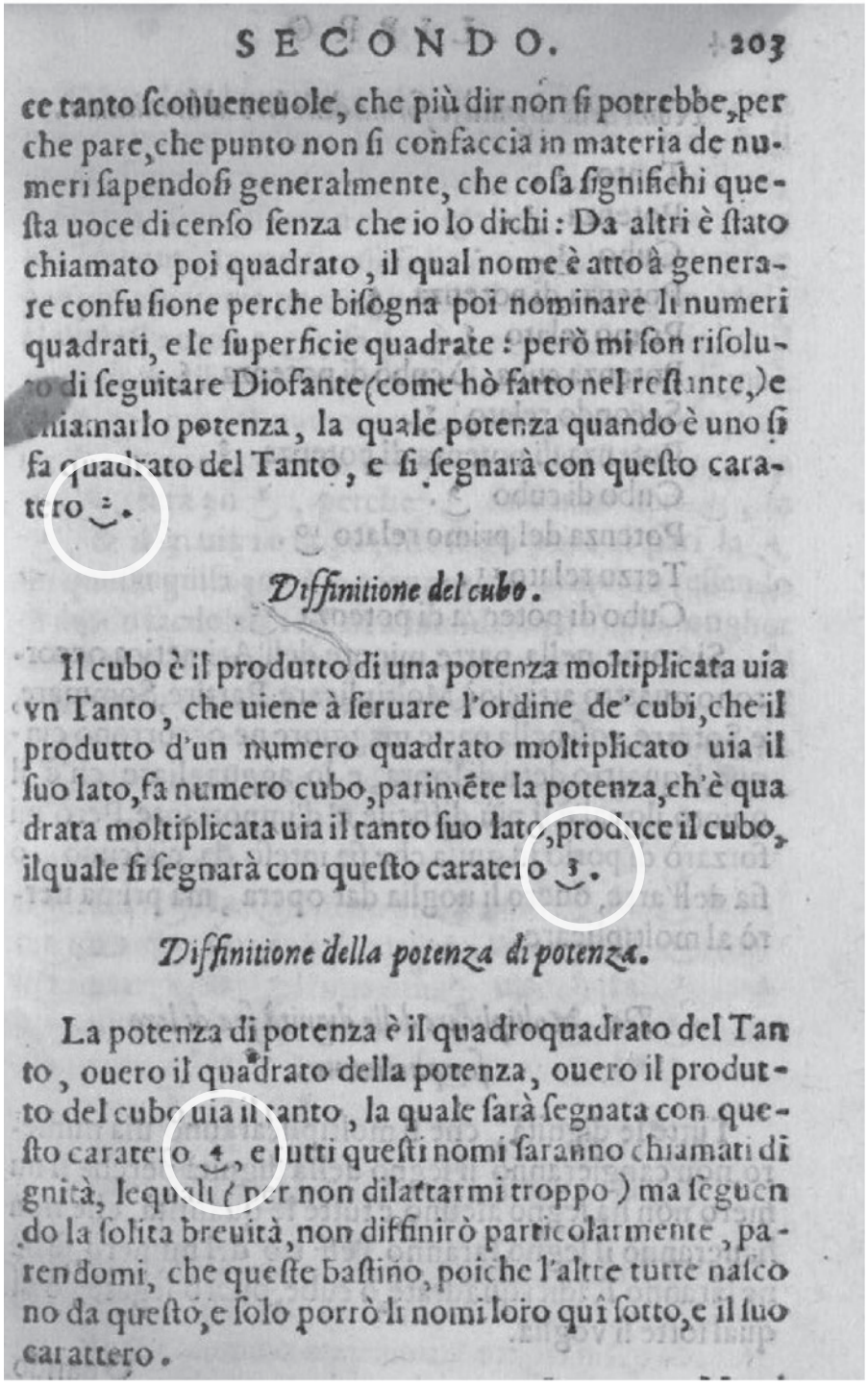


Si 1 $\overset{3}{\smile}$ eguale a 6 $\overset{1}{\smile}$ p. 4. e sia la q. la unità. Tirisi la m.e. e faccisi m.l. che sia pari alla q. cioè sia 1. e l.f. 6. cioè quanto è il numero delli tanti, e sopra detta l.f. si faccia un parallelogrammo che sia 4. di superficie, cioè quanto il numero, e sarà il parallelogrammo a.b.f. poi allonghisi la a.b. sino in d. ed' a.l. sino in r. poi habbiansi due squadri, delli quali l'uno si ponga con l'angolo sopra la linea r. e che l'uno delle braccia tocchi la estremità m, il qual squadro si alzi o abbassi tanto, che tirato dal angolo del squadro una linea, che tocchi la estremità f. che vada a toccare la b.d. in tal luogo, che mettendo un altro squadro con l'angolo al detto toccamento, e con l'uno delle braccia sopra la d.a. vadi a intersegare il braccio dell'altro squadro nella linea f.e. fatto questo dico che la linea, ch'è dal punto l. sino al angolo del squadro, è la valuta del tanto, e lo provo in questo modo. Prosupposto che si habbia alzato e abbassato lo squadro talmente, che in i. tirando la i.f. sino in c., e che il braccio dello squadro p. tagliassi con l'altro squadro in g. suso la linea g.e. fatto questo; dico la linea l.i. essere la valuta del tanto. Perche essendo la l.i. 1 $\overset{1}{\smile}$ et m.l.i (+ male credo impressum, lege: et m.l. 1. +) la l.g. sarà 1 $\overset{2}{\smile}$, perche tanto può la m.l. in l.m. (+ lege: in l.g. +) quanto l.i. in se stessa, essendo il angolo i. retto, il parallelogrammo i.l.g. sarà un cubo (+ vel y^3 +) et il parallelogrammo i.l.f. sarà 6 $\overset{1}{\smile}$, perche i.l. è 1 $\overset{1}{\smile}$, et l.f. 6. et il parallelogrammo h.f.g. sarà 4, perch'è pari al parallelogrammo a.l.f. ch'era 4, e essendo i.l.g. tutto insieme 6 $\overset{1}{\smile}$, e 4.; e per l'altra ragione è provato essere 1 $\overset{3}{\smile}$, dunque 1 $\overset{3}{\smile}$ sarà eguale à 6 $\overset{1}{\smile}$ p. 4., et la i.l. sarà 1 $\overset{1}{\smile}$, che per la agguaglia tione insegnata la l.i. sarà Rq.3. p. 1. la l.g. sarà 4. p. Rq.12. la f.g. sarà

◌͡◌ COMBINING BOMBELLI POWER MARK

A digit combined with a small bow below as introduced by R. Bombelli denotes the n-th power of a quantity. By this, Bombelli provides a different formalization of what is adressed otherwise by the use of cossic signs. Today, we write x, x^2, x^3, … instead.

LAA VII-2 p. 662, 663



SECONDO. 203

ce tanto ſconueneuole, che più dir non ſi potrebbe, per che pare, che punto non ſi confaccia in materia de numeri ſapendoſi generalmente, che coſa ſignifichi queſta uoce di cenſo ſenza che io lo dichi: Da altri è ſtato chiamato poi quadrato, il qual nome è atto à generare confuſione perche biſogna poi nominare li numeri quadrati, e le ſuperficie quadrate: però mi ſon riſoluto di ſeguitare Diofante (come hò fatto nel reſtante,) e chiamarlo potenza, la quale potenza quando è uno ſi fa quadrato del Tanto, e ſi ſegnarà con queſto carattero ☺².

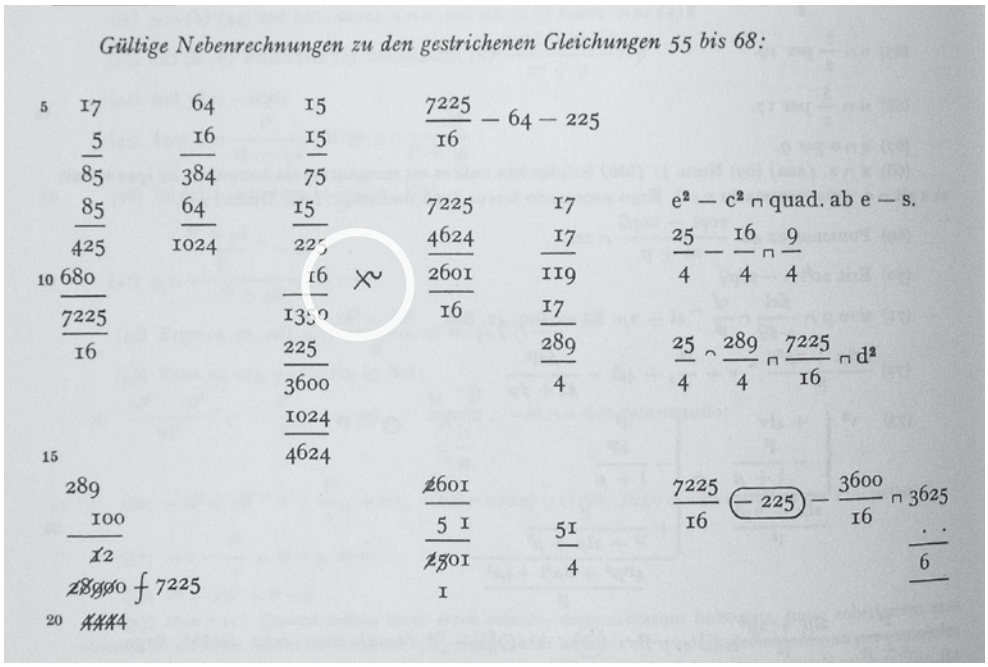
Diffinitione del cubo.

Il cubo è il produtto di una potenza moltiplicata uia vn Tanto, che uiene à ſeruare l'ordine de' cubi, che il produtto d'un numero quadrato moltiplicato uia il ſuo lato, fa numero cubo, parimẽte la potenza, ch'è quadrata moltiplicata uia il tanto ſuo lato, produce il cubo, ilquale ſi ſegnarà con queſto carattero ☺³.

Diffinitione della potenza di potenza.

La potenza di potenza è il quadroquadrato del Tanto, ouero il quadrato della potenza, ouero il produtto del cubo uia il tanto, la quale ſarà ſegnata con queſto carattero ☺⁴, e tutti queſti nomi ſaranno chiamati dignità, lequali (per non dilattarmi troppo) ma ſeguendo la ſolita breuità, non diffinirò particolarmente, parendomi, che queſte baſtino, poiche l'altre tutte naſcono da queſto, e ſolo porrò li nomi loro quì ſotto, e il ſuo carattero.

☺ COMBINING BOMBELLI POWER MARK
R. Bombelli, L'Algebra. Bologna 1579, p. 203



Gültige Nebenrechnungen zu den gestrichenen Gleichungen 55 bis 68:

5 17 / 5 / 85 / 85 / 425 — 64 / 16 / 384 / 64 / 1024 — 15 / 15 / 75 / 15 / 225 / 16 / 1350 / 225 / 3600 / 1024 / 4624 — $\frac{7225}{16} - 64 - 225$

10 680 / $\frac{7225}{16}$ — 7225 / 4624 / 2601 / 16 — 17 / 17 / 119 / 17 / 289 / 4

$e^2 - c^2 \sqcap$ quad. ab e — s.

$\frac{25}{4} - \frac{16}{4} \sqcap \frac{9}{4}$

$\frac{25}{4} \frown \frac{289}{4} \sqcap \frac{7225}{16} \sqcap d^2$

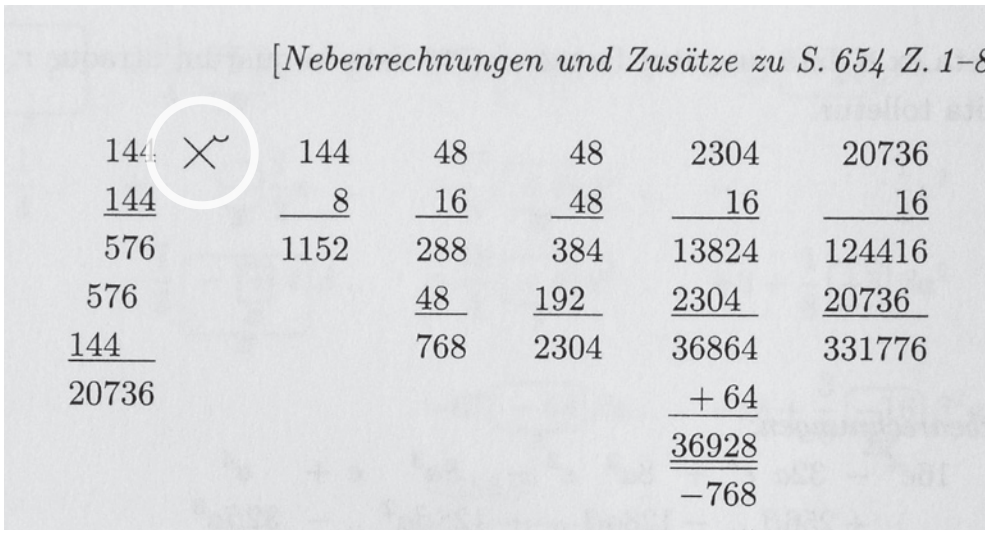
15 289 / 100 / ~~12~~ — ~~2601~~ / 5 1 / ~~2501~~ / 1 — 51 / 4

$\frac{7225}{16} - 225 - \frac{3600}{16} \sqcap 3625$

. . / 6

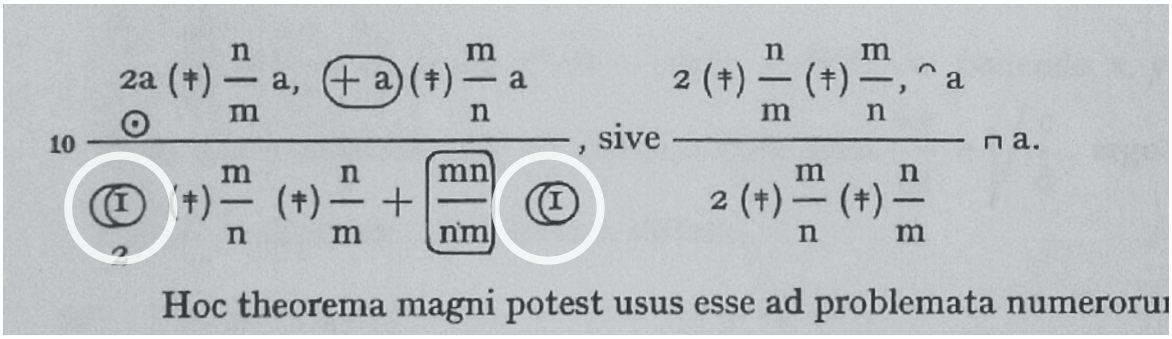
20 ~~4444~~ — ~~28900~~ ∫ 7225

⟡ CASTING-OUT-NINES
LAA VII-1 p. 408; VII-3 p.660 (below)



[Nebenrechnungen und Zusätze zu S. 654 Z. 1–8

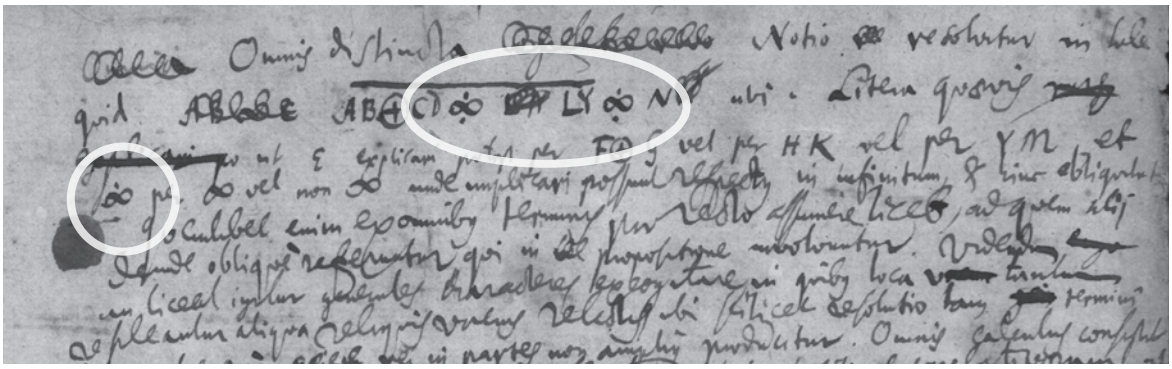
144	144	48	48	2304	20736
144	8	16	48	16	16
576	1152	288	384	13824	124416
576		48	192	2304	20736
144		768	2304	36864	331776
20736				+ 64	
				36928	
				−768	



$$10\quad \frac{2a\,(\dagger)\,\frac{n}{m}\,a,\ (+a)\,(\dagger)\,\frac{m}{n}\,a}{(\dagger)\,\frac{m}{n}\,(\dagger)\,\frac{n}{m} + \frac{mn}{nm}}\ \text{, sive}\ \frac{2\,(\dagger)\,\frac{n}{m}\,(\dagger)\,\frac{m}{n}\,\frown a}{2\,(\dagger)\,\frac{m}{n}\,(\dagger)\,\frac{n}{m}} \sqcap a.$$

Hoc theorema magni potest usus esse ad problemata numerorui

① LUNATE ENCIRCLED DIGIT ONE
LAA VII-1 p. 472



⟡ INFINITY WITH DOTS
LH 4 VII B 2, fol. 73v (top), LAA VI-4 p. 873 (below)

Omnia ad haec videntur revocari posse. Aliquidditas. Essentia. Existentia. Realitas. Perfectio. Uni[tas.] Convenientia. Veritas. Consequentia. Ordo. Causalitas. Mutatio. Magnitudo. Sensus. Appetitus. Cogitatio. Qualitates Sensibiles.

⟨ – ⟩ in characteristica omnia distincte cogitabilia revocari possunt ad $\overline{AB + CD}^{\infty}_{\text{non } \infty} LM \infty N$, hoc uno not⟨ato⟩ ⟨ – – – ⟩ et contra explicari ⟨ – – – ⟩ quod quaedam literae in ⟨ – ⟩ ut *Y* pro *S* pon⟨–⟩

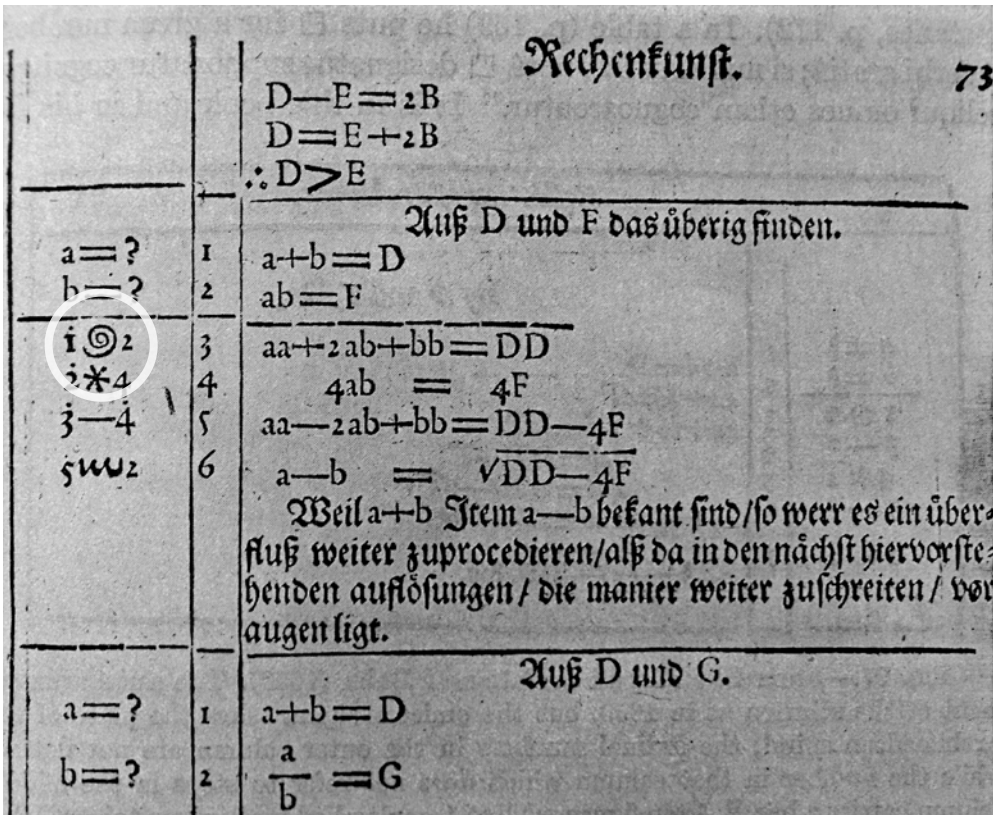
Omnis distincta Notio resolvitur in tale quid $\overline{AB \oplus CD} \Cdot \overline{LY} \Cdot N$ ubi Litera quaevis ut *E* explicari potest per $F \oplus G$ vel per *HK* vel per *YM* et ⟡ per ∞ vel *non* ∞ unde implicari possunt respectus in infinitum, et hinc obliquitat[es].

Quemlibet enim ex omnibus terminis pro recto assumere licet, ad quem alii deinde oblique referuntur, qui in propositione involvuntur. Videndum an liceat igitur generales

In a comment from SEW (2024) another solution has been proposed:

#8 INFINITY SIGN WITH DOTS This character could potentially already be accurately represented using the sequence ‹⟡›.

¶8 This solution would require a sequence of **three** different existing characters of which two are combing characters. In which succession they are to be arranged properly? This is not at all obvious and hence this model would result in at least two different sequences in practice, which makes the identification of the character difficult to impossible, because one can never be sure (in a search) to reach all instances, when they happen to be encoded differently (although looking the same visually). Moreover, there is precedence of structurally similar cases encoded. For instance, the Mathematical Operators block contains a range of characters whose glyphs are built of other well-known base glyphs and dots, in the range 2238 to 2255; e.g. HOMOTHETIC (223B) and GEOMETRICALLY EQUAL TO (2251) can be seen as analogue characters, which have their own meaning, despite being graphically composed of prevalent glyphs. There are also many characters which are represented by combinations of e.g. = or < with a SLASH or ‘solidus overlay’. It is a feature inherent in mathematical notation that new expressions are created by combining established elements, in the one way or the other. Nevertheless, it is justifiable to assign a separate codepoint for such a character, because its use and meaning are testified and this encoding is conformant with established encoding practice and principles.



Rechenkunſt. 73

D—E = 2B
D = E+2B
∴ D > E

Auß D und F das überig finden.

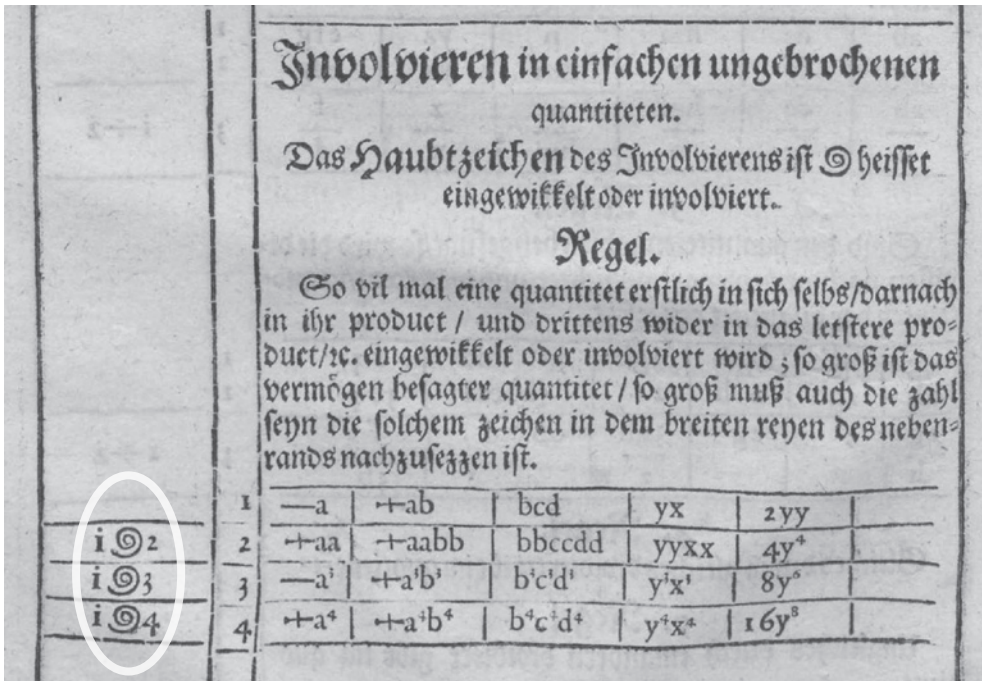
a = ?	1	a+b = D
b = ?	2	ab = F
i ◎ 2	3	aa+2ab+bb = DD
i ✱ 4	4	4ab = 4F
3—4	5	aa—2ab+bb = DD—4F
5 ww 2	6	a—b = √DD—4F

Weil a+b Item a—b bekant ſind/ſo werr es ein überfluß weiter zuprocedieren/alß da in den nächſt hiervorſtehenden auflöſungen/ die manier weiter zuſchreiten/ vor augen ligt.

Auß D und G.

a = ?	1	a+b = D
b = ?	2	a/b = G

◎ INVOLVED – J. H. Rahn, Teutsche Algebra, 1659 (after Cajori).
In expressions of the form *a* ◎ *b*, the sign ◎ is used to denote the exponentiation of *a* by the power of *b*. In his “Teutsche Algebra” from 1659, the swiss mathematician Johann Heinrich Rahn refers to the operation as “involvieren” (= to involve).



Involvieren in einfachen ungebrochenen quantiteten.

Das Haubtzeichen des Involvierens iſt ◎ heiſſet eingewikkelt oder involviert.

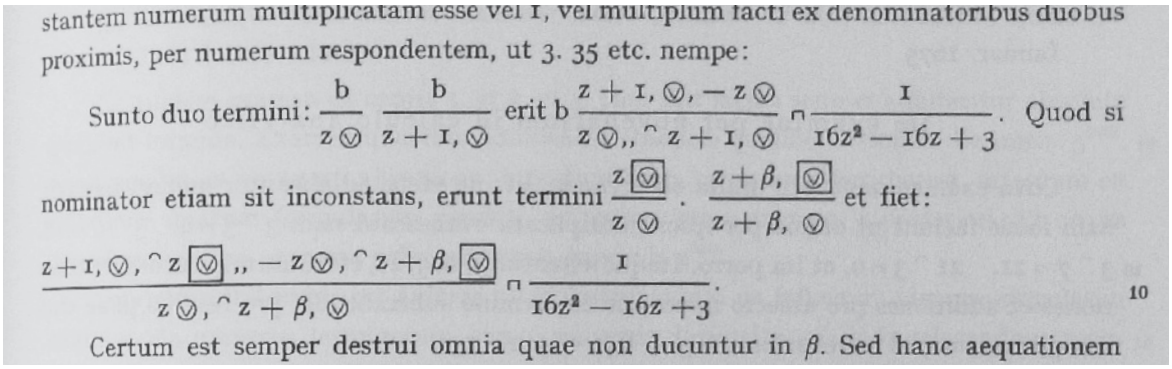
Regel.

So vil mal eine quantitet erſtlich in ſich ſelbs/darnach in ihr product / und drittens wider in das letſtere product/ꝛc. eingewikkelt oder involviert wird ; ſo groß iſt das vermögen beſagter quantitet / ſo groß muß auch die zahl ſeyn die ſolchem zeichen in dem breiten reyen des nebenrands nachzuſezzen iſt.

	1	—a	+ab	bcd	yx	2yy
i ◎ 2	2	+aa	+aabb	bbccdd	yyxx	$4y^4$
i ◎ 3	3	$-a^3$	$+a^3b^3$	$b^3c^3d^3$	y^3x^3	$8y^6$
i ◎ 4	4	$+a^4$	$+a^4b^4$	$b^4c^4d^4$	y^4x^4	$16y^8$

◎ INVOLVED – J. H. Rahn, Teutsche Algebra, 1659.

In the time of Leibniz, the usual way of referring to curves or magnitudes is by giving equations that describe their specific relations. The concept of mapping as it is used in modern mathematics is not yet developed. Leibniz writes the signs Ⓥ and 🄥 to the right of an expression (such as *x* Ⓥ and *y+1*, 🄥) in order to denote two different arbitrary rules by which the expressions given in the left position are treated. The result is an expression. By this, the meaning is similar to writing *f(x)* or *g(y+1)* in modern mathematical notation with *f* and *g* denoting arbitrary functions.
In a similar way, Johann Bernoulli uses the sign ℘ (see L-2535/N5335R) to denote a quantity depending on variables *x* and *a* (in modern terminology a function in *x* and *a*).



stantem numerum multiplicatam esse vel 1, vel multiplum facti ex denominatoribus duobus proximis, per numerum respondentem, ut 3. 35 etc. nempe:

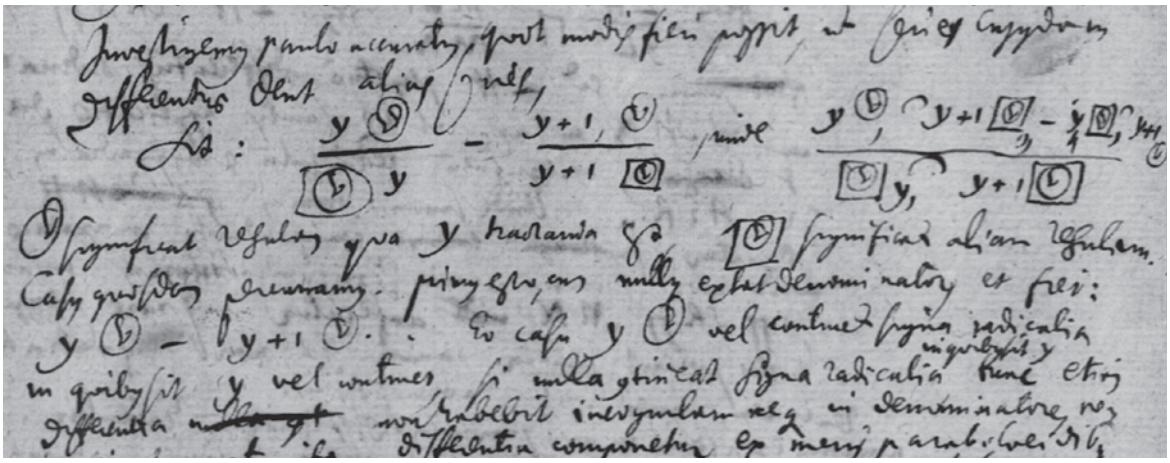
Sunto duo termini: $\frac{b}{z\,Ⓥ}\ \frac{b}{z+1,Ⓥ}$ erit $b\,\frac{z+1,Ⓥ,-z\,Ⓥ}{z\,Ⓥ,,\hat{}\,z+1,Ⓥ} \sqcap \frac{1}{16z^2-16z+3}$. Quod si nominator etiam sit inconstans, erunt termini $\frac{z\,🄥}{z\,Ⓥ}\ .\ \frac{z+\beta,\,🄥}{z+\beta,\,Ⓥ}$ et fiet:

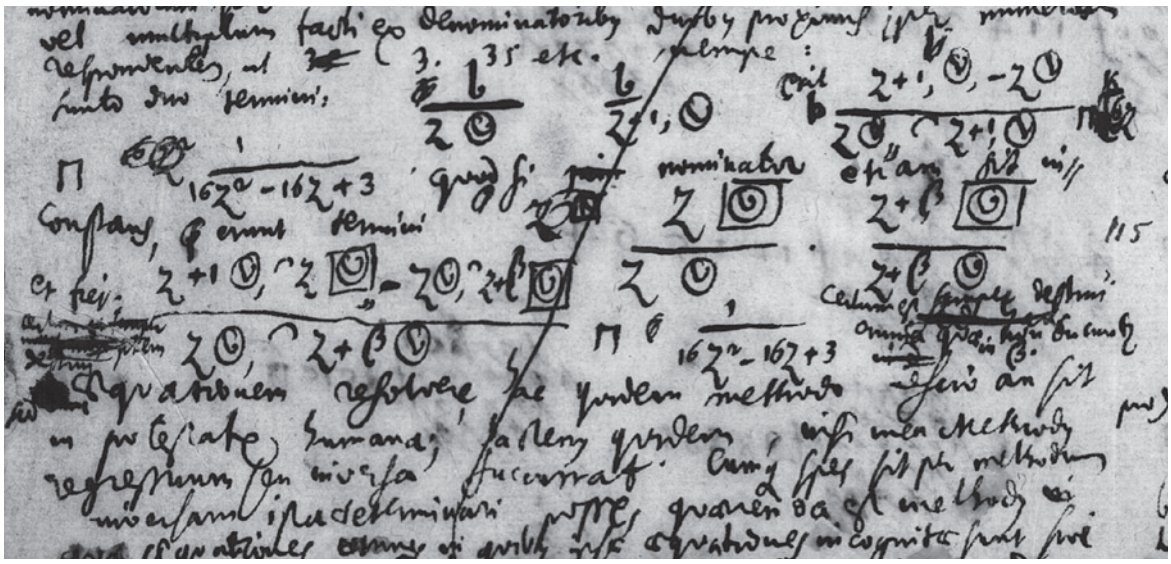
$$\frac{z+1,Ⓥ,\hat{}\,z\,🄥,,-z\,Ⓥ,\hat{}\,z+\beta,🄥}{z\,Ⓥ,\hat{}\,z+\beta,Ⓥ} \sqcap \frac{1}{16z^2-16z+3}.$$

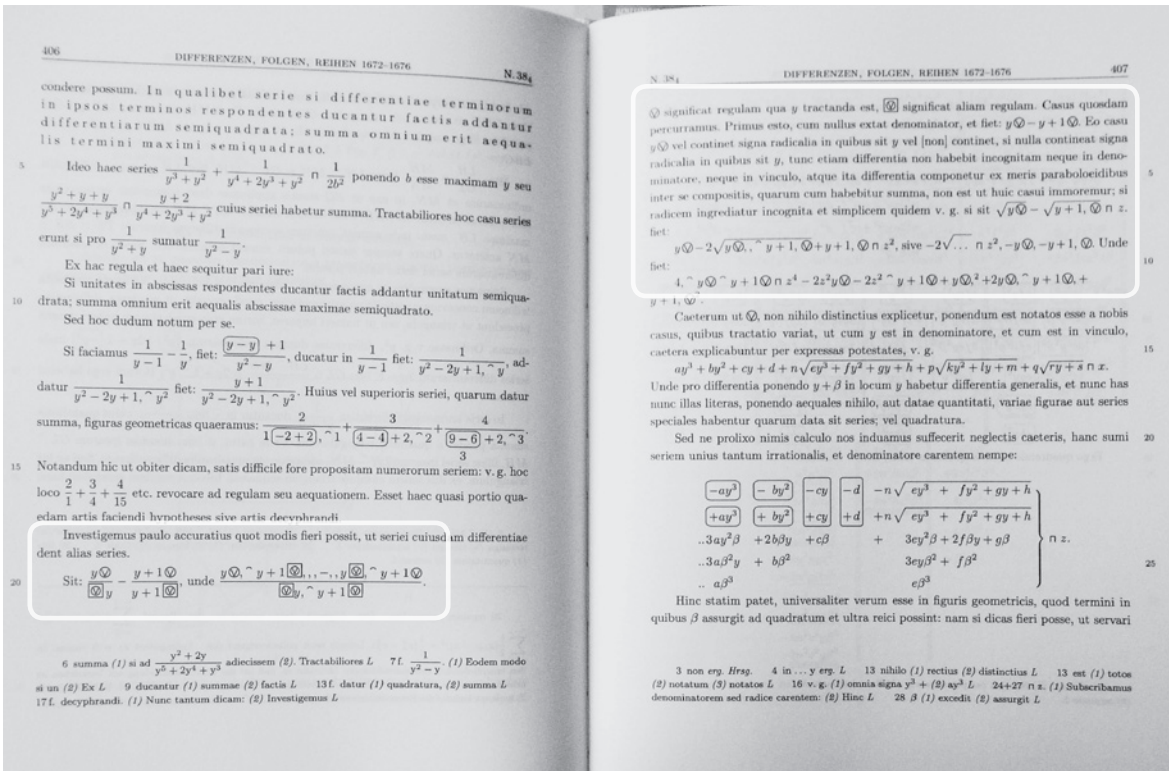
10

Certum est semper destrui omnia quae non ducuntur in β. Sed hanc aequationem

Ⓥ LEIBNIZIAN ENCIRCLED V, 🄥 LEIBNIZIAN ENCIRCLED V IN BOX
LAA VII-1 p. 527 (above), LH 35 V 4, fol. 6 (below), LH 35 VIII 30, fol. 115r (bottom)







406 DIFFERENZEN, FOLGEN, REIHEN 1672–1676 N. 38₄

condere possum. In qualibet serie si differentiae terminorum in ipsos terminos respondentes ducantur factis addantur differentiarum semiquadrata; summa omnium erit aequalis termini maximi semiquadrato.

Ideo haec series $\frac{1}{y^3+y^2} + \frac{1}{y^4+2y^3+y^2} \sqcap \frac{1}{2b^2}$ ponendo b esse maximam y seu $\frac{y^2+y+y}{y^5+2y^4+y^3} \sqcap \frac{y+2}{y^4+2y^3+y^2}$ cuius seriei habetur summa. Tractabiliores hoc casu series erunt si pro $\frac{1}{y^2+y}$ sumatur $\frac{1}{y^2-y}$.

Ex hac regula et haec sequitur pari iure:

Si unitates in abscissas respondentes ducantur factis addantur unitatum semiquadrata; summa omnium erit aequalis abscissae maximae semiquadrato.

Sed hoc dudum notum per se.

Si faciamus $\frac{1}{y-1} - \frac{1}{y}$, fiet: $\frac{\boxed{y-y}+1}{y^2-y}$, ducatur in $\frac{1}{y-1}$ fiet: $\frac{1}{y^2-2y+1,\hat{}\,y}$, addatur $\frac{1}{y^2-2y+1,\hat{}\,y^2}$ fiet: $\frac{y+1}{y^2-2y+1,\hat{}\,y^2}$. Huius vel superioris seriei, quarum datur summa, figuras geometricas quaeramus: $\frac{2}{1\overline{-2+2},\hat{}\,1} + \frac{3}{\overline{4-4}+2,\hat{}\,2} + \frac{4}{\overline{9-6}+2,\hat{}\,3}$.

Notandum hic ut obiter dicam, satis difficile fore propositam numerorum seriem: v. g. hoc loco $\frac{2}{1}+\frac{3}{4}+\frac{4}{15}$ etc. revocare ad regulam seu aequationem. Esset haec quasi portio quaedam artis faciendi hypotheses sive artis decyphrandi.

Investigemus paulo accuratius quot modis fieri possit, ut seriei cuiusd um differentiae dent alias series.

Sit: $\frac{y\textcircled{V}}{\boxed{\textcircled{V}}y} - \frac{y+1\textcircled{V}}{y+1\boxed{\textcircled{V}}}$, unde $\frac{y\textcircled{V},\hat{}\,y+1\boxed{\textcircled{V}},,,-,,y\boxed{\textcircled{V}},\hat{}\,y+1\textcircled{V}}{\boxed{\textcircled{V}}y,\hat{}\,y+1\boxed{\textcircled{V}}}$.

6 summa *(1)* si ad $\frac{y^2+2y}{y^5+2y^4+y^3}$ adiecissem *(2)*. Tractabiliores *L* 7 f. $\frac{1}{y^2-y}$. *(1)* Eodem modo si un *(2)* Ex *L* 9 ducantur *(1)* summae *(2)* factis *L* 13 f. datur *(1)* quadratura, *(2)* summa *L* 17 f. decyphrandi. *(1)* Nunc tantum dicam: *(2)* Investigemus *L*

N. 38₄ DIFFERENZEN, FOLGEN, REIHEN 1672–1676 407

$\textcircled{V}$ significat regulam qua y tractanda est, $\boxed{\textcircled{V}}$ significat aliam regulam. Casus quosdam percurramus. Primus esto, cum nullus extat denominator, et fiet: $y\textcircled{V} - y + 1\textcircled{V}$. Eo casu $y\textcircled{V}$ vel continet signa radicalia in quibus sit y vel [non] continet, si nulla contineat signa radicalia in quibus sit y, tunc etiam differentia non habebit incognitam neque in denominatore, neque in vinculo, atque ita differentia componetur ex meris paraboloeidibus inter se compositis, quarum cum habebitur summa, non est ut huic casui immoremur; si radicem ingrediatur incognita et simplicem quidem v. g. si sit $\sqrt{y\textcircled{V}} - \sqrt{y+1, \textcircled{V}} \sqcap z$. fiet:

$y\textcircled{V} - 2\sqrt{y\textcircled{V},\hat{}\,y+1,\textcircled{V}} + y + 1, \textcircled{V} \sqcap z^2$, sive $-2\sqrt{\ldots} \sqcap z^2, -y\textcircled{V}, -y+1, \textcircled{V}$. Unde fiet:

$4,\hat{}\,y\textcircled{V}\hat{}\,y+1\textcircled{V} \sqcap z^4 - 2z^2y\textcircled{V} - 2z^2\hat{}\,y+1\textcircled{V} + y\textcircled{V},^2 + 2y\textcircled{V},\hat{}\,y+1\textcircled{V},+ y+1,\textcircled{V}^2$.

Caeterum ut $\textcircled{V}$, non nihilo distinctius explicetur, ponendum est notatos esse a nobis casus, quibus tractatio variat, ut cum y est in denominatore, et cum est in vinculo, caetera explicabuntur per expressas potestates, v. g.

$ay^3 + by^2 + cy + d + n\sqrt{ey^3+fy^2+gy+h} + p\sqrt{ky^2+ly+m} + q\sqrt{ry+s} \sqcap x$.

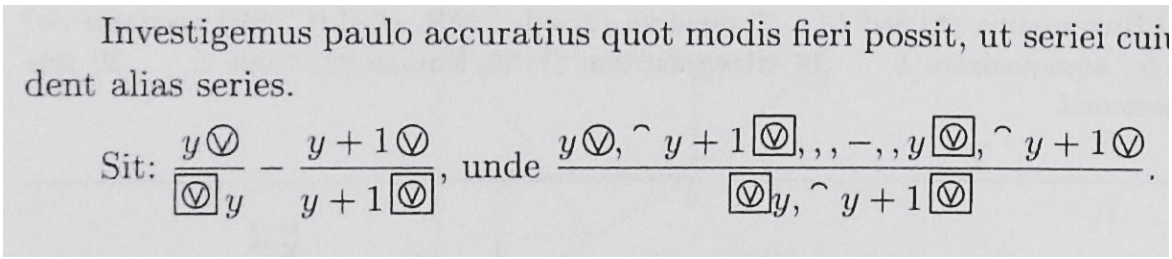
Unde pro differentia ponendo $y+\beta$ in locum y habetur differentia generalis, et nunc has nunc illas literas, ponendo aequales nihilo, aut datae quantitati, variae figurae aut series speciales habentur quarum data sit series; vel quadratura.

Sed ne prolixo nimis calculo nos induamus suffecerit neglectis caeteris, hanc sumi seriem unius tantum irrationalis, et denominatore carentem nempe:

$$\left.\begin{array}{llll} \boxed{-ay^3} & \boxed{-by^2} & \boxed{-cy} & \boxed{-d} \quad -n\sqrt{ey^3+fy^2+gy+h} \\ \boxed{+ay^3} & \boxed{+by^2} & \boxed{+cy} & \boxed{+d} \quad +n\sqrt{ey^3+fy^2+gy+h} \\ ..3ay^2\beta & +2b\beta y & +c\beta & + \quad 3ey^2\beta + 2fy\beta + g\beta \\ ..3a\beta^2 y & + b\beta^2 & & \quad 3ey\beta^2 + f\beta^2 \\ .. \; a\beta^3 & & & \quad e\beta^3 \end{array}\right\} \sqcap z.$$

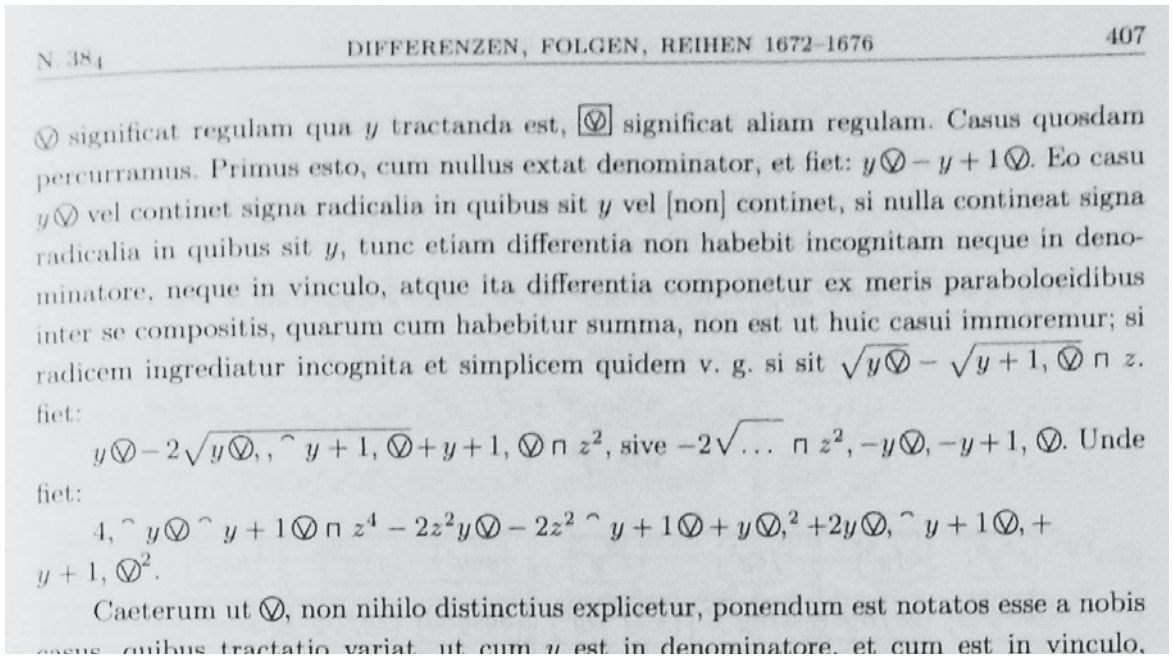
Hinc statim patet, universaliter verum esse in figuris geometricis, quod termini in quibus β assurgit ad quadratum et ultra reici possint: nam si dicas fieri posse, ut servari

3 non *erg. Hrsg.* 4 in ... y *erg. L* 13 nihilo *(1)* rectius *(2)* distinctius *L* 13 est *(1)* totos *(2)* notatum *(3)* notatos *L* 16 v. g. *(1)* omnia signa y^3 + *(2)* ay^3 *L* 24+27 $\sqcap z$. *(1)* Subscribamus denominatorem sed radice carentem: *(2)* Hinc *L* 28 β *(1)* excedit *(2)* assurgit *L*



Investigemus paulo accuratius quot modis fieri possit, ut seriei cui
dent alias series.

Sit: $\frac{y\textcircled{V}}{\boxed{\textcircled{V}}y} - \frac{y+1\textcircled{V}}{y+1\boxed{\textcircled{V}}}$, unde $\frac{y\textcircled{V},\hat{}\,y+1\boxed{\textcircled{V}},,,-,,y\boxed{\textcircled{V}},\hat{}\,y+1\textcircled{V}}{\boxed{\textcircled{V}}y,\hat{}\,y+1\boxed{\textcircled{V}}}$.



N. 38₄ DIFFERENZEN, FOLGEN, REIHEN 1672–1676 407

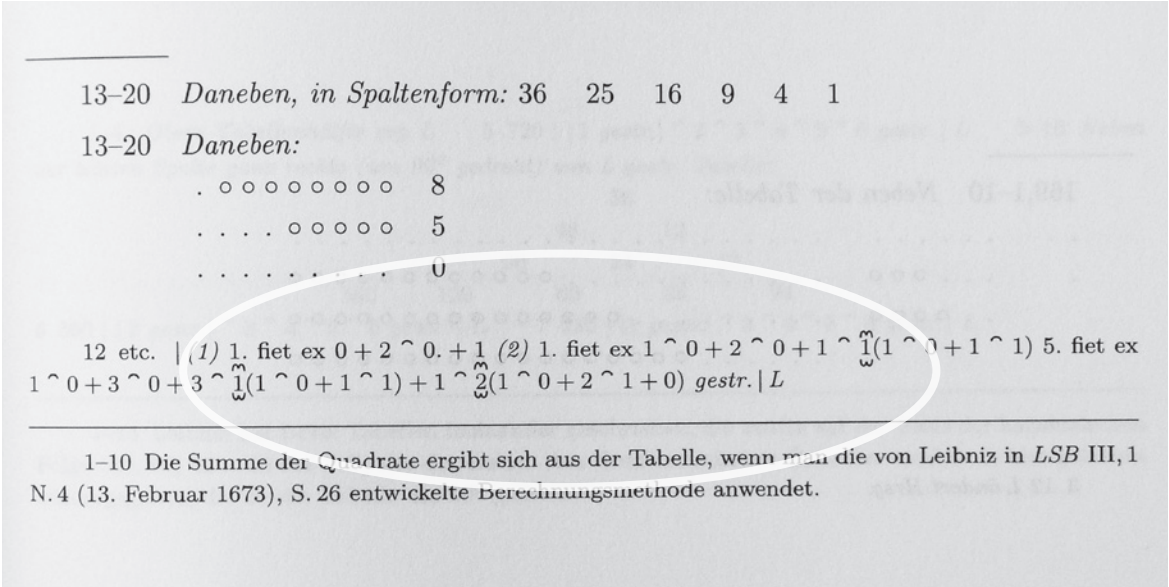
$\textcircled{V}$ significat regulam qua y tractanda est, $\boxed{\textcircled{V}}$ significat aliam regulam. Casus quosdam percurramus. Primus esto, cum nullus extat denominator, et fiet: $y\textcircled{V} - y + 1\textcircled{V}$. Eo casu $y\textcircled{V}$ vel continet signa radicalia in quibus sit y vel [non] continet, si nulla contineat signa radicalia in quibus sit y, tunc etiam differentia non habebit incognitam neque in denominatore, neque in vinculo, atque ita differentia componetur ex meris paraboloeidibus inter se compositis, quarum cum habebitur summa, non est ut huic casui immoremur; si radicem ingrediatur incognita et simplicem quidem v. g. si sit $\sqrt{y\textcircled{V}} - \sqrt{y+1, \textcircled{V}} \sqcap z$. fiet:

$y\textcircled{V} - 2\sqrt{y\textcircled{V},\hat{}\,y+1,\textcircled{V}} + y + 1, \textcircled{V} \sqcap z^2$, sive $-2\sqrt{\ldots} \sqcap z^2, -y\textcircled{V}, -y+1, \textcircled{V}$. Unde fiet:

$4,\hat{}\,y\textcircled{V}\hat{}\,y+1\textcircled{V} \sqcap z^4 - 2z^2y\textcircled{V} - 2z^2\hat{}\,y+1\textcircled{V} + y\textcircled{V},^2 + 2y\textcircled{V},\hat{}\,y+1\textcircled{V},+ y+1,\textcircled{V}^2$.

Caeterum ut $\textcircled{V}$, non nihilo distinctius explicetur, ponendum est notatos esse a nobis casus, quibus tractatio variat, ut cum y est in denominatore, et cum est in vinculo,

$\textcircled{V}$ LEIBNIZIAN ENCIRCLED V, $\boxed{\textcircled{V}}$ LEIBNIZIAN ENCIRCLED V IN BOX
The typographical solution of these characters in the edition is bad. In fact the round shape has to be connected with the *v*, similar as in @ (0040). – LAA VII-3 p. 406-407



13–20 *Daneben, in Spaltenform:* 36 25 16 9 4 1

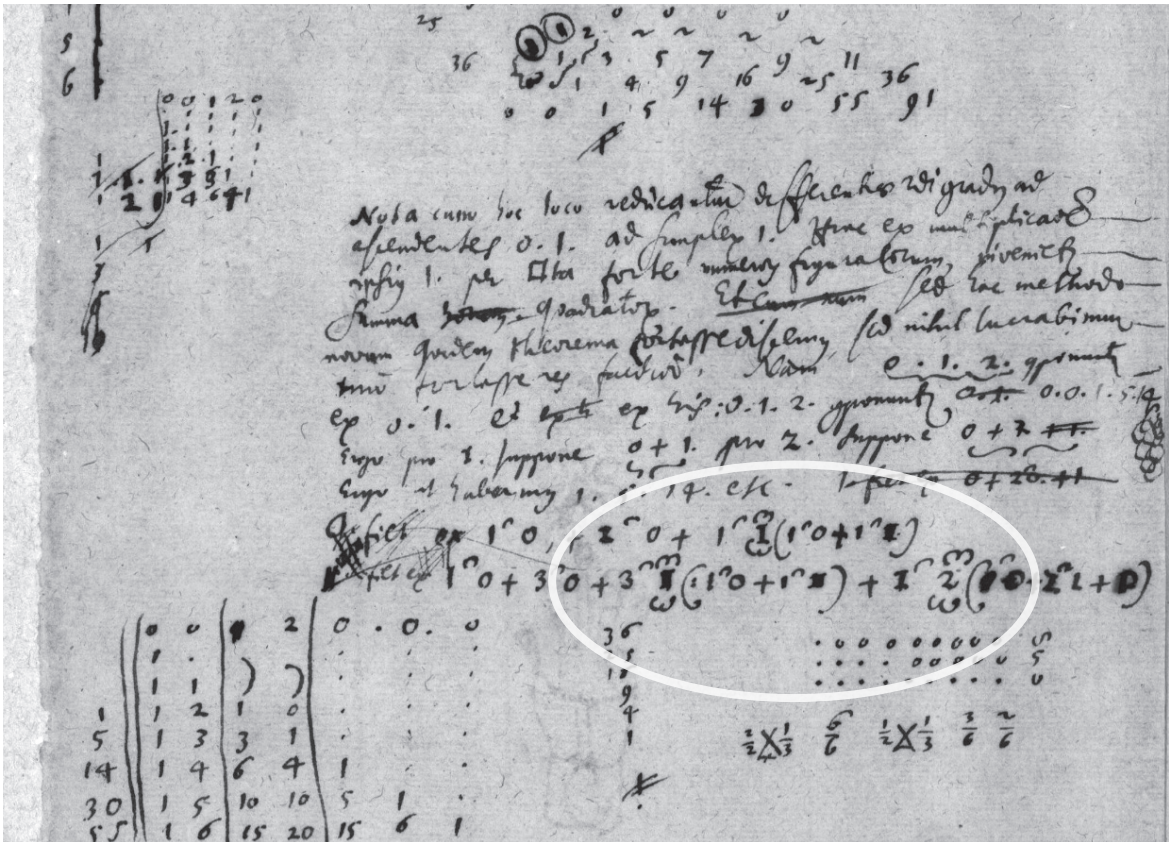
13–20 *Daneben:*

. o o o o o o o o o 8
. . . . o o o o o 5
. . . . 0

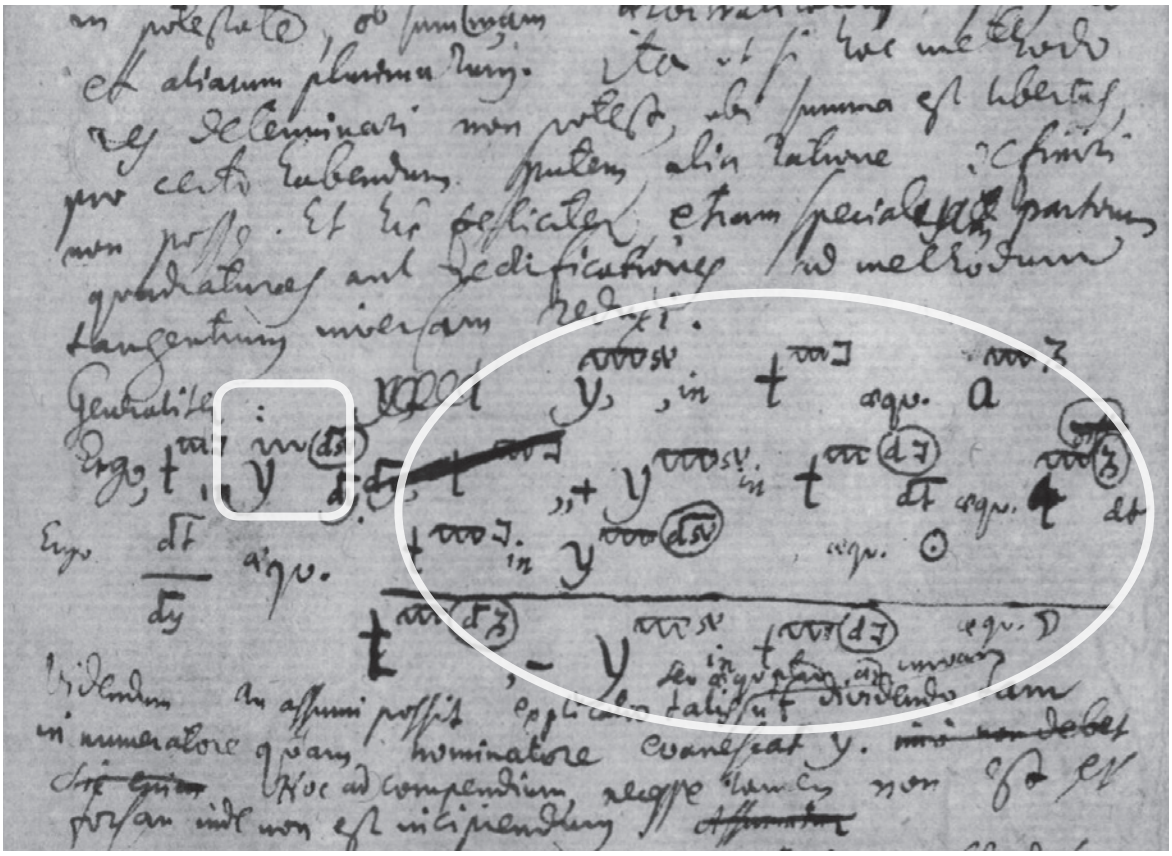
12 etc. | *(1)* 1. fiet ex $0+2\hat{}0.+1$ *(2)* 1. fiet ex $1\hat{}0+2\hat{}0+1\hat{}\overset{\text{m}}{\underset{\omega}{1}}(1\hat{}0+1\hat{}1)$ 5. fiet ex $1\hat{}0+3\hat{}0+3\hat{}\overset{\text{m}}{\underset{\omega}{1}}(1\hat{}0+1\hat{}1)+1\hat{}\overset{\text{m}}{\underset{\omega}{2}}(1\hat{}0+2\hat{}1+0)$ *gestr.* | *L*

1–10 Die Summe der Quadrate ergibt sich aus der Tabelle, wenn man die von Leibniz in *LSB* III, 1 N. 4 (13. Februar 1673), S. 26 entwickelte Berechnungsmethode anwendet.

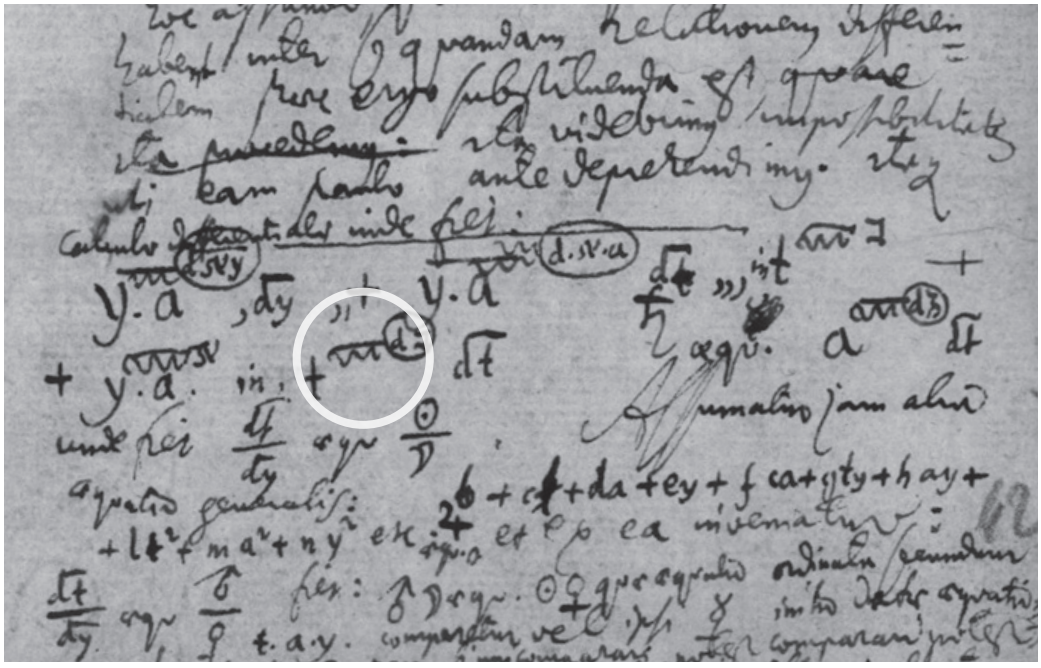
COMBINING FACTOR MARK
LAA VII-3 p. 167



COMBINING FACTOR MARK
LH 35 XII 1, fol. 138r



U+0360 ◌͠ COMBINING TILDE , U+033E ◌̾ COMBINING VERTICAL TILDE
LH 35 VII 1, fol. 39r. *The edition of this manuscript is currently in progress*.



U+033E ◌̾ COMBINING VERTICAL TILDE
LH 35 VII 1, fol. 41v. *The edition of this manuscript is currently in progress*.

5. Unicode Character Properties

```
xf01;CASTING-OUT-NINES;Sm;0;ON;;;;;N;;;;;
xf02;LUNATE ENCIRCLED DIGIT ONE;Sm;0;ON;;;;;N;;;;;
xf03;PROPORTION WITH ONE STROKE;Sm;0;ON;;;;;N;;;;;
xf04;PROPORTION WITH TWO STROKES;Sm;0;ON;;;;;N;;;;;
xf05;INFINITY WITH TWO DOTS;Sm;0;ON;;;;;N;;;;;
xf06;INVOLVED;Sm;0;ON;;;;;N;;;;;
xf07;LEIBNIZIAN ENCIRCLED V;Sm;0;ON;;;;;N;;;;;
xf08;LEIBNIZIAN ENCIRCLED V IN BOX;Sm;0;ON;;;;;N;;;;;
xf09;SUPERSCRIPT WAVE SYMBOL;Sm;0;ON;;;;;N;;;;;
xf10;SUPERSCRIPT WAVE WITH OVERLINE SYMBOL;Sm;0;ON;;;;;N;;;;;
xf11;COMBINING BOMBELLI POWER MARK;Mn;220;NSM;;;;;N;;;;;
xf12;COMBINING HALF CIRCLE BELOW;Mn;220;NSM;;;;;N;;;;;
xf13;COMBINING FACTOR MARK;Mn;1;NSM;;;;;N;;;;;
```

6. Bibliography

LAA – refers to: Leibniz, Gottfried Wilhelm: Sämtliche Schriften und Briefe. (‘Leibniz-Akademie-Ausgabe’, many volumes)
LH – refers to: Leibniz’s original manuscripts, GWLB Hanover

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ISO/IEC JTC 1/SC 2/WG 2
PROPOSAL SUMMARY FORM TO ACCOMPANY SUBMISSIONS
FOR ADDITIONS TO THE REPERTOIRE OF ISO/IEC 10646[1]
Please fill all the sections A, B and C below.
Please read Principles and Procedures Document (P & P) from http://std.dkuug.dk/JTC1/SC2/WG2/docs/principles.html **for guidelines and details before filling this form.**
Please ensure you are using the latest Form from http://std.dkuug.dk/JTC1/SC2/WG2/docs/summaryform.html**.**
See also http://std.dkuug.dk/JTC1/SC2/WG2/docs/roadmaps.html **for latest *Roadmaps*.**

A. Administrative

1. **Title:** **Proposal to encode 21 miscellaneous scientific symbols**
2. Requester's name: Uwe Mayer, Siegmund Probst, David Rabouin, Elisabeth Rinner, Andreas Stötzner, Achim Trunk, Charlotte Wahl
3. Requester type (Member body/Liaison/Individual contribution): Individual (work group)
4. Submission date: 2025-11.25.
5. Requester's reference (if applicable): LUCP L-2536
6. Choose one of the following:
   This is a complete proposal: Yes
   (or) More information will be provided later:

B. Technical – General

1. Choose one of the following:
   a. This proposal is for a new script (set of characters): No
      Proposed name of script:
   b. The proposal is for addition of character(s) to an existing block: No
      Name of the existing block: [Miscellaneous Symbols Supplement]
2. Number of characters in proposal: 13
3. Proposed category (select one from below - see section 2.2 of P&P document):
   A-Contemporary ; B.1-Specialized (small collection) Yes ; B.2-Specialized (large collection) ;
   C-Major extinct ; D-Attested extinct ; E-Minor extinct ;
   F-Archaic Hieroglyphic or Ideographic ; G-Obscure or questionable usage symbols
4. Is a repertoire including character names provided? Yes
   a. If YES, are the names in accordance with the "character naming guidelines" in Annex L of P&P document? Yes
   b. Are the character shapes attached in a legible form suitable for review? Yes
5. Fonts related:
   a. Who will provide the appropriate computerized font to the Project Editor of 10646 for publishing the standard?
      Andreas Stötzner
   b. Identify the party granting a license for use of the font by the editors (include address, e-mail, ftp-site, etc.):
      Andreas Stötzner Gestaltung, Klauflügelweg 21, 88400 Biberach/R., Germany, as@signographie.de
6. References:
   a. Are references (to other character sets, dictionaries, descriptive texts etc.) provided? Yes
   b. Are published examples of use (such as samples from newspapers, magazines, or other sources) of proposed characters attached? Yes
7. Special encoding issues:
   Does the proposal address other aspects of character data processing (if applicable) such as input, presentation, sorting, searching, indexing, transliteration etc. (if yes please enclose information)? No

8. Additional Information:

Submitters are invited to provide any additional information about Properties of the proposed Character(s) or Script that will assist in correct understanding of and correct linguistic processing of the proposed character(s) or script. Examples of such properties are: Casing information, Numeric information, Currency information, Display behaviour information such as line breaks, widths etc., Combining behaviour, Spacing behaviour, Directional behaviour, Default Collation behaviour, relevance in Mark Up contexts, Compatibility equivalence and other Unicode normalization related information. See the Unicode standard at http://www.unicode.org for such information on other scripts. Also see Unicode Character Database (http://www.unicode.org/reports/tr44/) and associated Unicode Technical Reports for information needed for consideration by the Unicode Technical Committee for inclusion in the Unicode Standard.

[1] Form number: N4502-F (Original 1994-10-14; Revised 1995-01, 1995-04, 1996-04, 1996-08, 1999-03, 2001-05, 2001-09, 2003-11, 2005-01, 2005-09, 2005-10, 2007-03, 2008-05, 2009-11, 2011-03, 2012-01)

C. Technical - Justification

1. Has this proposal for addition of character(s) been submitted before? Yes
   If YES explain *see N5336 (L-2521), N5277 (L-2402n)*
2. Has contact been made to members of the user community (for example: National Body, user groups of the script or characters, other experts, etc.)? Yes
   If YES, with whom? Leibniz-Archiv, Forschungsstelle der Leibniz-Edition, Niedersächsische Landesbibliothek (GWLB), Hanover, Göttingen Academy of Science and Humanities in Lower Saxony (DE), Philiumm research group of CNRS (UMR 7219, laboratoire SPHERE) / Université de Paris VII; general: scholars, researchers, authors and editors working in the field of science history and upon editions of historic text corpora (e.g. of G. W. Leibniz, but also many others)
   If YES, available relevant documents: L-2409, L-2410
3. Information on the user community for the proposed characters (for example: size, demographics, information technology use, or publishing use) is included? Yes
   Reference:
4. The context of use for the proposed characters (type of use; common or rare) Common
   Reference: mainly specialist usage, scholarly, worldwide
5. Are the proposed characters in current use by the user community? Yes
   If YES, where? Reference: mainly Europe, Americas; other countries
6. After giving due considerations to the principles in the P&P document must the proposed characters be entirely in the BMP? No
   If YES, is a rationale provided?
   If YES, reference:
7. Should the proposed characters be kept together in a contiguous range (rather than being scattered)? No
8. Can any of the proposed characters be considered a presentation form of an existing character or character sequence? No
   If YES, is a rationale for its inclusion provided?
   If YES, reference:
9. Can any of the proposed characters be encoded using a composed character sequence of either existing characters or other proposed characters? No
   If YES, is a rationale for its inclusion provided?
   If YES, reference:
10. Can any of the proposed character(s) be considered to be similar (in appearance or function) to, or could be confused with, an existing character? No
    If YES, is a rationale for its inclusion provided?
    If YES, reference:
11. Does the proposal include use of combining characters and/or use of composite sequences? Yes
    If YES, is a rationale for such use provided? *Yes*
    If YES, reference: *see explanations*
    Is a list of composite sequences and their corresponding glyph images (graphic symbols) provided? No
    If YES, reference:
12. Does the proposal contain characters with any special properties such as control function or similar semantics? No
    If YES, describe in detail (include attachment if necessary)
13. Does the proposal contain any Ideographic compatibility characters? No
    If YES, are the equivalent corresponding unified ideographic characters identified?
    If YES, reference: