Unification of Cuneiform Numbers

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Names for Number Signs

I have complimented Steve Tinney on the overall extreme clarity, systematicity and descriptive helpfulness of the names assigned to numbers in the web page he referred us to. He says they are based on the CDLI conventions, of which he was one of the architects, not the only one. If I had my druthers, even a bit more transparency could be added by systematizing the names for different arrangements / groupings of components of the signs, as for example these, which will handle the overwhelming majority of instances.
FOUR ASH LINEAR (all in one line)
FOUR ASH IN PAIRS (in square, two sets of two)
FOUR ASH IN TRIPLES (three-and-one)
EIGHT ASH IN PAIRS (2 and 2 and 2 and 2)
EIGHT ASH IN TRIPLES (3 and 3 and 2)

Unifying Number Signs across Times and Styles

I will be drawing analogies to how we have handled other cases of character identity which are similar to this one. The core question is whether a distinction which is obvious to all is best handled as a distinction of individual characters, or as a distinction in formatting or in some other dimension. I believe the evidence is clearly in favor of the latter. The traditional scholarship is unambiguously clear that the later forms are historically descended from the earlier, and preserved most functions intact. In *Archaic Bookkeeping*, by Hans Nissen, Peter Damerow, and Robert Englund, the authors show the equivalences in a small table with arrows indicating the transitions (p.140). Beyond that, there were a very few signs lost, and a very few added, but the core is the same between earlier and later. The older forms persisted into the UrIII period.

Below is an approximate table of such corresponding forms, with more detail. This shows which forms from different time periods would be unified with each other since they are lineal descendants (namely, all those in the same row, except when different arrangements are encoded distinctly, as for the leftmost two under ESH '30'.). The second line shows one substantial deviation from equivalence of function across time of the unified sets of shapes, namely a switch between the "ASH" family and the "DISH" family for certain uses (as noted in *Archaic Bookkeeping*). Column labels are not always completely accurate on the time period for the forms entered in those columns, but that does not affect unification.

Uruk archaic	Intermed. earlier	Intermed. later	"Classic"	Neo-Assyrian	sign name		Typical value	Fara (LAK)
					ASH (surface, capacity)	B001	1	820, 839, 855
					(sexagesimal system)		1	
					DISH (sexagesimal sys.)	B748	1 or 60	822 = 841
						B824 (v)	600	823 = 842
					U	B661	10	821, 840 {857 as '1'? BUR)}
					MAN	B708	20	
					ESH	B711	30	? 191
					NIMIN	B712	40	
					NINNU	B714	50	
					SHAR2	B632	3600	825 = 844 = 859 (& 809?)
					SHAR2 x U (sexag. sys.)	B653 B654	10 x 3600 = 36,000	826 = 846 = 860
					(surface measurement)	****	? value = 'Great Shar' 60 x 3600	827 = 861
					(surface measurement)	****		858; old lig. in 824, 845,

Fractions

Uruk archaic	Intermed. earlier	Intermed. later	"Classic"	Neo-Assyrian	sign name		Typical value	Fara (LAK)
					SHUSHANA	B826	fraction 1/3	817
					SHANABI	B832	fraction 2/3	818
					KINGUSILI	B573	fraction 5/6	819

Measures

Uruk archaic	Intermed. earlier	Intermed. later	"Classic"	Neo-Assyrian	sign name		Typical value	Fara (LAK)
					MASH	B120	1/2	816
					BAN2	B122	1 BAN (measure)	829
					BANMIN	B465	2 BAN (measure)	830
					BANESH	B549	3 BAN (measure)	831
					BANLIMMU	B550 (v)	4 BAN (measure)	832 & var't. w/ four horizontals
					BANIA	B551 (v)	5 BAN (measure)	833 & var't. w/ five horizontals
					2 BARIGA	****		835
					NIESH 3 BARIGA	B850		836
					LIMMU5 4 BARIGA	B852		837

It is absolutely the case that different styles of number can occur on the same tablet. In *Archaic Bookkeeping*, there are examples of this discussed at some length around pp.62-3 and pp.138-9. Those who have access to this book are encouraged to look there. There were also what we might think of as partial transitions, as in Gudea statues E and G, where later wedge forms were used for the units, but the earlier small round form was used for the decades instead of the later Winkelhaken. This Gudea usage does not in any way contradict unification of distinctions across time.

The View of Traditional Scholarship

The authors describe the process thus: "Although the core of third millennium arithmetic remained the same [as in the preceding period], its appearance exhibits considerable changes. In a long process which only concluded in the Ur III period at the end of the millennium, numerical signs, traditionally impressed into the clay with a round stylus, were increasingly replaced by cuneiform signs which imitated their form, but which were impressed with the same stylus as that used for ideograms."

and "For an extended period, both curvilinear and cuneiform numerical signs did appear together in a number of tablets, exploiting the apparent differences in graphic form to indicate specific entries, for example, those to be included in a subsequent summation."

To give a flavor from the example on page 62, I give the transcription here, indicating in angles the style of numeral. The non-numeral signs were not normally differentiated in this way.
<wedge shapes>
1 (U$) 1.5 (e$2) 5 gi, equal lengths
3 (e$2) 6 gi, equal widths
field area involved
<curved shapes>
1 (bur3) 1 (e$e3) 1.5 iku. It is ploughed.

This is an example of how the different styles of number sign could be used in different contexts to make a visual contrast. It is important to remember that the number signs (except some discussed below as using "textural" diacritics) do not normally carry in their individual signs the meaning of what they are used to measure. The contrast of styles can be used for different contrasts of content. A numerical expression in one style has an exact equivalent in another style.

Number Styles as Formatting Rather than as Distinct Characters

There is no question that both styles were used.
They are produced with different tools. One was older, and originally dominated exclusively. The other is newer, and later dominated exclusively. There was a period of overlap and use of both. But the arithmetical system remained the same in its core. And almost every one of the newer signs can be recognized as descending lineally from the older ones.
(There was some fluctuation in earlier periods in the relative numerical value of some signs, but that is not a question of identity of signs. See *Archaic Bookkeeping* p.61.)

Consider possible analogies in English, if we were to use contrast of styles of sans-serif vs. serif (or plain vs. bold, or even better plain vs. Fraktur styles) or "Arabic" vs. Roman numerals to signal differences.

<plain numbers>
a field 5 meters long by 25 meters wide
<Fraktur numbers>
yields an area of 125 square meters

or
<"Arabic" numbers>
a field 5 meters long by 25 meters wide
<Roman numbers>
yields an area of CXXV square meters.

It should be clear in the second example that Roman numerals have a structure very different from that of "Arabic" numerals,
and of course we have encoded them differently. On the other hand, the use of Fraktur numbers (assuming they are visually as different as Fraktur letters are from our usual Latin) does not imply a different set of encoded characters.

The crucial determinant in discussions of this kind of thing in the past has normally been whether the distinction being considered had its span limited to single characters, or rather normally spanned larger extents of text. The case before us is of the second kind, since the wedge linear and the curved number signs are not normally mixed together in a single numerical expression. However, it is slightly borderline, since the span which is covered by the distinction is not a long one. It is normally as many signs as there are significant places of precision in a particular numerical expression. Normally that can be three or four signs in a span. There is also an interesting occurrence on the Gudea Statues E and G where the sign U (10's or 6's) is in the older form, a small round depression rather than the later Winkelhaken angle form, while the units are represented by the later wedge shapes. In administrative texts of the same period, however, Sumerologist Wolfgang Heimpel has indicated that the styles are consistent.

I think this is closest to what we regard as formatting, rather than distinctions of individual characters. The CDLI names given to the characters also reflect this, adding the word "curved" to virtually the entire range of linear wedge signs to yield the names for the older style. If that analysis is correct, it will be best handled by using a different typographic style (implemented via choice of font for example). There are style contrasts needed for Cuneiform trump this, such as normal vs. monumental (usually rendered outline vs. solid, and perhaps best handled via distinct fonts). It would be a great pity to break the cultural connections and the linear descent of one set of signs from the other by encoding them as different characters, when there is no requirement for that.

Redistribution of Functions across Signs

There is one place where a systematic distinction of wedge orientation (ASH and multiples vs. DISH and multiples) means that only one of these two sets in later number signs can be unified with the earlier signs for the smallest units (see the arrows for the first two transitions in the small table *Archaic Bookkeeping* p.140). My previous table of equivalences did not point to this split. The number signs DISH and multiples of the later typography carry in part the same functions as did the intermediate-stage GESH2 and multiples (fatter wedges in that intermediate stage), and carry in part some specialized functions of the smaller curved unit signs ASH curved and multiples. But this small re-allocation of functions does not alter the lineal historical descent of the sign forms.

The Few Extras Needed for Uruk Style Numbers.

Because the UrIII period still includes the archaic style (and whether or not the curved and linear number signs are unified), there is only a tiny number of well-understood number characters to add to a standard in order to cover older numbers used at Uruk. These fall mostly into the following categories: A. the Bisexagesimal system; B. Texture-marked numbers; C. Ligatures of numbers with text signs; D. Additional Fractions, and E. Marks to signal measure of length vs. of width.

A. The Bisexagesimal system
We need to add three base signs and three with textural diacritics (using the ZATU numbering)
N-51 (120)
N-52 and N-53 (diacritic-marked versions of N-51),
N-54 (1200)
N-55 (diacritic-marked version of N-54)
N-56 (7200)
*

B. Texture-marked numbers ("diacritic-marked")
There are several number signs whose numerical significance is identical to their corresponding basic numbers, but which are marked with what we may call "textures" in addition. Only a single one of these textural markings ever occurs on a number sign, never more than one, and only numbers with the same textural markings occur together in number expressions (? this last statement ?). If there are contexts (as I think there are) where these markings are omitted but the thing counted is the same, then it might well be best to encode these as diacritics? Or as a styling? The most prominent of these I list here with names drawn from their usage as defined in *Archaic Bookkeeping* in the text and in charts pp.28-29, and other publications. If other names are already standard, we should use those instead.

B1. "MALT" MARK ? A single (diagonal) mark.
The SHE system $-prime. Capacity measures of a grain, probably germinated barley (malt).
Occurs in N-3, N-18, N-40, N-45', N24'

B2 "LINEAR TALLY" (probably not a good name here) not diagonal.
The Sexagesimal system S-prime.
Occurs in N-2, N-15, N-35 One of the Sexagesimal system.

B3. SPLIT LINEAR TALLY One stroke on each side, not diagonal.
Usage?
Occurs in N-16

B4. "EMMER" MARK ? two short strokes on each side of the base sign.
The SHE system $-doubleprime. Capacity measures of a certain grain, probably emmer.
Occurs in N-4, N-13, N-19, N-36, N-41, N-46, N-49
Perhaps in the bisexagesimal signs N-53, N-55

B5. "BARLEY GROATS" MARK? Used to signal something used in production of beer.
The SHE system $-asterisk. Capacty measures of grain, probably barley groats.
Occurs in N-5, N-20, N-25, N-27, N-28*, N-37, N-42, N-47

B6. Multiple slightly longer strokes on one side of the base sign.

The Bisexagesimal system B-asterisk. Counts rations of unclear nature, perhaps a type of fish.
Occurs in N-6, N-21, N-38, N-52, N-60 (in listing) = N-54 (in chart p.28)
Probably in the sign N-43.

Not listed above: N-17, N-44.

C. Ligatures of Numbers with Text Signs

In *Archaic Bookkeeping* there are two nice examples of the subtraction sign LAL with a
number nestled under it, on pages 62 (beginning of column 2) and on page 139 (second
illustration). It is shaped as an upside-down L, which can be diagrammed thus:

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 ____
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|
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Since this is productive with any number, it is almost certainly better to handle these as ligatures.
(But non-numerical signs with this same sign as Container are like other cuneiform or Han
character enclosures, non-productive, and should be encoded as a single character and named by
our general rule, as LAL x SIGN. For examples, please click here and scroll down a page.) There
are quite a quantity of other cuneiform signs which have a number infixed in some other sign.
When these are fully productive combinations, again I would suggest ligaturing not
separateencoding, although our encoding of all Container-x-Infix signs individually would seem
to suggest going the other way. A detailed list of these will have to be separate from this
message.

D. Fractions

Uruk has several fraction signs of types which do NOT continue into later signs, although many
fraction signs do continue throughout the time span.

These might or might not be encoded at this time. Their values and distinctiveness cannot in all
cases be determined with certainty, because there are simply not enough examples. (This
confirmed recently by Robert Englund, private communication.) But some of them we are
certain of, and one or two of those in the ranges noted here do descend to later signs.

N-7, value known as one sixteenth of N-1, see *Archaic Bookkeeping* page 28.
N-9 through N-12, value as fractions of N-7.
    But their values are not demonstrable, nor their distinctness from each other. *Archaic
Bookkeeping* page 28
N-23 through N-33. *Archaic Bookkeeping* page 29
    The values of N-31, N-32, N-33 and some under N-30 are not demonstrable in every case,
    nor their distinctness from each other..

E. Marks to Signal Measure of Length vs. Width

These are illustrated on the drawing of *Archaic Bookkeeping* page 57, for example. The first
column has a horizontal stroke to indicate "length", while the second column has a vertical stroke
to indicate "width". I think these marks should be treated as characters, and encoded as distinct
units of the writing system. In ZATU they are entered as if copies of ASH and DISH, but they
have no relation to those that I can detect. (Do they occur in any complex signs when their
original meaning is still present, so are demonstrably the same sign?)