L2/17-219R

Proposal to encode Ancient Chinese Mathematical Symbols

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1 Introduction

1.1 Counting Rods

Since the *Shang* period, the Chinese had already fully developed a decimal system. Since early times, Chinese understood basic arithmetic, algebra, equations, and negative numbers with counting rods, which were used for calculating. Some of them were added to UCS, but there is also something left. Both L2/04-227 and L2/17-187 have several mistakes, and after discussing we failed to reach a consensus. The amendments are list below.

1.2 Mathematical Symbols During *Ming* and *Qing* Dynasty

China continued to use vertical text for a long time, so when a lot of western mathematics books were translated verbally and introduced into China by western missionaries in *Ming* and *Qing* Dynasty, the horizontal text and formulas should be converted to vertical ones. These changes are mainly reflected in the following aspects:

- Mathematical Symbols. Some of them were reformed.
- Names of unknowns. The Latin letters and the Greek letters would be translated into Heavenly Stems, Earthly Branches, Twenty-Eight Mansions or Sixty-Four Hexagrams.
- Equations. The simple equations would be rotated, and the complex equations would be written horizontally, where the numerator is below the denominator. Sometimes the column was as long as the equation, but sometimes an equation could cross several columns.

1.3 New Block - Ancient Chinese Mathematical Symbols

Ancient Chinese mathematical symbols are self-contained, and they should be collect in a special block. The new block, Ancient Chinese Mathematical Symbols, will be set from U+1EF00 to U+1EF3F. If other symbols found in ancient Chinese books, they will be put in this block.

2 Character Information

2.1 Counting Rods

First, the values in the Character properties are obviously wrong. The writer of those two proposals think that “the current name reflects the fact that the digits have different value … it’s just that they take different values depending on their position”. This is a common misconception. In fact, the value has no relation with its position. For example, “𝍠” in Fig.7 mean “4”, “𝍡𝍠𝍢” in Fig.7 means “945”, “𝍠𝍧𝍨” in Fig.7 means “476”, “𝍠𝍧〇𝍡” in Fig.1 means “4608”. In these cases, the value of “𝍠” should be “4” instead of “40”, “400”, “4000” …

Therefore, the value 10 to 90 should be corrected as 1 to 9.

Second, from the description above, the use of vertical or horizontal counting rods has no limit. People would use them interchangeably to make the structure compact, but the specific digits to put vertical or horizontal counting rods were not stipulated. Many mathematical books in the series listed below can prove it.

Therefore, the character names of “UNIT” and “TENS” should be corrected as “HORIZONTAL” and “VERTICAL”.

Third, Suzhou numerals appears much later than the counting rods. So, all the Suzhou numerals should be directed to Counting Rods instead of the opposite.

Suzhou Numerals	𝍩	𝍪	𝍫	𝍬	𝍭	𝍮	𝍯	𝍰	𝍱
Counting Rods	𝍠	𝍡	𝍢	𝍣	𝍤	𝍥	𝍦	𝍧	𝍨

2.2 Combining Diacritical Marks

[◌́ , ◌̋ , ◌̏] Like “point *A*′”, “point *A*″” in western symbol system, ancient Chinese books used some left-falling strokes (撇). One to three left-falling strokes were found in *Qiuyishu Tongjie* (求一術通解).

[◌⃞] In the process of copying and printing, mistakes were made occasionally. Later when people corrected the content of books, the wrong words would be marked by rectangles with cut corners. Lots of examples were found in *Siyuan Yujian* (四元玉鑑).

2.3 Enclosed Ideographic

To give numbers to the order of formulas, the ideographic enclosed by circles and squares were used.

The former can be found in *Pingsanjiao Bianjiao Huqiushu* (平三角邊角互求術), *Weiji Suyuan* (微積溯源), *Qiuyishu Tongjie* (求一術通解), *Duishu Xiangjie* (對數詳解), *Daishushu* (代數術) and *Zhusuan* (珠算).

The latter can be found in *Yuanlü Kaozhen Tujie* (圓率攷眞圖解), *Daishu Beizhi* (代數備旨), *Subu Yancao* (粟布演草),

Duishu Xiangjie (對數詳解) and *Siyuan Yujian Xicao* (四元玉鑑細草).

2.4 Ancient Chinese Mathematical Symbols

[⊥, ⊤, ∓] Found in *Daiweiji Sheji* (代微積拾級). ⊥ means "plus" (+), ⊤ means "minus" (-), and ∓ means "plus or minus" (±).

[⩊, ⩏] Found in *Xingxue Beizhi* (形學備旨) and *Daishu Beizhi* (代數備旨). ⩊ is the variant of ⊥, and ⩏ is the variant of ⊤.

[ᖶ] Found in *Xuesuan Bitan* (學算筆談). ᖶ means "find positive difference". For example, 5 ᖶ 3 = 3 ᖶ 5 = 2.

[⍈] Found in *Xuesuan Bitan* (學算筆談). ⍈ means "factorial" (!).

[⦃, ⦄] Found in *Duishu Xiangjie* (對數詳解). They are the variants of "{" and "}".

[⋯] Found in *Duishu Xiangjie* (對數詳解). ⋯ means "et cetera" (……). The explanatory notes of it enumerated this symbol particularly, for it has not been used before.

[ⅆ, ∱] Found in *Weiji Suyuan* (微積溯源). ⅆ means "differential" (d), and ∱ means "integral" (∫). Considering that ∱ and ⅆ would grow much larger than surrounding letters and be used symbolically (Fig.28 to Fig.30), they should be encoded separately as mathematical operators and cannot be unified with the Kangxi radicals 彳 and 禾.

Some of the text examples reflect complex notation, which will need higher level representation. Here are the example.

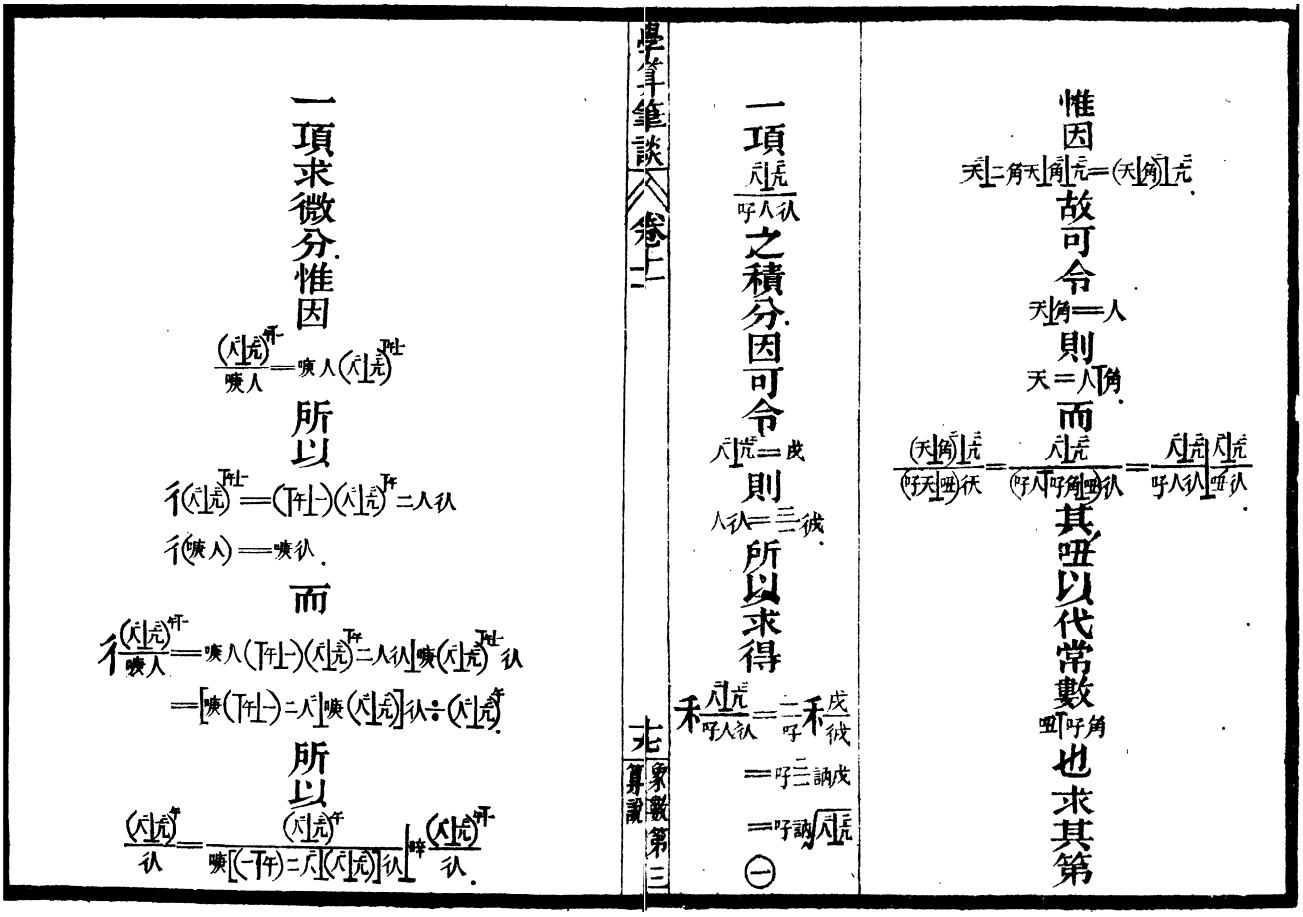

Figure 29. Example from Hua Hengfang's (華蘅芳) book *Xuesuan Bitan* (學算筆談) vol.11 p.19.

惟因

$$天^{二}⊥二角天⊥角^{二}⊥亢^{二}=(天⊥角)^{二}⊥亢^{二}.$$

故可令 $天⊥角=人$ 則 $天=人⊤角$. 而

$$\frac{(天⊥角)^{二}⊥亢^{二}}{(呼天⊥咀)彳天}=\frac{人^{二}⊥亢^{二}}{(呼人⊤呼角⊥咀)彳人}=\frac{人^{二}⊥亢^{二}}{呼人彳人}⊥\frac{人^{二}⊥亢^{二}}{\dot{咀}彳人}$$

其 $\dot{咀}$ 以代常數 $咀⊤呼角$ 也. 求其第一項 $\frac{人^{二}⊥亢^{二}}{呼人彳人}$ 之積分. 因可令 $人^{二}⊥亢^{二}=戌$ 則 $人彳人=\frac{二}{二}彳戌$

所以求得

$$禾\frac{人^{二}⊥亢^{二}}{呼人彳人}=\frac{一}{呼}禾\frac{戌}{彳戌}=呼\frac{二}{二}訥戌=呼訥\sqrt{人^{二}⊥亢^{二}} \qquad ㈠$$

一項求微分. 惟因

$$\frac{(人^{二}⊥亢^{二})^{午}⊤一}{嗅人}=嗅人(人^{二}⊥亢^{二})^{⊤午}⊥一$$

所以

$$彳(人^{二}⊥亢^{二})^{⊤午}⊥一=(⊤午⊥一)(人^{二}⊥亢^{二})^{⊤午}二人彳人，\ 彳(嗅人)=嗅彳人.$$

而

$$彳\frac{(人^{二}⊥亢^{二})^{午}⊤一}{嗅人}=嗅人(⊤午⊥一)(人^{二}⊥亢^{二})^{⊤午}二人彳人⊥嗅(人^{二}⊥亢^{二})^{⊤午}⊥一彳人$$

$$=[嗅(⊤午⊥一)二人^{二}⊥嗅(人^{二}⊥亢^{二})]彳人÷(人^{二}⊥亢^{二})^{午}.$$

所以

$$\frac{(人^{二}⊥亢^{二})^{午}}{彳人}=\frac{(人^{二}⊥亢^{二})^{午}}{嗅[(⊤午⊥一)二人^{二}⊥(人^{二}⊥亢^{二})]彳人}⊥啐\frac{(人^{二}⊥亢^{二})^{午}⊤一}{彳人}.$$

3. Character Data

3.1 Character position and name

Combining Diacritical Marks Extended (U+1AB0 - U+1AFF)

◌ʻ	1AC0	COMBINING CHINESE PRIME
◌ʻʻ	1AC1	COMBINING CHINESE DOUBLE PRIME
◌ʻʻʻ	1AC2	COMBINING CHINESE TRIPLE PRIME

Combining Diacritical Marks for Symbols (U+20D0 - U+20FF)

◌⃱	20F1	COMBINING ENCLOSING CUT-CORNER RECTANGLE

Ancient Chinese Mathematical Symbols (U+1EF00 - U+1EF3F)

	1EF00	CHINESE PLUS SIGN
	1EF01	CHINESE MINUS SIGN
	1EF02	CHINESE PLUS-OR-MINUS SIGN
	1EF03	CHINESE VARIANT PLUS SIGN
	1EF04	CHINESE VARIANT MINUS SIGN
	1EF05	CHINESE POSITIVE DIFFERENCE SIGN
	1EF06	CHINESE FACTORIAL SIGN
	1EF07	CHINESE LEFT CURLY BRACKET
	1EF08	CHINESE RIGHT CURLY BRACKET
	1EF09	CHINESE ELLIPSIS
	1EF0A	CHINESE DIFFERENTIAL SIGN
	1EF0B	CHINESE INTEGRAL SIGN

Enclosed Ideographic Supplement (U+1F200 - U+1F2FF)

	1F23C	SQUARED CJK UNIFIED IDEOGRAPH-554F
	1F23D	SQUARED CJK UNIFIED IDEOGRAPH-9700
	1F23E	SQUARED CJK UNIFIED IDEOGRAPH-8A1F
	1F23F	SQUARED CJK UNIFIED IDEOGRAPH-5E2B
	1F252	CIRCLED CJK UNIFIED IDEOGRAPH-7532
	1F253	CIRCLED CJK UNIFIED IDEOGRAPH-4E59
	1F254	CIRCLED CJK UNIFIED IDEOGRAPH-4E19
	1F255	CIRCLED CJK UNIFIED IDEOGRAPH-4E01
	1F256	CIRCLED CJK UNIFIED IDEOGRAPH-620A

🈑	1F257	CIRCLED CJK UNIFIED IDEOGRAPH-5DF1
🈒	1F258	CIRCLED CJK UNIFIED IDEOGRAPH-5E9A
🈓	1F259	CIRCLED CJK UNIFIED IDEOGRAPH-8F9B
🈔	1F25C	CIRCLED CJK UNIFIED IDEOGRAPH-89D2
🈕	1F25D	CIRCLED CJK UNIFIED IDEOGRAPH-4EA2
🈖	1F25E	CIRCLED CJK UNIFIED IDEOGRAPH-6C10
🈗	1F25F	CIRCLED CJK UNIFIED IDEOGRAPH-623F
🈘	1F260	CIRCLED CJK UNIFIED IDEOGRAPH-5341 4E00
🈙	1F261	CIRCLED CJK UNIFIED IDEOGRAPH-5341 4E8C
🈚	1F262	CIRCLED CJK UNIFIED IDEOGRAPH-5341 4E09
🈛	1F263	CIRCLED CJK UNIFIED IDEOGRAPH-5341 56DB
🈜	1F264	CIRCLED CJK UNIFIED IDEOGRAPH-5341 4E94
🈝	1F265	CIRCLED CJK UNIFIED IDEOGRAPH-5341 516D
🈞	1F266	CIRCLED CJK UNIFIED IDEOGRAPH-5341 4E03
🈟	1F267	CIRCLED CJK UNIFIED IDEOGRAPH-5341 516B
🈠	1F268	CIRCLED CJK UNIFIED IDEOGRAPH-5341 4E5D
🈡	1F269	CIRCLED CJK UNIFIED IDEOGRAPH-58F9
🈢	1F26A	CIRCLED CJK UNIFIED IDEOGRAPH-8CB3
🈣	1F26B	CIRCLED CJK UNIFIED IDEOGRAPH-53C1
🈤	1F26C	CIRCLED CJK UNIFIED IDEOGRAPH-5343
🈥	1F26D	CIRCLED CJK UNIFIED IDEOGRAPH-5929
🈦	1F26E	CIRCLED CJK UNIFIED IDEOGRAPH-5730
🈧	1F26F	CIRCLED CJK UNIFIED IDEOGRAPH-4EBA

🈀	1F270	CIRCLED CJK UNIFIED IDEOGRAPH-5B50
🈁	1F271	CIRCLED CJK UNIFIED IDEOGRAPH-4E11
🈂	1F272	CIRCLED CJK UNIFIED IDEOGRAPH-5BC5
🈃	1F273	CIRCLED CJK UNIFIED IDEOGRAPH-536F
🈄	1F274	CIRCLED CJK UNIFIED IDEOGRAPH-8FB0
🈅	1F275	CIRCLED CJK UNIFIED IDEOGRAPH-5DF3
🈆	1F276	CIRCLED CJK UNIFIED IDEOGRAPH-5348
🈇	1F277	CIRCLED CJK UNIFIED IDEOGRAPH-672A
🈈	1F278	CIRCLED CJK UNIFIED IDEOGRAPH-7533
🈉	1F279	CIRCLED CJK UNIFIED IDEOGRAPH-9149
🈊	1F27B	CIRCLED CJK UNIFIED IDEOGRAPH-4EA5
🈋	1F27C	CIRCLED CJK UNIFIED IDEOGRAPH-8853
🈌	1F27D	CIRCLED CJK UNIFIED IDEOGRAPH-53C8
🈍	1F280	CIRCLED CJK UNIFIED IDEOGRAPH-5477
🈎	1F281	CIRCLED CJK UNIFIED IDEOGRAPH-20B99
🈏	1F282	CIRCLED CJK UNIFIED IDEOGRAPH-20C33
🈐	1F283	CIRCLED CJK UNIFIED IDEOGRAPH-53EE
🈑	1F284	CIRCLED CJK UNIFIED IDEOGRAPH-NNNNN
🈒	1F285	CIRCLED CJK UNIFIED IDEOGRAPH-NNNNN
🈓	1F286	CIRCLED CJK UNIFIED IDEOGRAPH-NNNNN
🈔	1F287	CIRCLED CJK UNIFIED IDEOGRAPH-3595
🈕	1F28A	CIRCLED CJK UNIFIED IDEOGRAPH-4E8C 7533
🈖	1F28B	CIRCLED CJK UNIFIED IDEOGRAPH-4E8C 9149

	1F28C	CIRCLED CJK UNIFIED IDEOGRAPH-4E8C 4EA5
	1F28D	CIRCLED CJK UNIFIED IDEOGRAPH-4E8C 5929
	1F28E	CIRCLED CJK UNIFIED IDEOGRAPH-7532 4E8C
	1F28F	CIRCLED CJK UNIFIED IDEOGRAPH-4E59 4E8C
	1F290	CIRCLED CJK UNIFIED IDEOGRAPH-4E19 4E8C
	1F291	CIRCLED CJK UNIFIED IDEOGRAPH-4E01 4E8C
	1F292	CIRCLED CJK UNIFIED IDEOGRAPH-620A 4E8C
	1F293	CIRCLED CJK UNIFIED IDEOGRAPH-5DF3 4E8C
	1F294	CIRCLED CJK UNIFIED IDEOGRAPH-5E9A 4E8C
	1F295	CIRCLED CJK UNIFIED IDEOGRAPH-8F9B 4E8C
	1F296	CIRCLED CJK UNIFIED IDEOGRAPH-5B50 4E8C
	1F297	CIRCLED CJK UNIFIED IDEOGRAPH-4E11 4E8C
	1F298	CIRCLED CJK UNIFIED IDEOGRAPH-5BC5 4E8C
	1F299	CIRCLED CJK UNIFIED IDEOGRAPH-536F 4E8C
	1F2A0	SQUARED IDEOGRAPHIC NUMBER ZERO
	1F2A1	SQUARED CJK UNIFIED IDEOGRAPH-56DB
	1F2A2	SQUARED CJK UNIFIED IDEOGRAPH-4E94
	1F2A3	SQUARED CJK UNIFIED IDEOGRAPH-516D
	1F2A4	SQUARED CJK UNIFIED IDEOGRAPH-4E03
	1F2A5	SQUARED CJK UNIFIED IDEOGRAPH-516B
	1F2A6	SQUARED CJK UNIFIED IDEOGRAPH-4E5D
	1F2A7	SQUARED CJK UNIFIED IDEOGRAPH-5341
	1F2A9	SQUARED CJK UNIFIED IDEOGRAPH-5341 4E8C

🈻	1F2AB	SQUARED CJK UNIFIED IDEOGRAPH-5341 56DB
🈽	1F2AD	SQUARED CJK UNIFIED IDEOGRAPH-5341 516D
🈾	1F2AE	SQUARED CJK UNIFIED IDEOGRAPH-5341 4E03
🈿	1F2AF	SQUARED CJK UNIFIED IDEOGRAPH-4E7E
🉀	1F2B0	SQUARED CJK UNIFIED IDEOGRAPH-5426
🉁	1F2B1	SQUARED CJK UNIFIED IDEOGRAPH-5C65
🉂	1F2B2	SQUARED CJK UNIFIED IDEOGRAPH-5C6F
🉃	1F2B3	SQUARED CJK UNIFIED IDEOGRAPH-6B65
🉄	1F2B4	SQUARED CJK UNIFIED IDEOGRAPH-6BD4
🉅	1F2B5	SQUARED CJK UNIFIED IDEOGRAPH-6CF0
🉆	1F2B6	SQUARED CJK UNIFIED IDEOGRAPH-4749
🉇	1F2B7	SQUARED CJK UNIFIED IDEOGRAPH-96A8

㊂ is different from it in the evidence figures, because there were mistakes in books in the process of transcribing.

According to the Earthly Branches order, the correct character should be “己”.

The “NNNNN” in the names of ㊀㊁㊃ means that the characters in the circle have not been included in the UCS yet:

- “□口己” was in Extension B. VNPF decided to change it into “□口巳” in IRGN2240, though Eiso Chan mentioned that the original Unihan property information for it can be moved to the new code point in the feedback.
- “□口戊”(UK-10022) and “□口庚”(UK-10023) have been included in WS 2017.

Blank codes are set to be filled in the future.

2.2. Character properties

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1AC0;COMBINING CHINESE PRIME;Mn;230;NSM;;;;;N;;;;;
1AC1;COMBINING CHINESE DOUBLE PRIME;Mn;230;NSM;;;;;N;;;;;
1AC2;COMBINING CHIENSE TRIPLE PRIME;Mn;230;NSM;;;;;N;;;;;
20F1;COMBINING ENCLOSING CUT-CORNER RECTANGLE;Me;0;NSM;;;;;N;;;;;
1EF00;CHINESE PLUS SIGN;Sm;0;L;;;;;N;;;;;
1EF01;CHINESE MINUS SIGN;Sm;0;L;;;;;N;;;;;
1EF02;CHINESE PLUS-OR-MINUS SIGN;Sm;0;L;;;;;N;;;;;
1EF03;CHINESE VARIANT PLUS SIGN;Sm;0;L;;;;;N;;;;;
1EF04;CHINESE VARIANT MINUS SIGN;Sm;0;L;;;;;N;;;;;
1EF05;CHINESE POSITIVE DIFFERENCE SIGN;Sm;0;L;;;;;N;;;;;
1EF06;CHINESE FACTORIAL SIGN;Mn;230;NSM;;;;;N;;;;;
1EF07;CHINESE LEFT CURLY BRACKET;Sm;0;L;;;;;N;;;;;
1EF08;CHINESE RIGHT CURLY BRACKET;Sm;0;L;;;;;N;;;;;
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1EF09;CHINESE ELLIPSIS;Sm;0;L;;;;;N;;;;;
1EF0A;CHINESE DIFFERENTIAL SIGN;Sm;0;L;;;;;N;;;;;
1EF0B;CHINESE INTEGRAL SIGN;Sm;0;L;;;;;N;;;;;
1F23C;SQUARED CJK UNIFIED IDEOGRAPH-554F;So;0;L;<square> 554F;;;;N;;;;;
1F23D;SQUARED CJK UNIFIED IDEOGRAPH-9700;So;0;L;<square> 9700;;;;N;;;;;
1F23E;SQUARED CJK UNIFIED IDEOGRAPH-8A1F;So;0;L;<square> 8A1F;;;;N;;;;;
1F23F;SQUARED CJK UNIFIED IDEOGRAPH-5E2B;So;0;L;<square> 5E2B;;;;N;;;;;
1F252;CIRCLED CJK UNIFIED IDEOGRAPH-7532;So;0;L;<circle> 7532;;;;N;;;;;
1F253;CIRCLED CJK UNIFIED IDEOGRAPH-4E59;So;0;L;<circle> 4E59;;;;N;;;;;
1F254;CIRCLED CJK UNIFIED IDEOGRAPH-4E19;So;0;L;<circle> 4E19;;;;N;;;;;
1F255;CIRCLED CJK UNIFIED IDEOGRAPH-4E01;So;0;L;<circle> 4E01;;;;N;;;;;
1F256;CIRCLED CJK UNIFIED IDEOGRAPH-620A;So;0;L;<circle> 620A;;;;N;;;;;
1F257;CIRCLED CJK UNIFIED IDEOGRAPH-5DF1;So;0;L;<circle> 5DF1;;;;N;;;;;
1F258;CIRCLED CJK UNIFIED IDEOGRAPH-5E9A;So;0;L;<circle> 5E9A;;;;N;;;;;
1F259;CIRCLED CJK UNIFIED IDEOGRAPH-8F9B;So;0;L;<circle> 8F9B;;;;N;;;;;
1F25C;CIRCLED CJK UNIFIED IDEOGRAPH-89D2;So;0;L;<circle> 89D2;;;;N;;;;;
1F25D;CIRCLED CJK UNIFIED IDEOGRAPH-4EA2;So;0;L;<circle> 4EA2;;;;N;;;;;
1F25E;CIRCLED CJK UNIFIED IDEOGRAPH-6C10;So;0;L;<circle> 6C10;;;;N;;;;;
1F25F;CIRCLED CJK UNIFIED IDEOGRAPH-623F;So;0;L;<circle> 623F;;;;N;;;;;
1F260;CIRCLED CJK UNIFIED IDEOGRAPH-5341 4E00;So;0;L;<circle> 5341 4E00;;;;N;;;;;
1F261;CIRCLED CJK UNIFIED IDEOGRAPH-5341 4E8C;So;0;L;<circle> 5341 4E8C;;;;N;;;;;
1F262;CIRCLED CJK UNIFIED IDEOGRAPH-5341 4E09;So;0;L;<circle> 5341 4E09;;;;N;;;;;
1F263;CIRCLED CJK UNIFIED IDEOGRAPH-5341 56DB;So;0;L;<circle> 5341 56DB;;;;N;;;;;
1F264;CIRCLED CJK UNIFIED IDEOGRAPH-5341 4E94;So;0;L;<circle> 5341 4E94;;;;N;;;;;
1F265;CIRCLED CJK UNIFIED IDEOGRAPH-5341 516D;So;0;L;<circle> 5341 516D;;;;N;;;;;
1F266;CIRCLED CJK UNIFIED IDEOGRAPH-5341 4E03;So;0;L;<circle> 5341 4E03;;;;N;;;;;
1F267;CIRCLED CJK UNIFIED IDEOGRAPH-5341 516B;So;0;L;<circle> 5341 516B;;;;N;;;;;
1F268;CIRCLED CJK UNIFIED IDEOGRAPH-5341 4E5D;So;0;L;<circle> 5341 4E5D;;;;N;;;;;
1F269;CIRCLED CJK UNIFIED IDEOGRAPH-58F9;So;0;L;<circle> 58F9;;;;N;;;;;
1F26A;CIRCLED CJK UNIFIED IDEOGRAPH-8CB3;So;0;L;<circle> 8CB3;;;;N;;;;;
1F26B;CIRCLED CJK UNIFIED IDEOGRAPH-53C1;So;0;L;<circle> 53C1;;;;N;;;;;
1F26C;CIRCLED CJK UNIFIED IDEOGRAPH-5343;So;0;L;<circle> 5343;;;;N;;;;;
1F26D;CIRCLED CJK UNIFIED IDEOGRAPH-5929;So;0;L;<circle> 5929;;;;N;;;;;
1F26E;CIRCLED CJK UNIFIED IDEOGRAPH-5730;So;0;L;<circle> 5730;;;;N;;;;;
1F26F;CIRCLED CJK UNIFIED IDEOGRAPH-4EBA;So;0;L;<circle> 4EBA;;;;N;;;;;
1F270;CIRCLED CJK UNIFIED IDEOGRAPH-5B50;So;0;L;<circle> 5B50;;;;N;;;;;
1F271;CIRCLED CJK UNIFIED IDEOGRAPH-4E11;So;0;L;<circle> 4E11;;;;N;;;;;
1F272;CIRCLED CJK UNIFIED IDEOGRAPH-5BC5;So;0;L;<circle> 5BC5;;;;N;;;;;
1F273;CIRCLED CJK UNIFIED IDEOGRAPH-536F;So;0;L;<circle> 536F;;;;N;;;;;
1F274;CIRCLED CJK UNIFIED IDEOGRAPH-8FB0;So;0;L;<circle> 8FB0;;;;N;;;;;
1F275;CIRCLED CJK UNIFIED IDEOGRAPH-5DF3;So;0;L;<circle> 5DF3;;;;N;;;;;
1F276;CIRCLED CJK UNIFIED IDEOGRAPH-5348;So;0;L;<circle> 5348;;;;N;;;;;
1F277;CIRCLED CJK UNIFIED IDEOGRAPH-672A;So;0;L;<circle> 672A;;;;N;;;;;
1F278;CIRCLED CJK UNIFIED IDEOGRAPH-7533;So;0;L;<circle> 7533;;;;N;;;;;
1F279;CIRCLED CJK UNIFIED IDEOGRAPH-9149;So;0;L;<circle> 9149;;;;N;;;;;
1F27B;CIRCLED CJK UNIFIED IDEOGRAPH-4EA5;So;0;L;<circle> 4EA5;;;;N;;;;;
1F27C;CIRCLED CJK UNIFIED IDEOGRAPH-8853;So;0;L;<circle> 8853;;;;N;;;;;
1F27D;CIRCLED CJK UNIFIED IDEOGRAPH-53C8;So;0;L;<circle> 53C8;;;;N;;;;;
1F280;CIRCLED CJK UNIFIED IDEOGRAPH-5477;So;0;L;<circle> 5477;;;;N;;;;;
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Script: Common
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3. References

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[第一百十四款，補劉彝程、沈善蒸、崔朝慶算式]

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華蘅芳. 學算筆談 (十二卷). 原刻本.

…… (There are also 75 books collected in the series but not put to use for examples)

4. Acknowledgement

This proposal is improved with great help of Eduardo Marin, Andrew West and Eiso Chan. The glyphs of the symbols are modified from Symbola and Sursong.

5. Figures

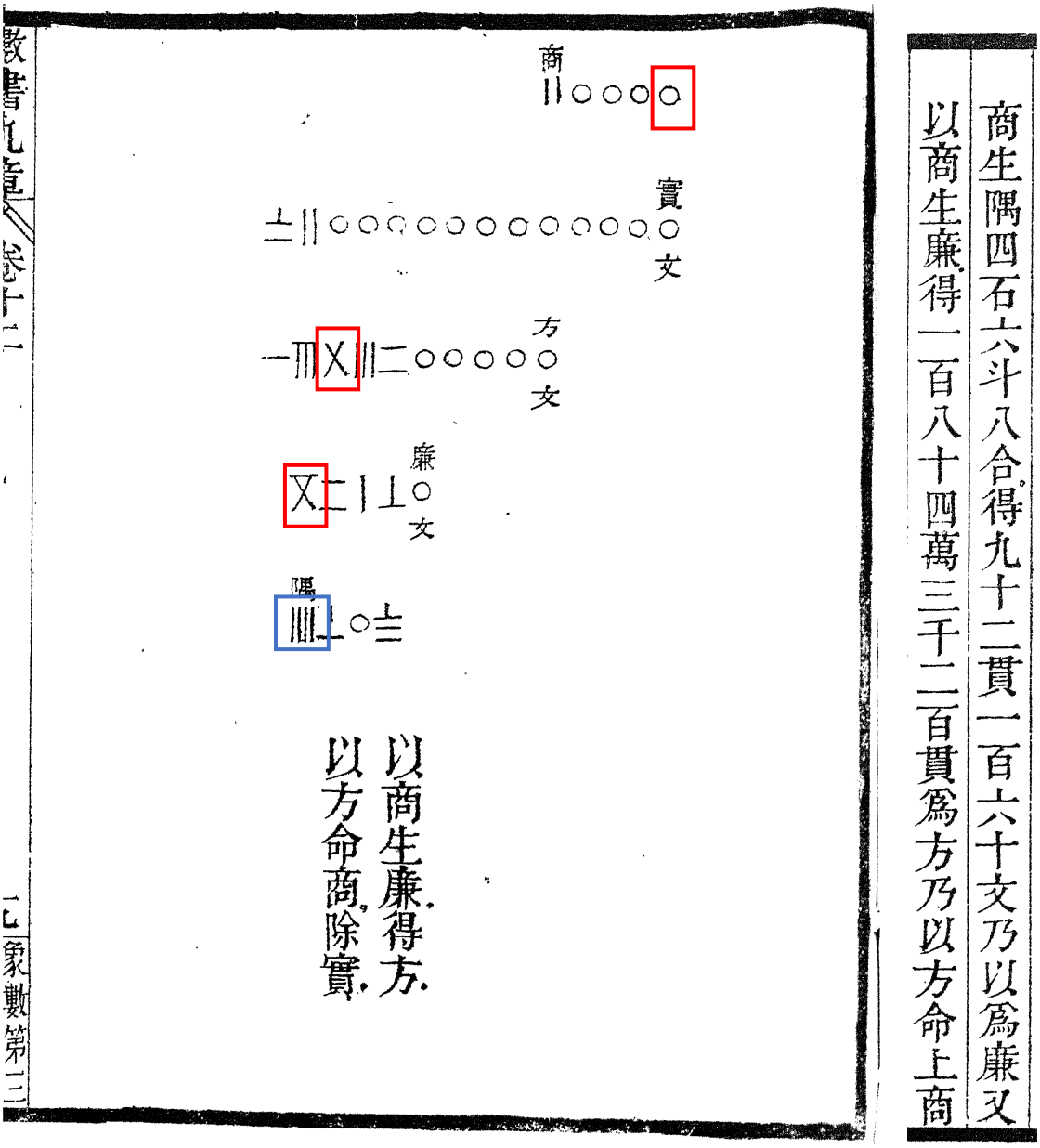


Figure 1. Example from Qin Jiushao's (秦九韶) book *Shushu Jiuzhang* (數書九章) vol.12 p.19 and p.24 (Shanghai: Arithmetic Publishing House (算學書局), 1898).

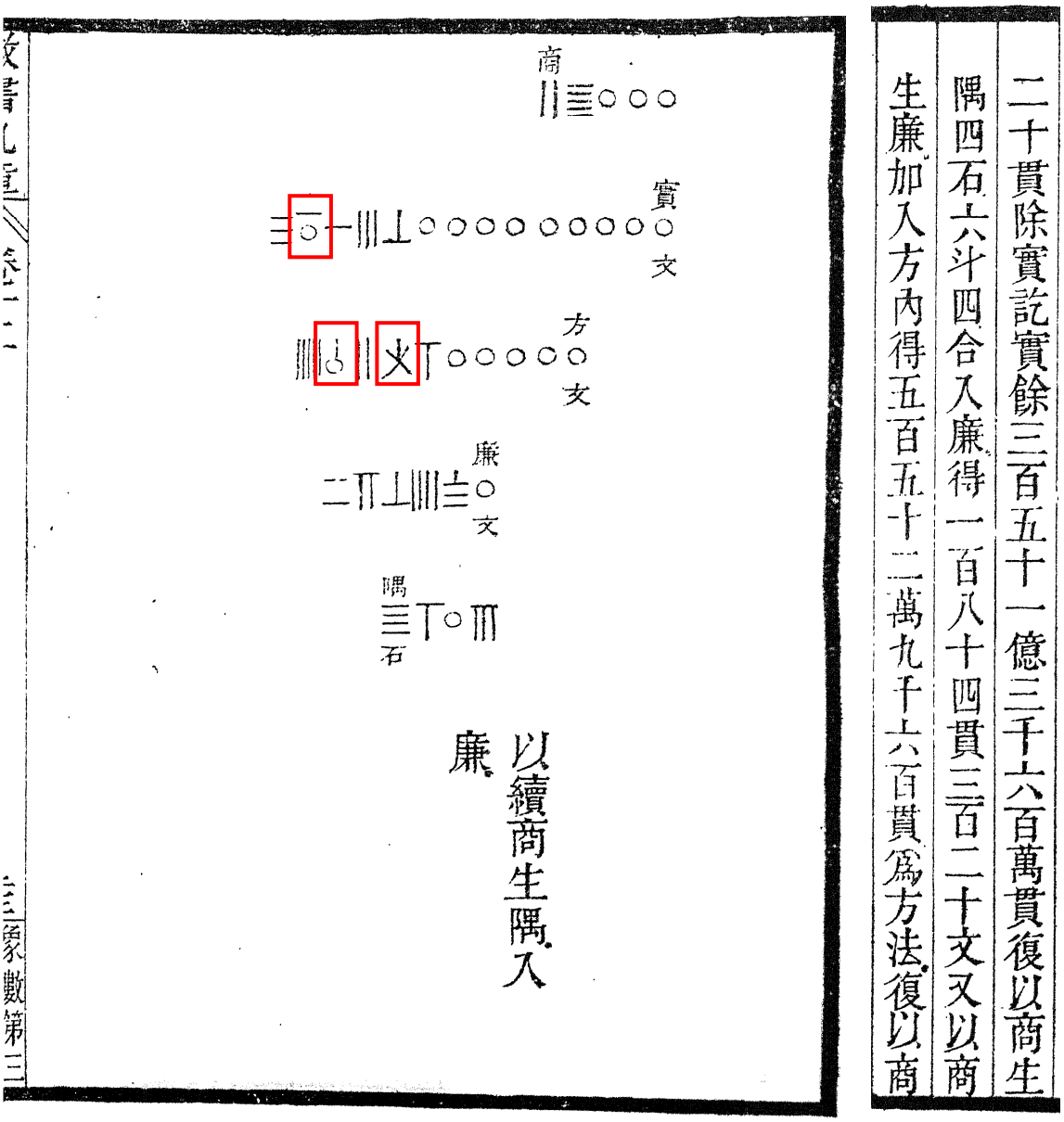


Figure 2. Example from Qin Jiushao's (秦九韶) book *Shushu Jiuzhang* (數書九章) vol.12 p.22 and p.24 (Shanghai: Arithmetic Publishing House (算學書局), 1898).

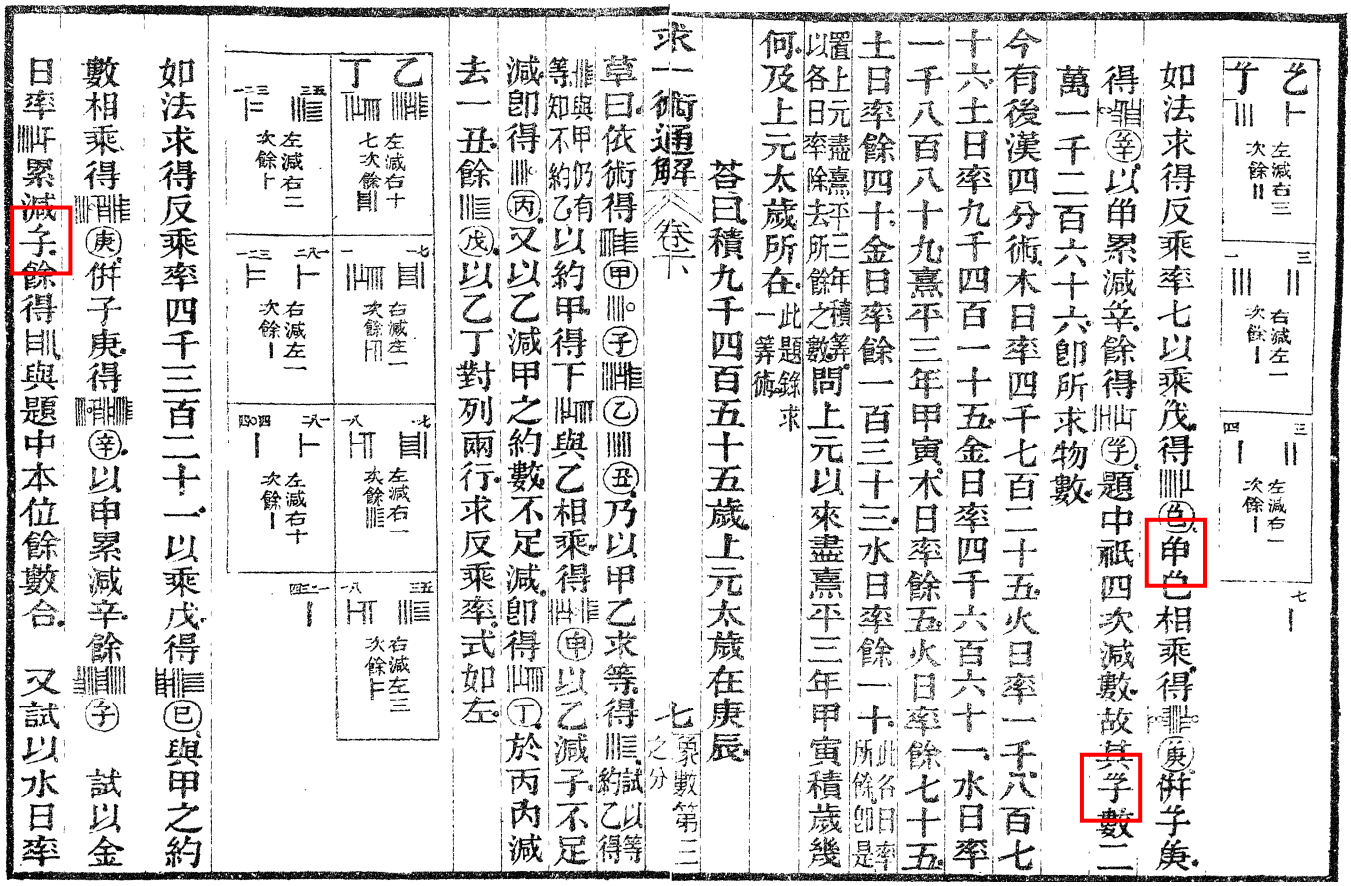


Figure 3. Example from Huang Zongxian's (黃宗憲) book *Qiuyishu Tongjie* (求一術通解) vol.2 p.7 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing ◌́, ◌̋ and ◌̋ in text.

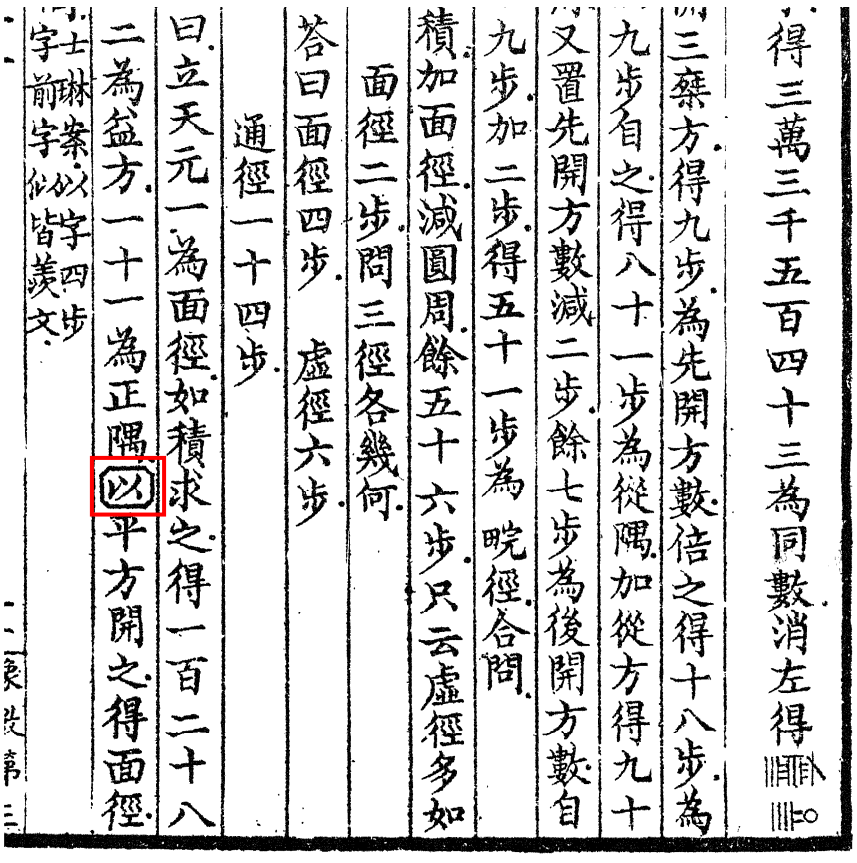


Figure 4. Example from Zhu Shijie's (朱世傑) book *Siyuan Yujian* (四元玉鑑) vol.1.2 p.11 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing ◎ in text.

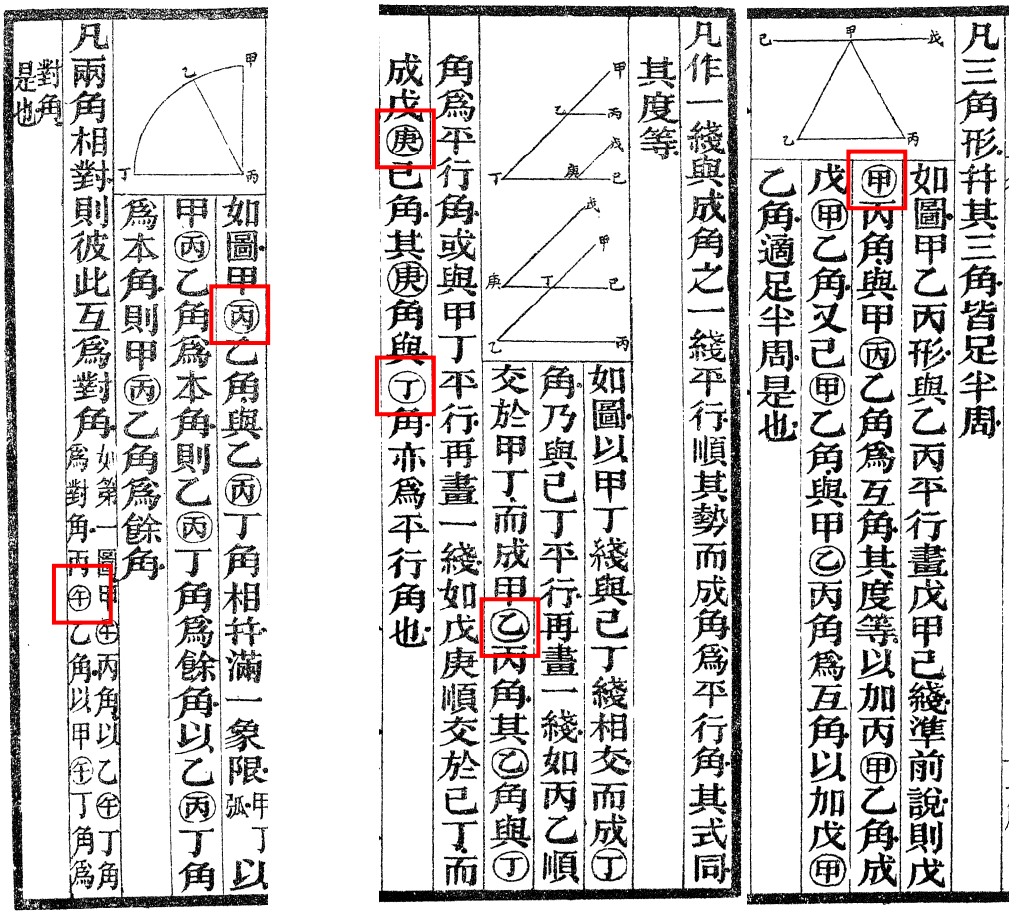


Figure 5. Example from Wu Jiashan's (吳嘉善) book *Pingsanjiao Bianjiao Huqiushu* (平三角邊角互求術) p.1 and p.2 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing ㊥, 乙, ㊦, 丁, ㊧ and ㊌ in text.

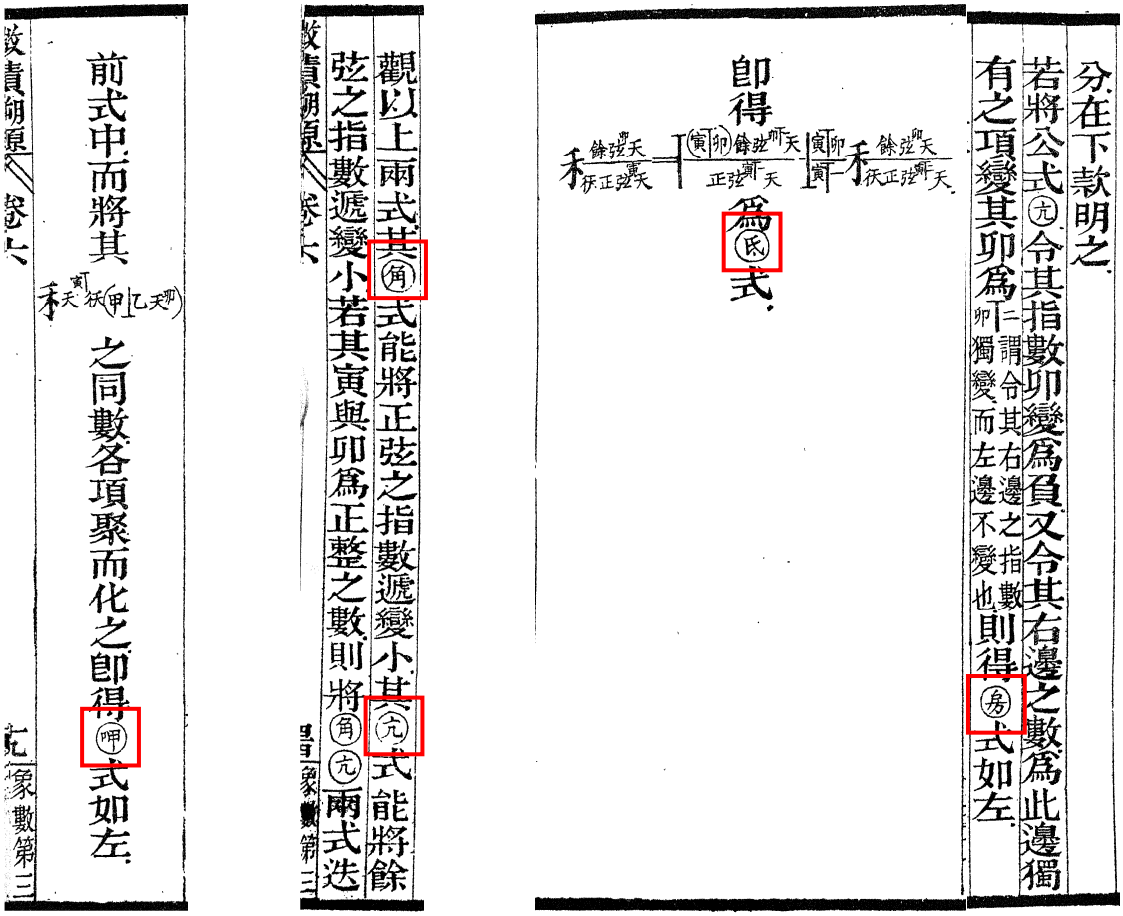


Figure 6. Example from Hua Hengfang's (華蘅芳) book *Weiji Suyuan* (微積溯源) vol.6 p.19, p.44 and p.45 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing ㊎, 角, ㊆, ㊍ and ㊏ in text.

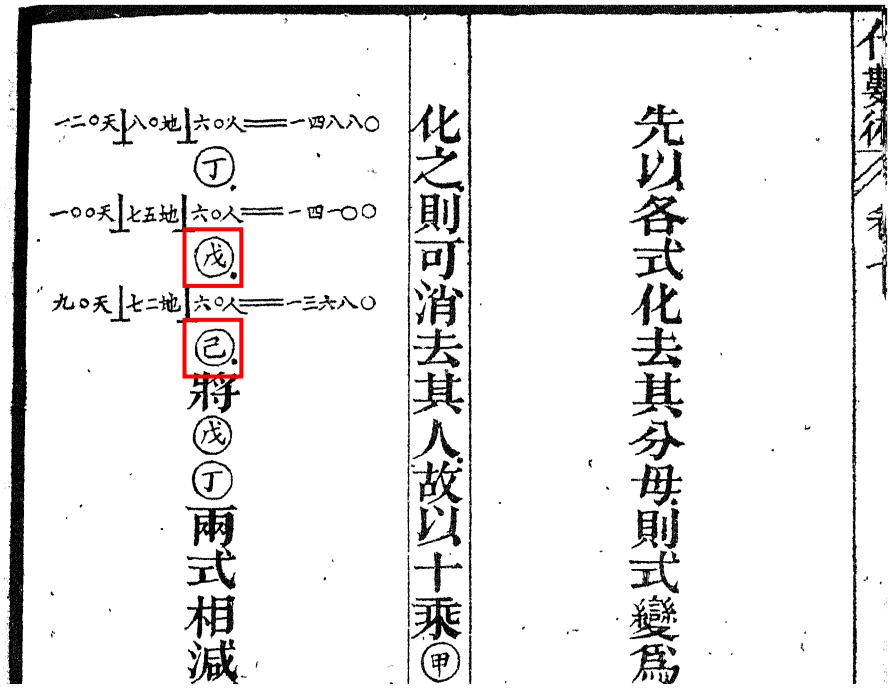


Figure 7. Example from Hua Hengfang's (華蘅芳) book *Daishushu* (代數術) vol.7 p.7 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing ㊇ and ㊈ in text.*

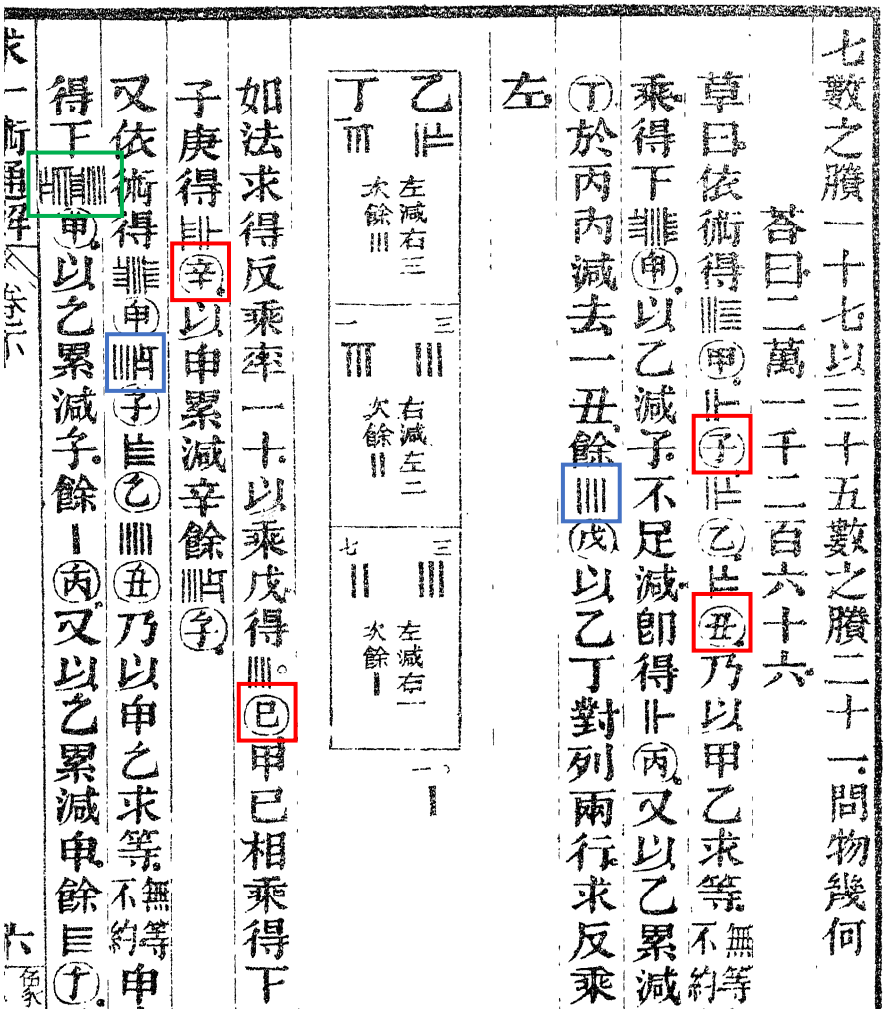


Figure 8. Example from Huang Zongxian's (黃宗憲) book *Qiuyishu Tongjie* (求一術通解) vol.2 p.7 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing ㊦, ㊣, ㊤ and ㊥ in text.†

* The dots are punctuations. The usage of them is like "." or ",".

† In the green box these are counting rods. "𝍠 𝍧𝍩 𝍨𝍩𝍩" means "17955".

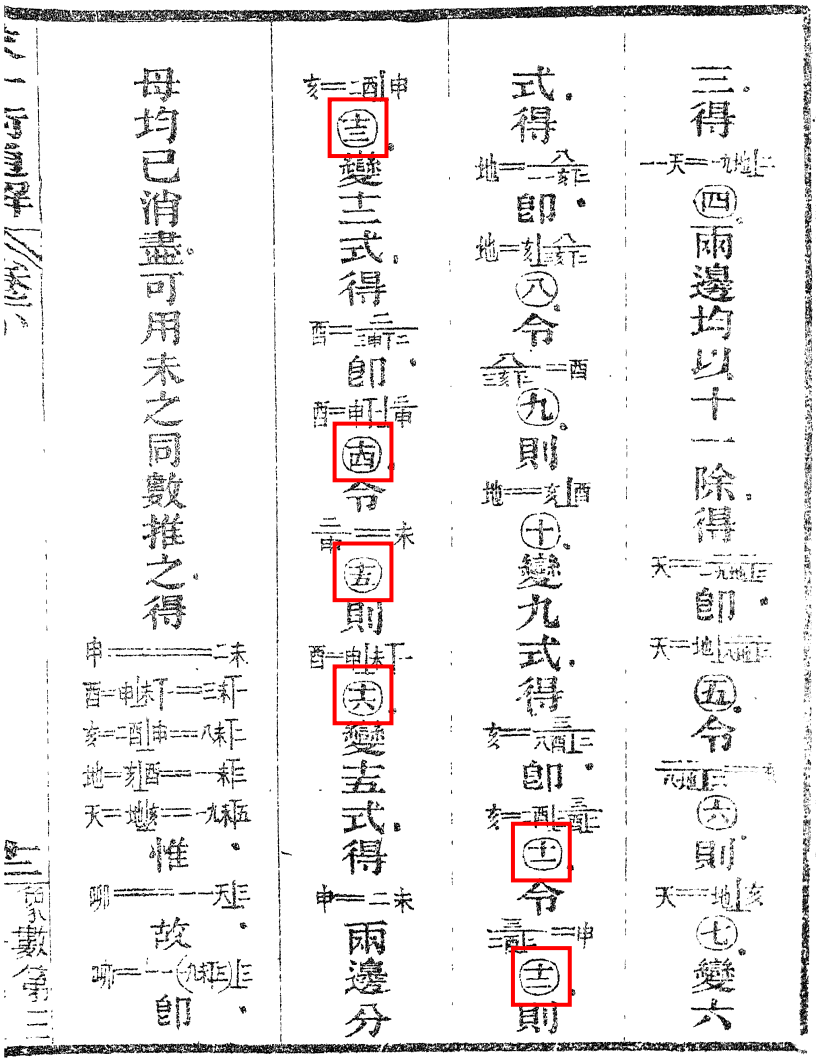


Figure 9. Example from Huang Zongxian's (黃宗憲) book *Qiuyishu Tongjie* (求一術通解) vol.2 p.12 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing ㊀, ㊁, ㊂, ㊃, ㊄ and ㊅ in text.

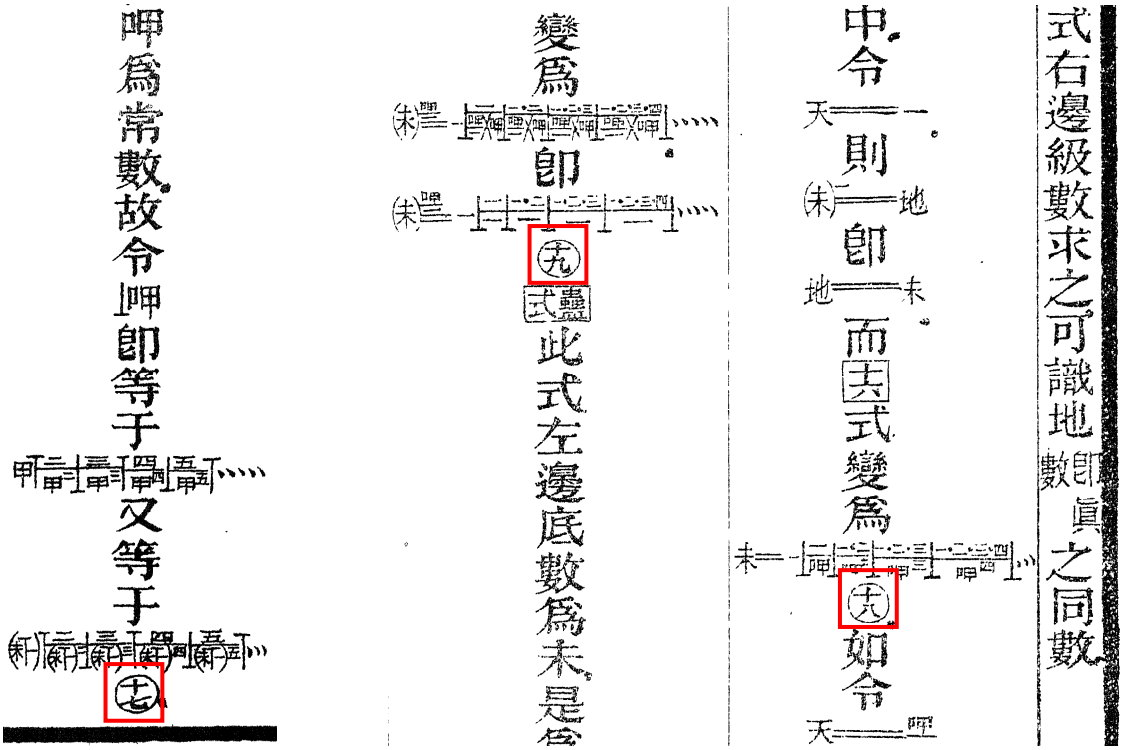


Figure 10. Example from Zeng Jihong's (曾紀鴻) book *Duishu Xiangjie* (對數詳解) vol.4 p.3 and p.4 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing ㊆, ㊇ and ㊈ in text.

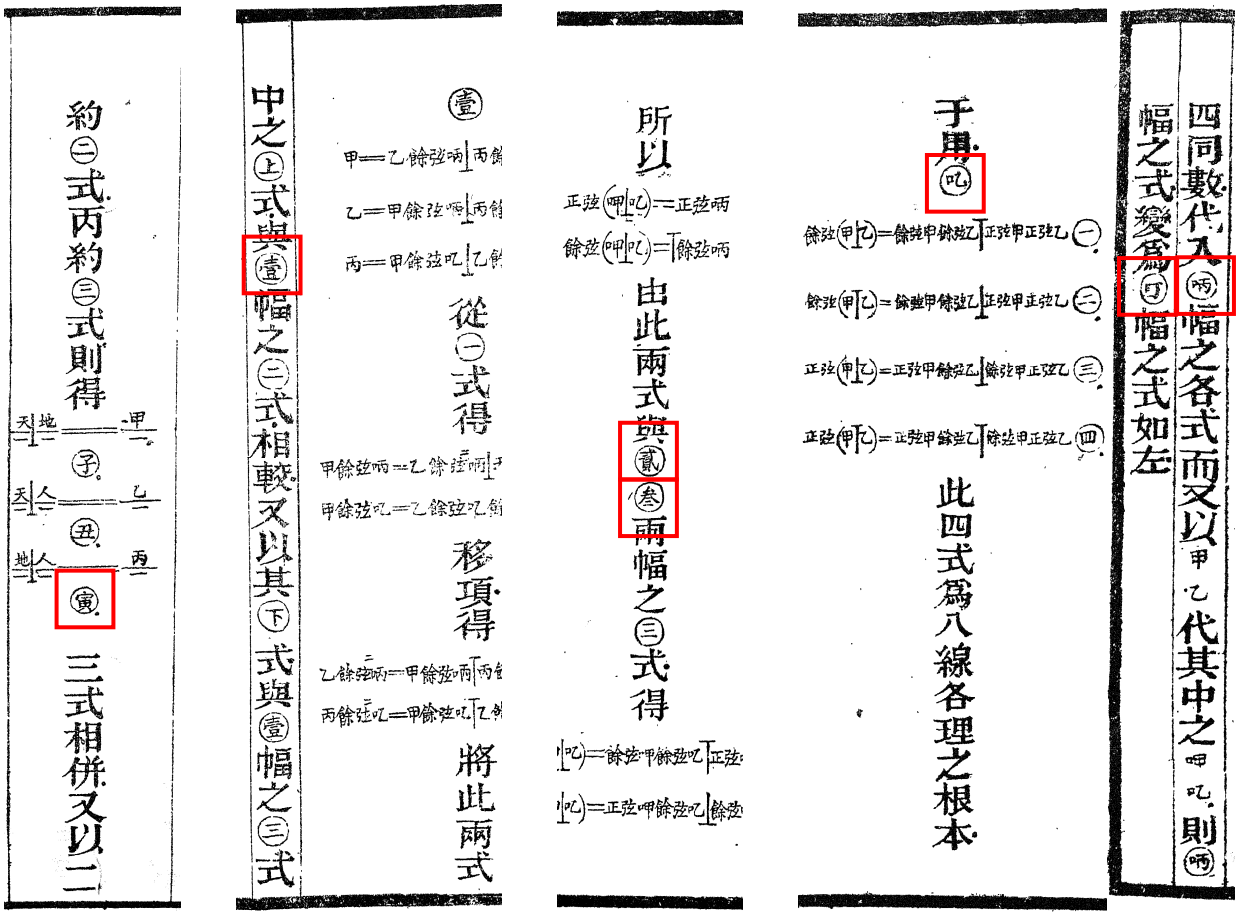


Figure 11. Example from Hua Hengfang's (華蘅芳) book *Daishushu* (代數術) vol.8 p.10; vol.24 p.13, p.15, p.19 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing 寅, 壹, 貳, 叁, 叺, 㒳 and 叮 in text.

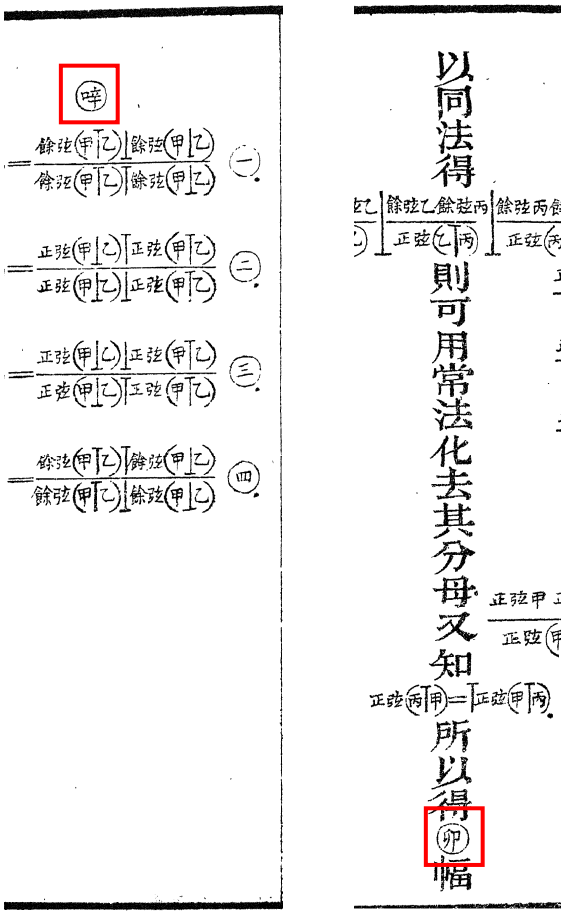


Figure 12. Example from Hua Hengfang's (華蘅芳) book *Daishushu* (代數術) vol.24 p.22, p.26 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing 晬 and 卯 in text.

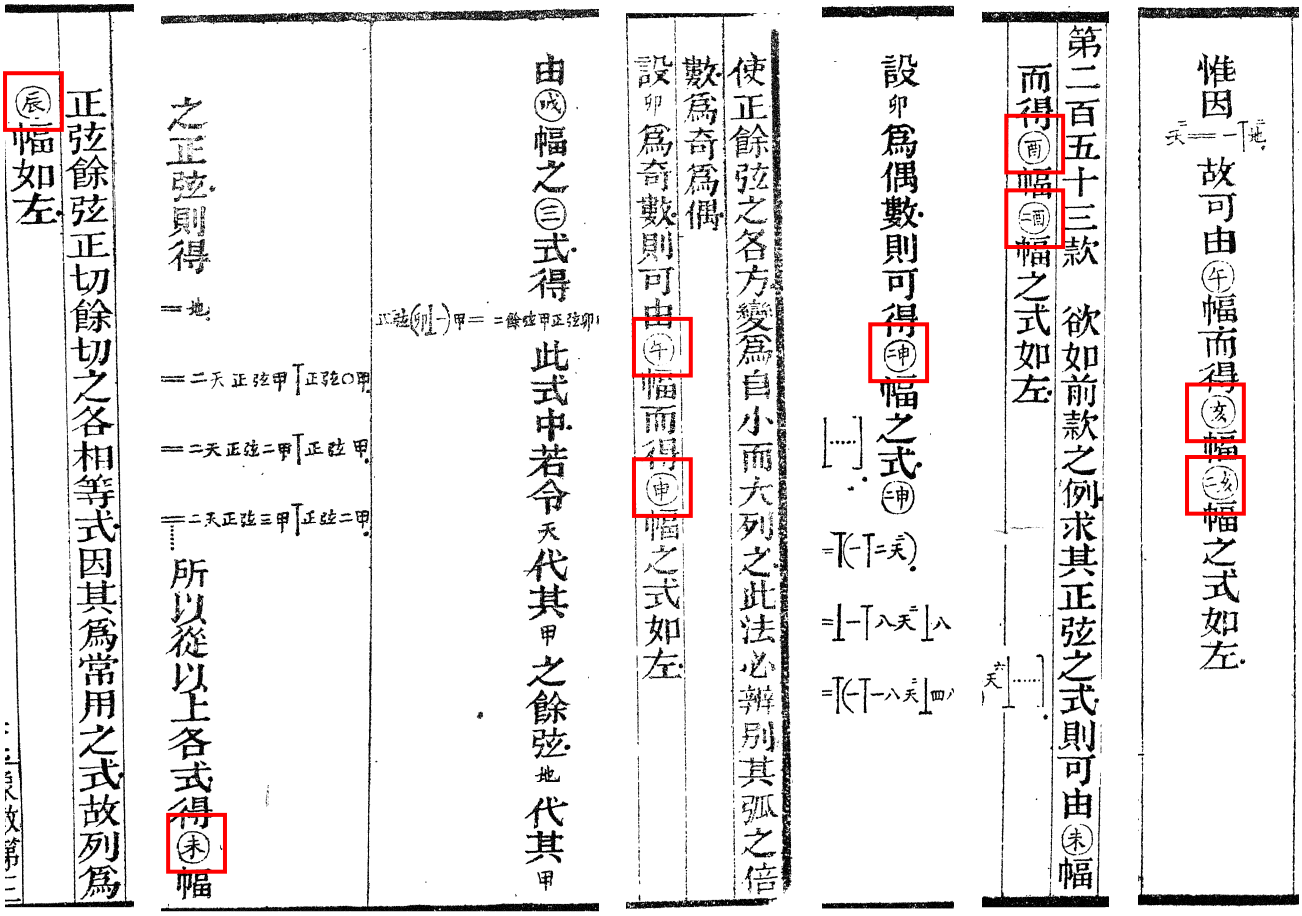


Figure 13. Example from Hua Hengfang's (華蘅芳) book *Daishushu* (代數術) vol.24 p.27, p.29, p.31, p.32, p.33, p.34 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing 辰, 午, 未, 申, 申, 酉, 酉, 亥 and 亥 in text.

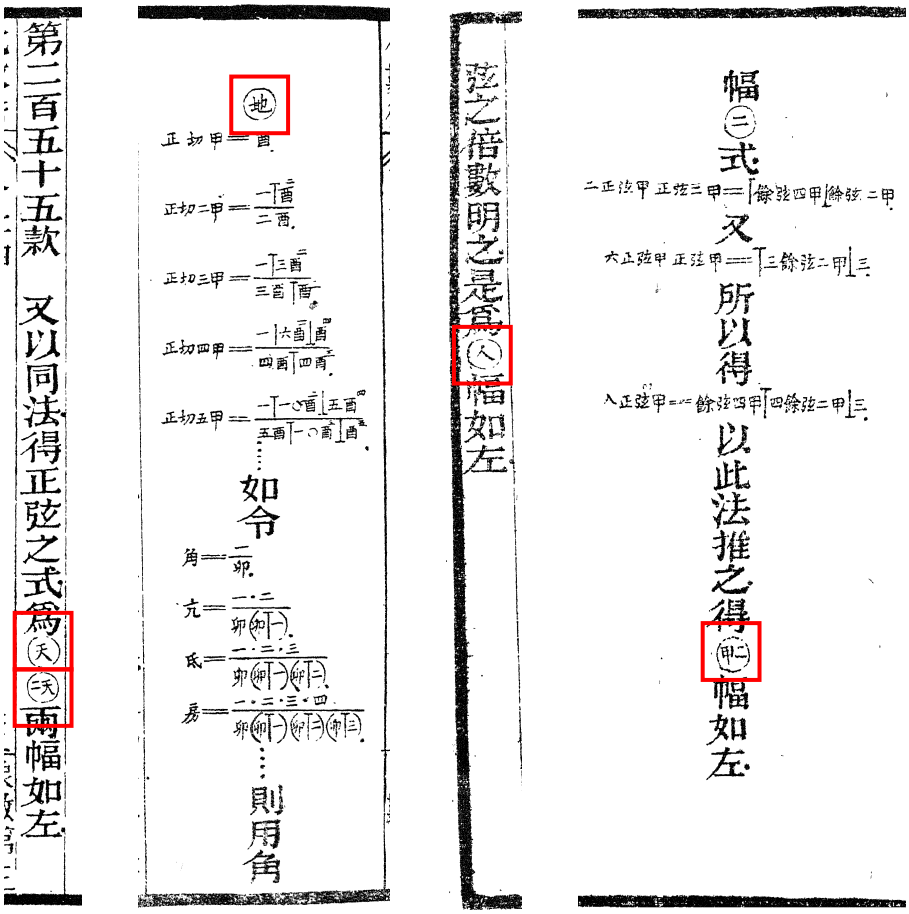


Figure 14. Example from Hua Hengfang's (華蘅芳) book *Daishushu* (代數術) vol.24 p.35, p.37, p.39, p.40 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing 天, 天, 地, 人 and 甲 in text.

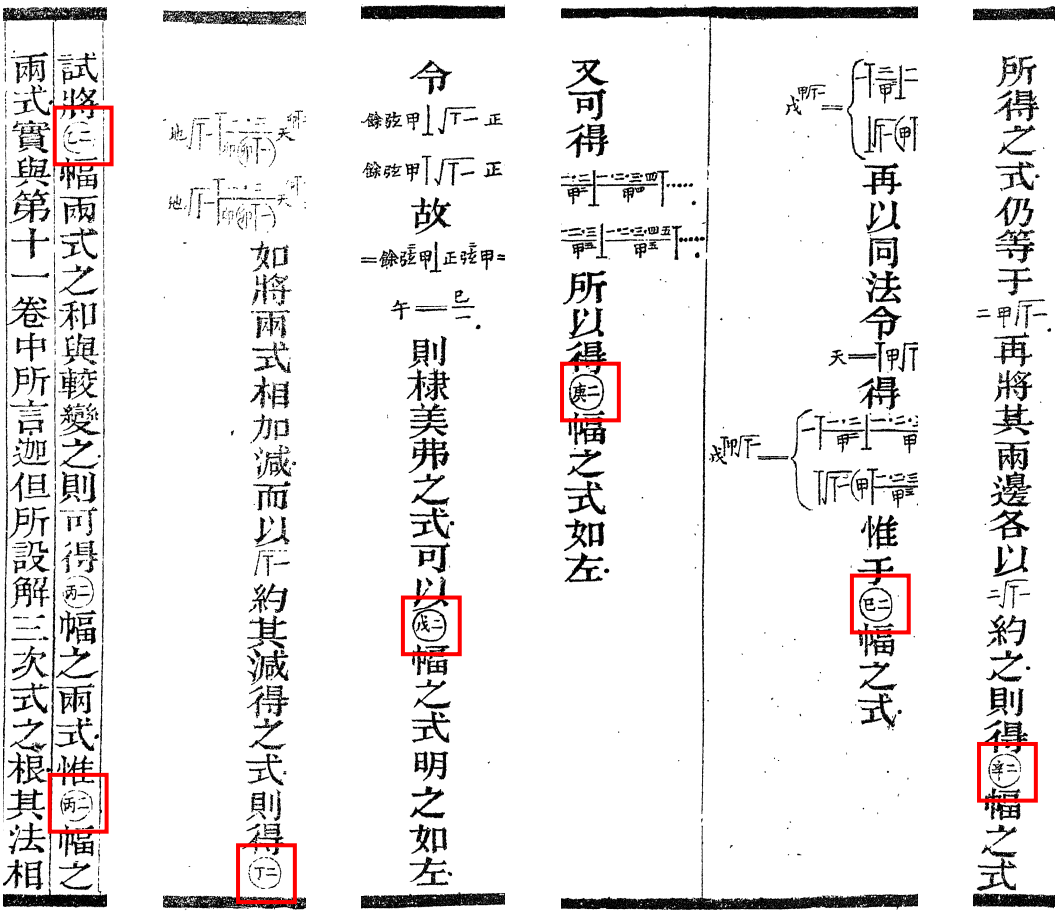


Figure 15. Example from Hua Hengfang's (華蘅芳) book *Daishushu* (代數術) vol.25 p.6, p.7, p.8, p.17, p.19 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing 乙, 丙, 丁, 戊, 己, 庚 and 辛 in text.‡

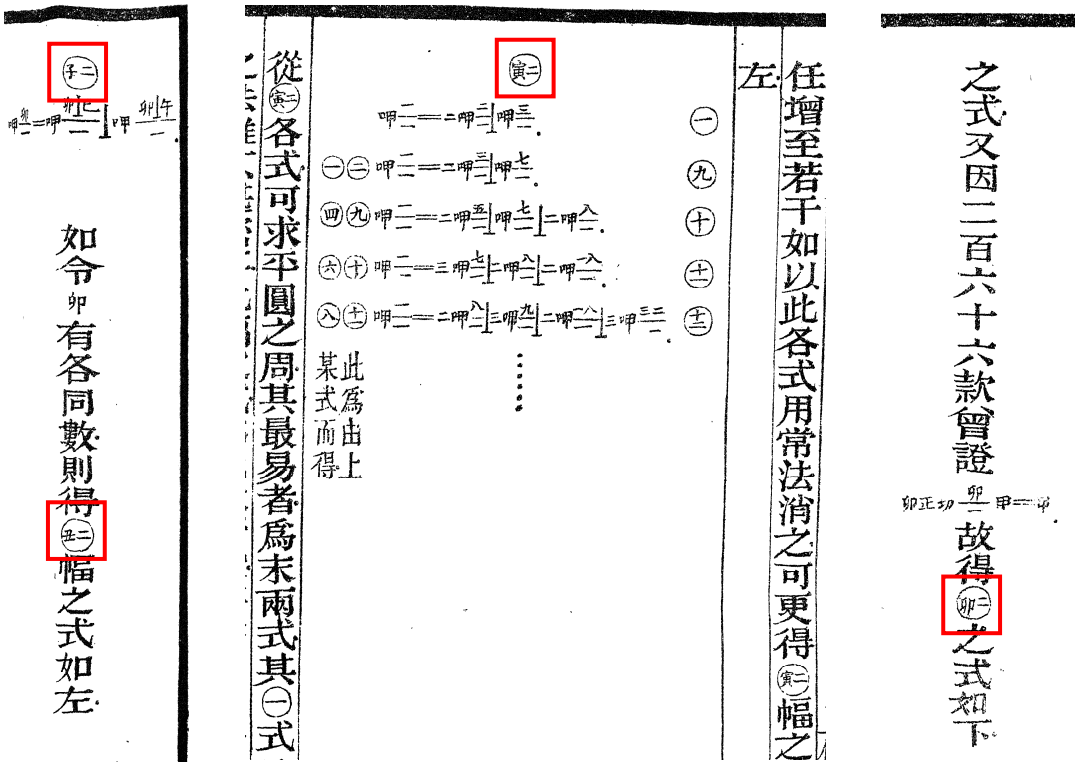
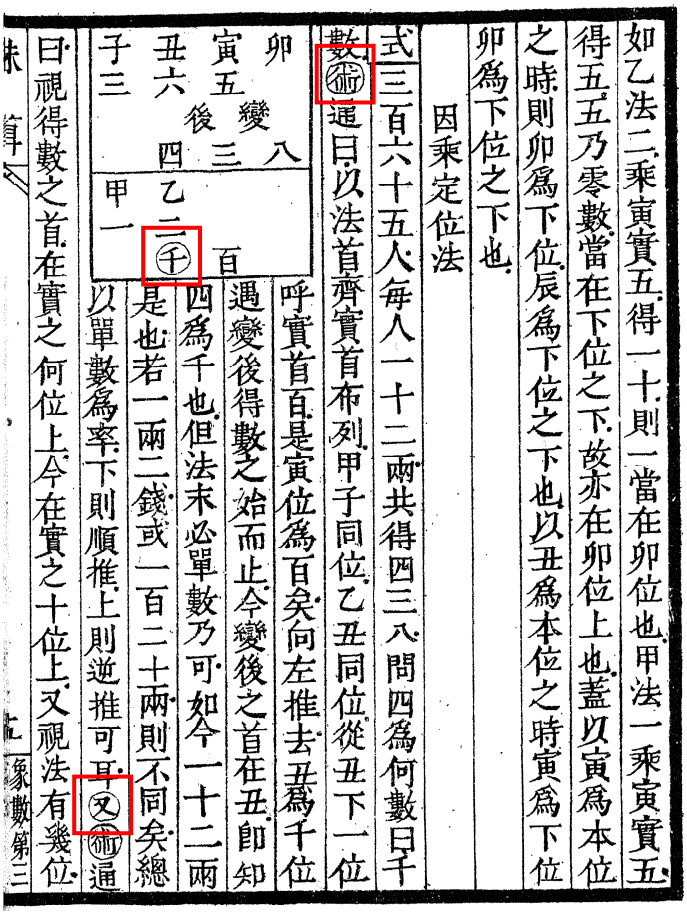


Figure 16. Example from Hua Hengfang's (華蘅芳) book *Daishushu* (代數術) vol.25 p.22, p.25 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing 子, 丑, 寅 and 卯 in text.

‡ 餘弦甲⊥$\sqrt{丁一}$ means cos *a*+$\sqrt{}$i, where "i" is the imaginary unit.



如乙法二乘寅實五得一十則一當在卯位也甲法一乘寅實五得五五乃零數當在下位之下故亦在卯位上也蓋以寅爲本位之時則卯爲下位辰爲下位之下也以丑爲本位之時寅爲下位卯爲下位之下也

因乘定位法

式三百六十五人每人一十二兩共得四三八問四爲何數曰千數㊣通曰以法首齊實首布列甲子同位乙丑同位從丑下一位呼實首百是寅位爲百矣向左推去丑爲千位遇變後得數之始而止今變後之首在丑卽知四爲千也但法末必單數乃可如今一十二兩是也若一兩二錢或一百二十兩則不同矣總以單數爲率下則順推上則逆推可耳㊣通曰視得數之首在實之何位上今在實之十位上又視法有幾位

卯	寅	丑	子
八	五	三	四
變後			
甲	乙	百	
一	二千		

Figure 17. Example from Fang Zhongtong's (方中通) book *Zhusuan* (珠算) p.5 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing ㊣, 又 and 千 in text.

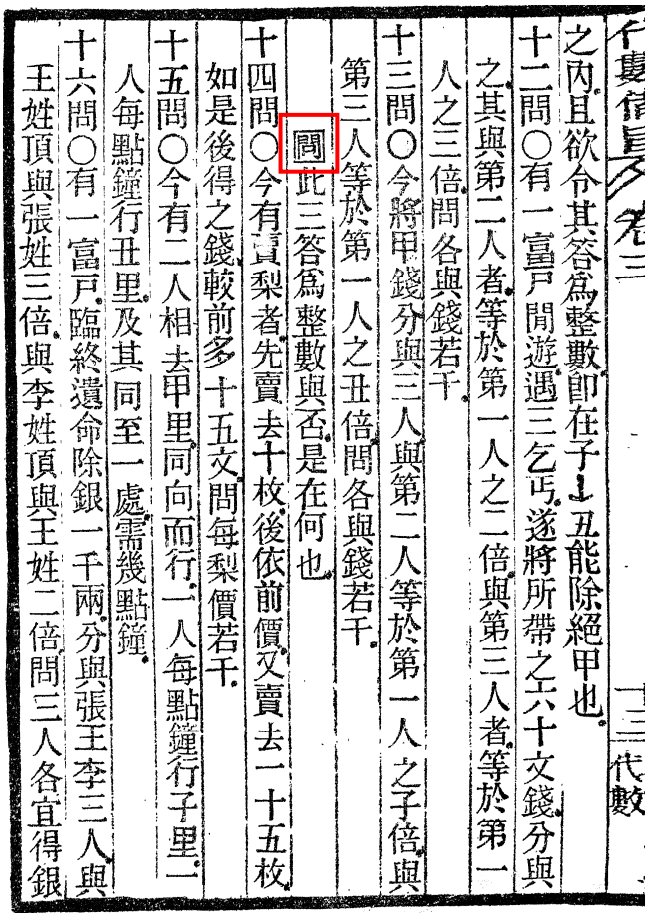
數以下坤九數坤十數皆由二十五除得之

一除坤一數爲丑一數正　三除坤二數爲丑二數負　五除坤三數爲丑三數正　七除坤四數爲丑四數負　九除坤五數爲丑五數正　十一除坤六數爲丑六數負　十三除坤七數爲丑七數正　十五除坤八數爲丑八數負以下十七除坤九數爲丑九數正等項可例推

五四除單一爲艮一數　二九一六卽五四除或以六除二次之異　可艮一數爲艮二數　二九一六除艮二數爲艮三數　二九一六除艮三數爲艮四數　二九一六除艮四數爲艮五數　一九一六除艮五數爲艮六數　二九一六除艮六數爲艮七數　二九一六除艮七數爲艮八數以下艮九數艮十數皆由二九一六除得之

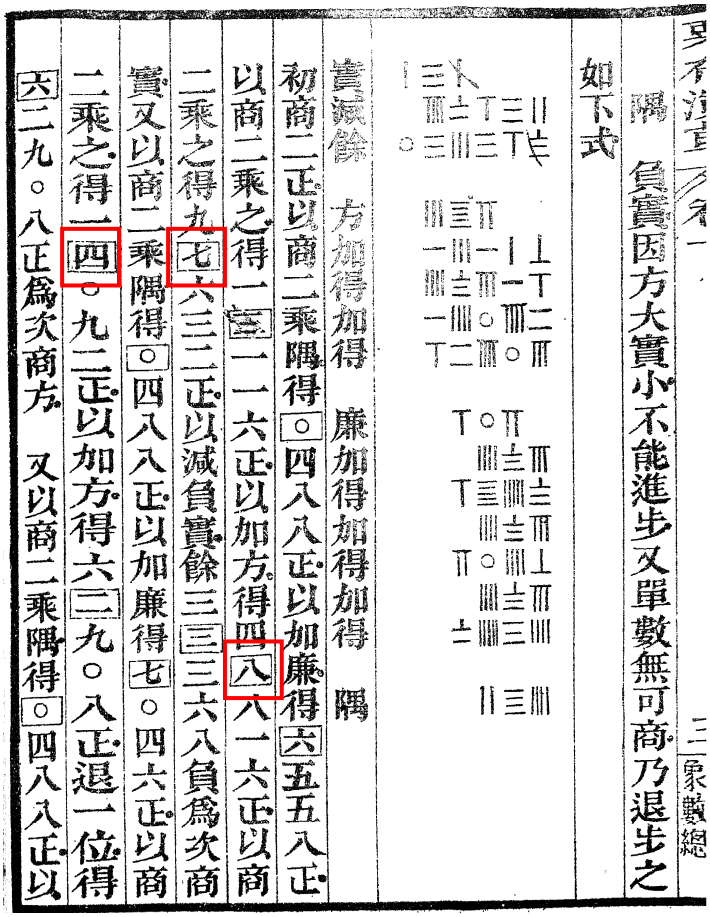
一除艮一數爲寅一數正　三除艮二數爲寅二數負　五除艮三數爲寅三數正　七除艮四數爲寅四數負　九除艮五數爲寅五數正　十一除艮六數爲寅六數負　十三除艮七數爲寅

Figure 18. Example from Zeng Jihong's (曾紀鴻) book *Yuanlü Kaozhen Tujie* (圓率攷眞圖解) p.13 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing 〇, 五, 六 and 九 in text.



之內且欲令其答爲整數即在子丑能除絕甲也
十二問○有一富戶閒遊遇三乞丐遂將所帶之六十文錢分與之其與第二人者等於第一人之二倍與第三人者等於第一人之三倍問各與錢若干
十三問○今將甲錢分與三人與第二人等於第一人之子倍與第三人等於第一人之丑倍問各與錢若干
圓此三答爲整數與否是在何也
十四問○今有賣梨者先賣去十枚後依前價又賣去一十五枚如是後得之錢較前多十五文問每梨價若干
十五問○今有二人相去甲里同向而行一人每點鐘行子里一人每點鐘行丑里及其同至一處需幾點鐘
十六問○有一富戶臨終遺命除銀一千兩分與張王李三人與王姓頂與張姓三倍與李姓頂與王姓二倍問三人各宜得銀

Figure 19. Example from Zou Liwen's (鄒立文) book *Daishu Beizhi* (代數備旨) vol.3 p.13 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing 圓 in text.



隅負實因方大實小不能進步又單數無可商乃退步之
如下式
實減餘方加得加得廉加得加得加得隅
初商二正以商二乘隅得〇四八正以加廉得六五五八正
以商二乘之得一三一一六正以加方得四八八一六正以商
二乘之得九七六三二正以減負實餘三三三六八負爲次商
實又以商二乘隅得〇四八八正以加廉得七〇四六正以商
二乘之得一四〇九二正以加方得六三九〇八正退一位得
六二九〇八正爲次商方又以商二乘隅得〇四八八正以

Figure 20. Example from Zeng Jihong's (曾紀鴻) book *Subu Yancao* (粟布演草) vol.1 p.3 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing 四, 七 and 八 in text.

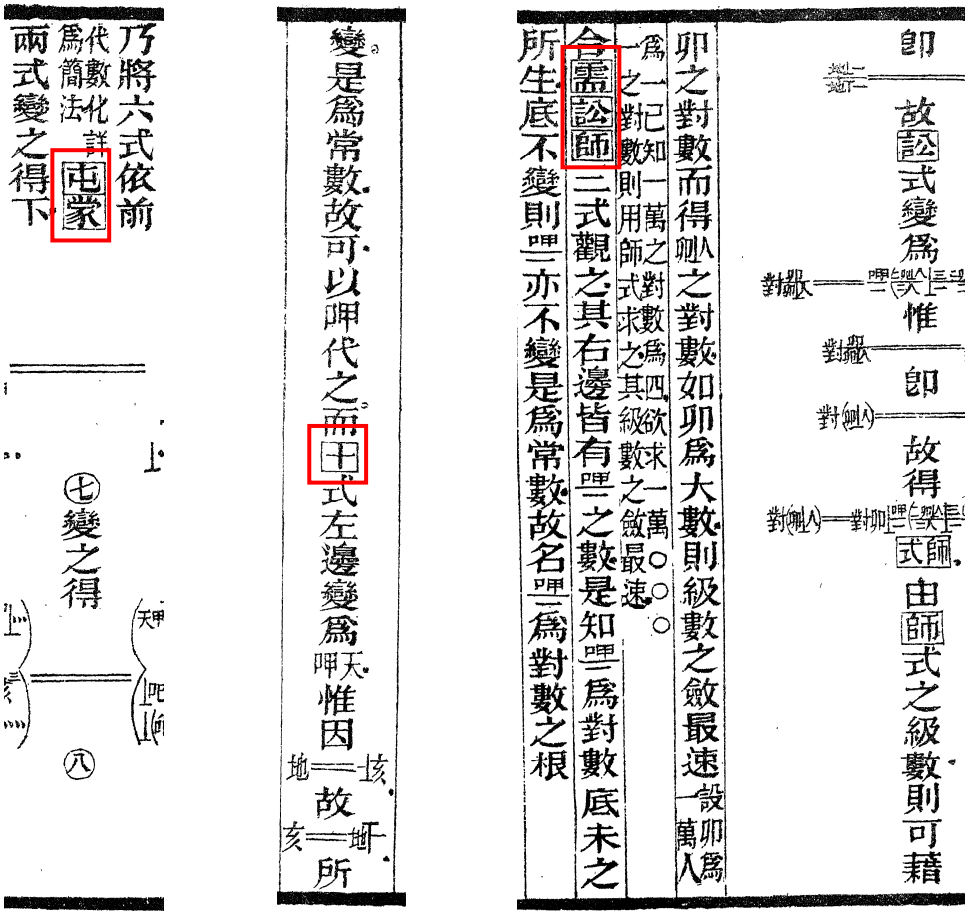


Figure 21. Example from Zeng Jihong's (曾紀鴻) book *Duishu Xiangjie* (對數詳解) vol.2 p.7, p.8 and p.10 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing 屯, 蒙, 屮, 需, 訟 and 師 in text.

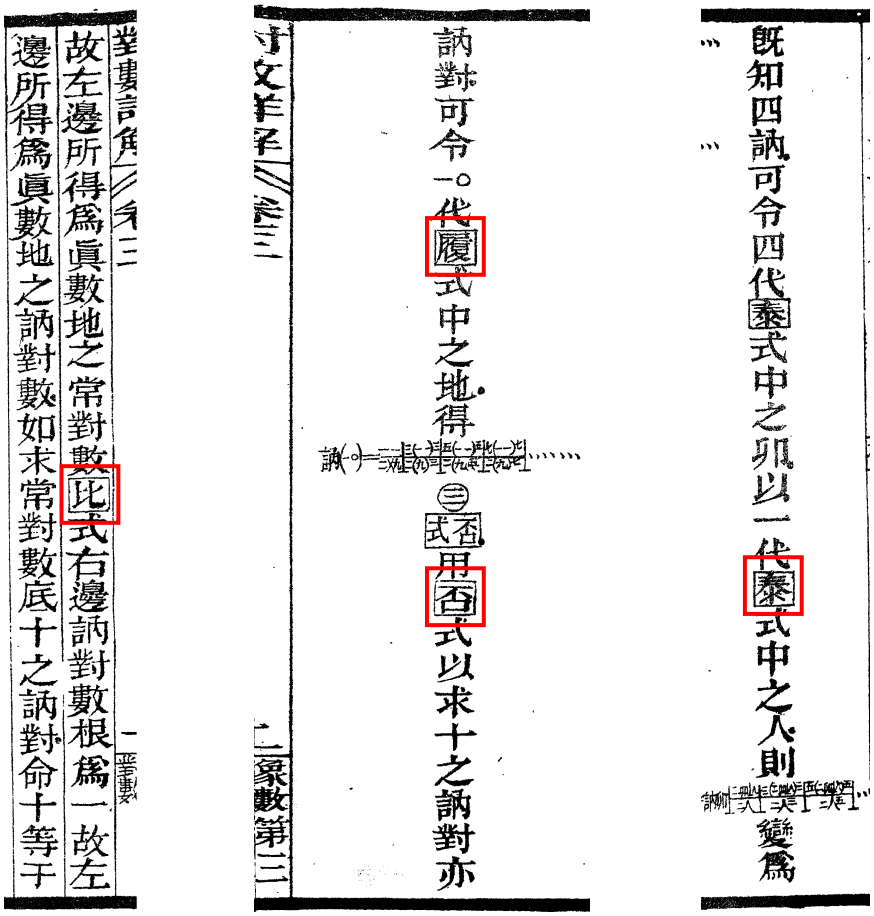


Figure 22. Example from Zeng Jihong's (曾紀鴻) book *Duishu Xiangjie* (對數詳解) vol.3 p.1, p.2 and p.3 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing 比, 履, 否 and 泰 in text.

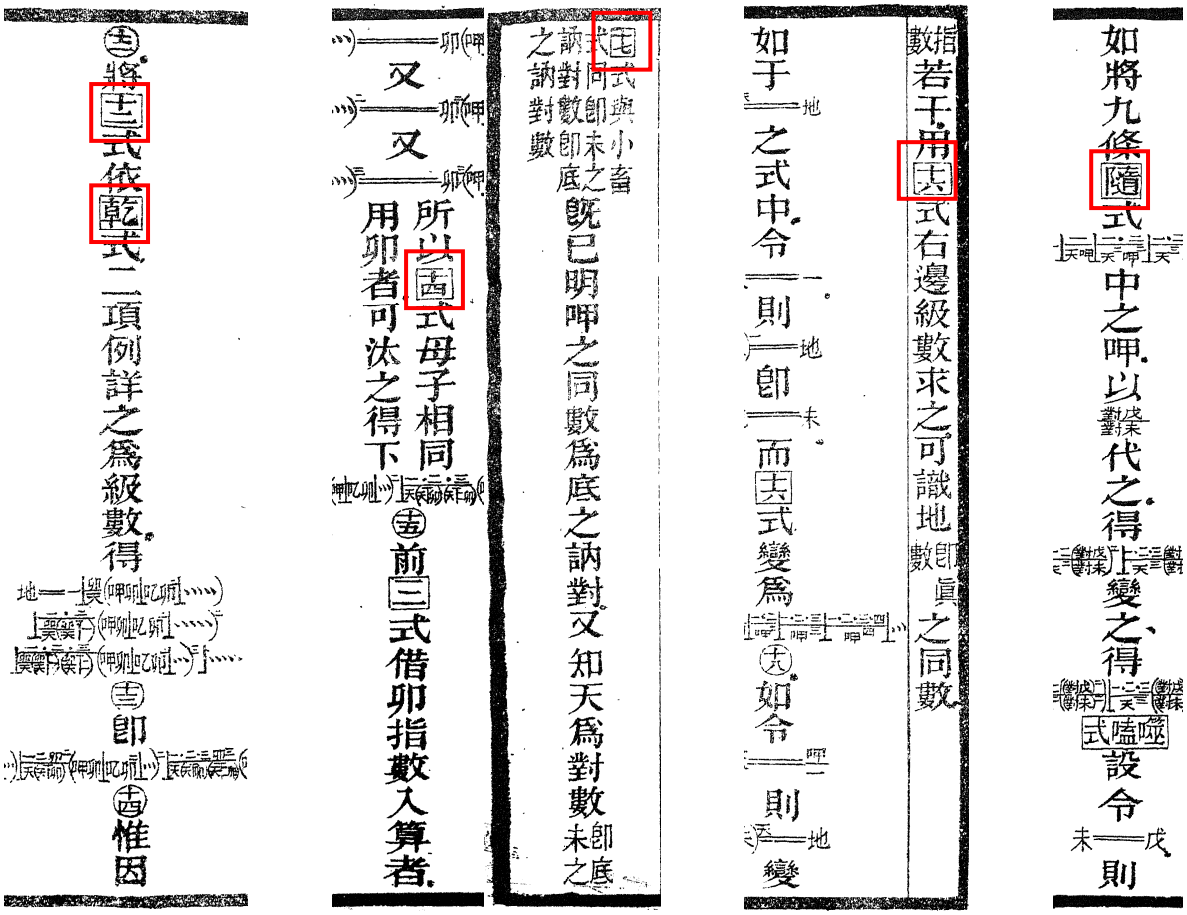


Figure 23. Example from Zeng Jihong's (曾紀鴻) book *Duishu Xiangjie* (對數詳解) vol.4 p.2, p.3, p.4 and p.6 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing 㐄, 㐅, 㐆, 㐇, 乾 and 隨 in text.

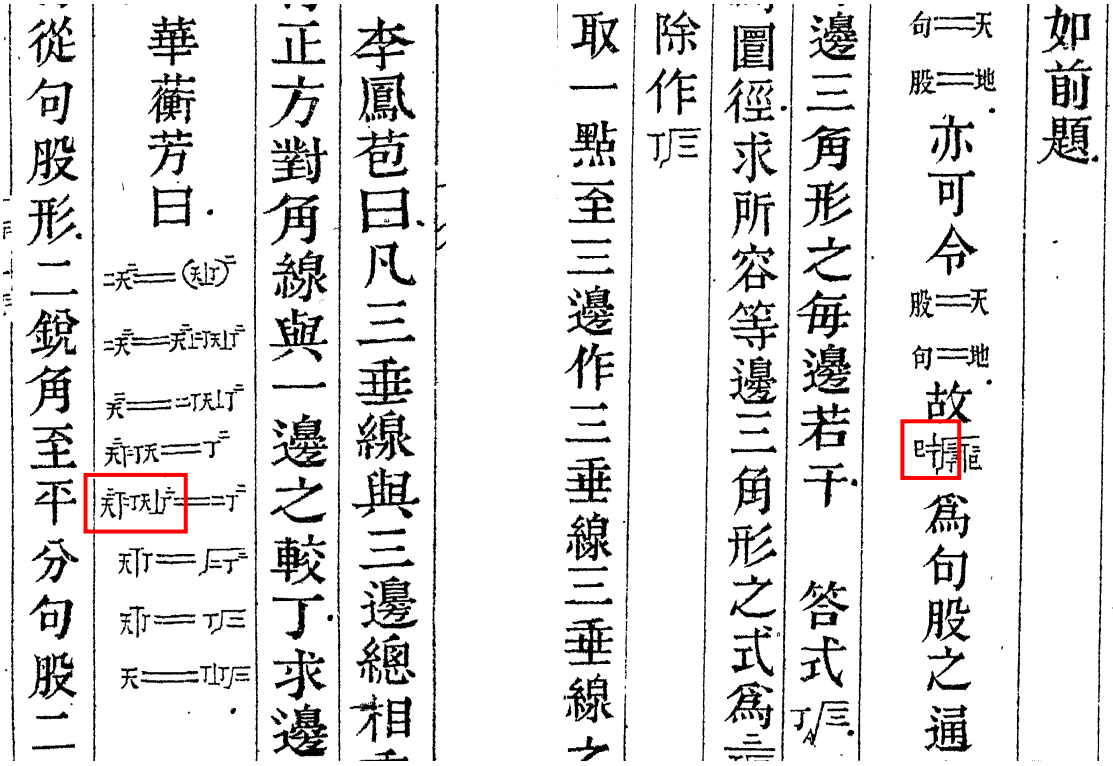


Figure 24. Example from Alexander Wylie and Li Shanlan's (李善蘭) book *Daiweiji Sheji Zhu* (代微積拾級注) vol.1 p.10 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing ⊥, 丅, and ⫟ in text.

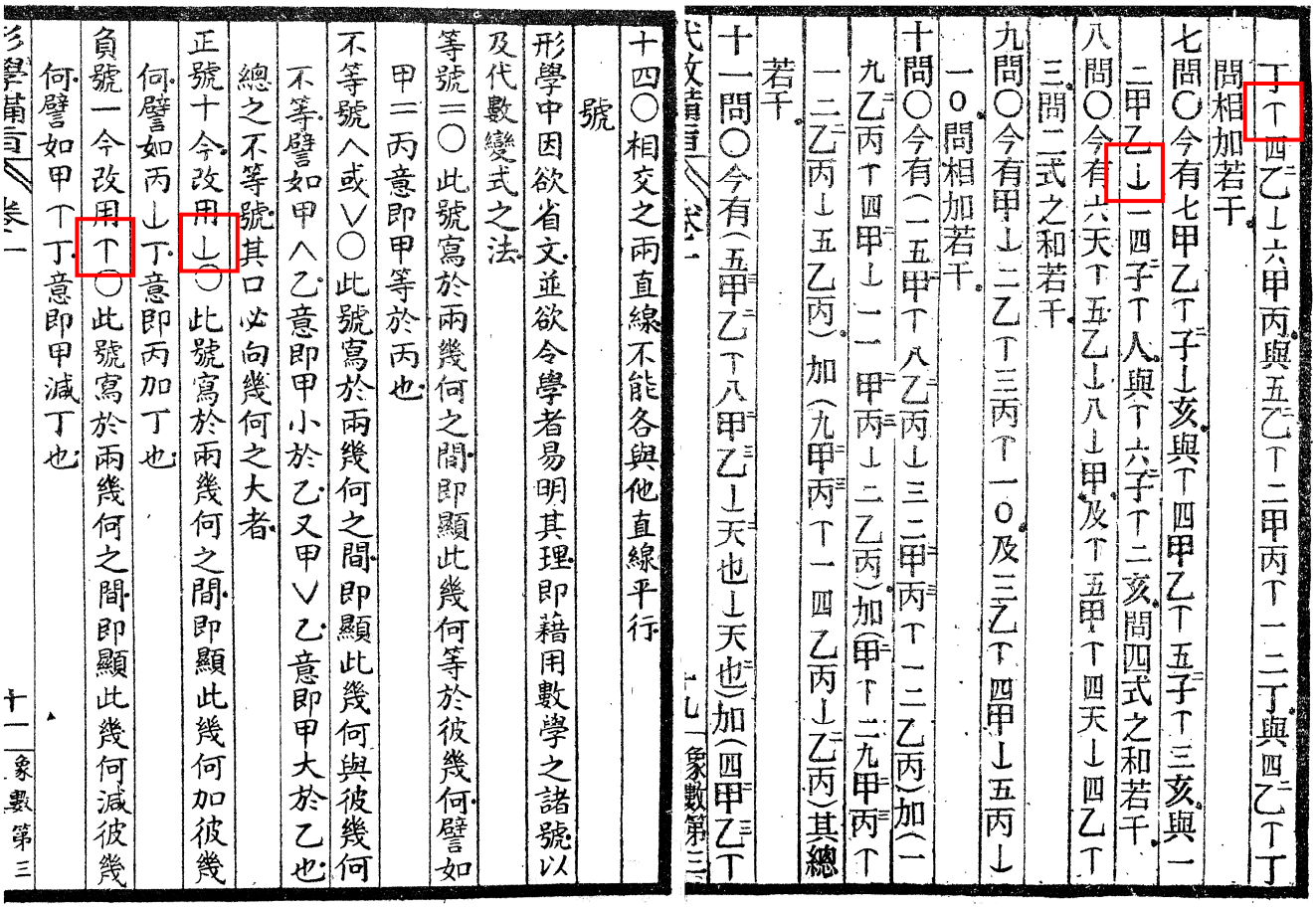


Figure 25. Example from Zou Liwen's (鄒立文) book *Xingxue Beizhi* (形學備旨) vol.1 p.11 and *Daishu Beizhi* (代數備旨) vol.1 p.19 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing ⊥ and ⊤ in text.

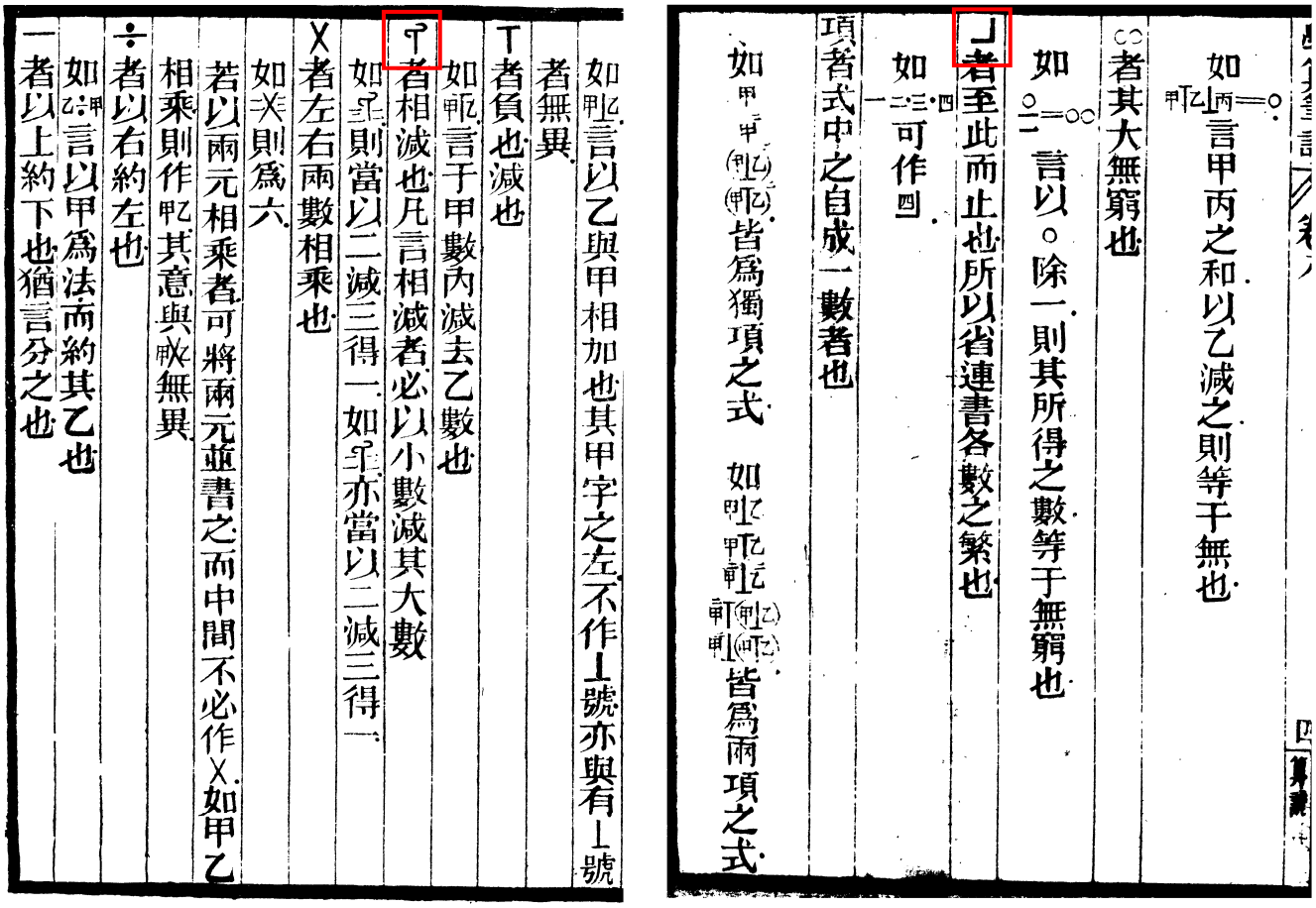


Figure 26. Example from Hua Hengfang's (華蘅芳) book *Xuesuan Bitan* (學算筆談) vol.8 p.1 and p.4 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing ㄒ, ㄩ in text.

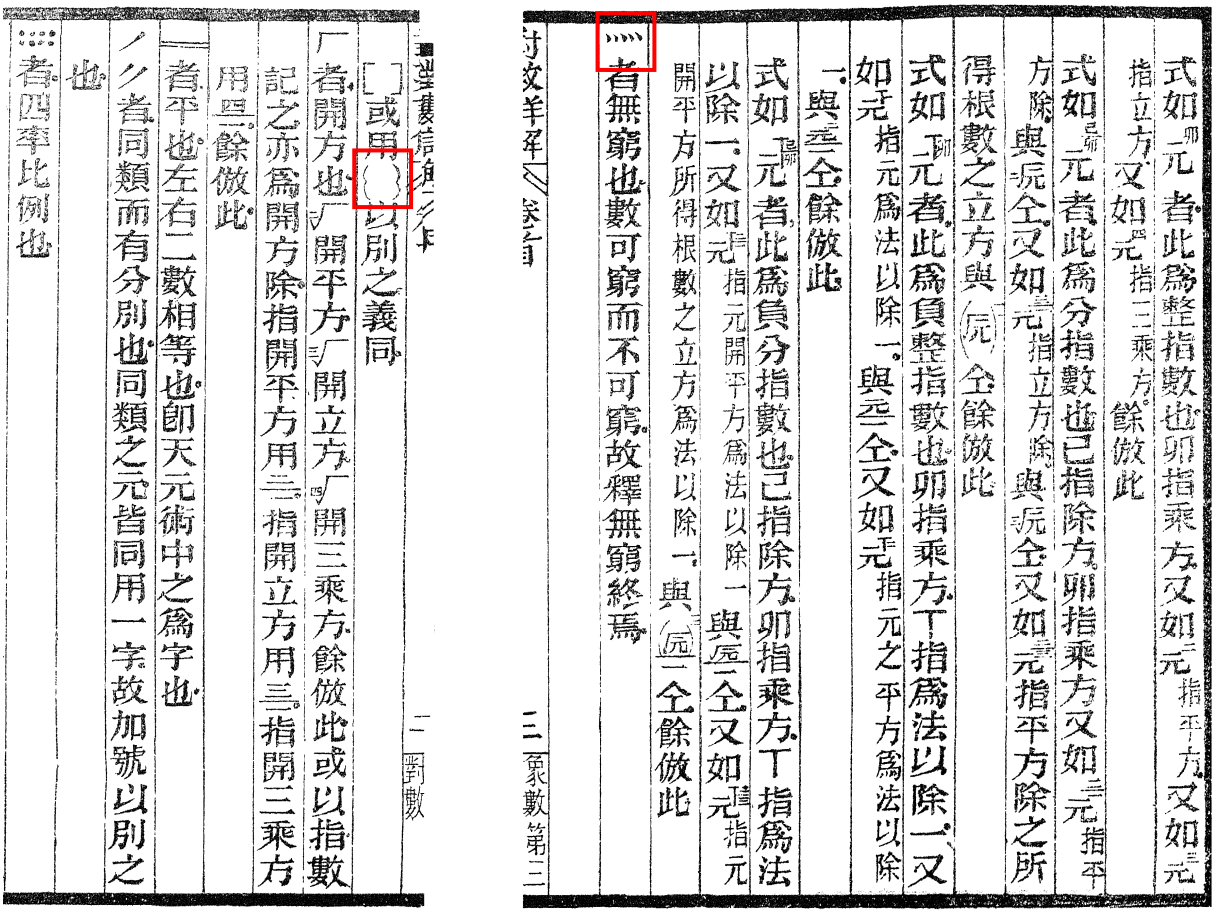


Figure 27. Example from Zeng Jihong's (曾紀鴻) book *Duishu Xiangjie* (對數詳解) vol.1 p.2 and p.3 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing ｛, ｝ and ⁘ in text.

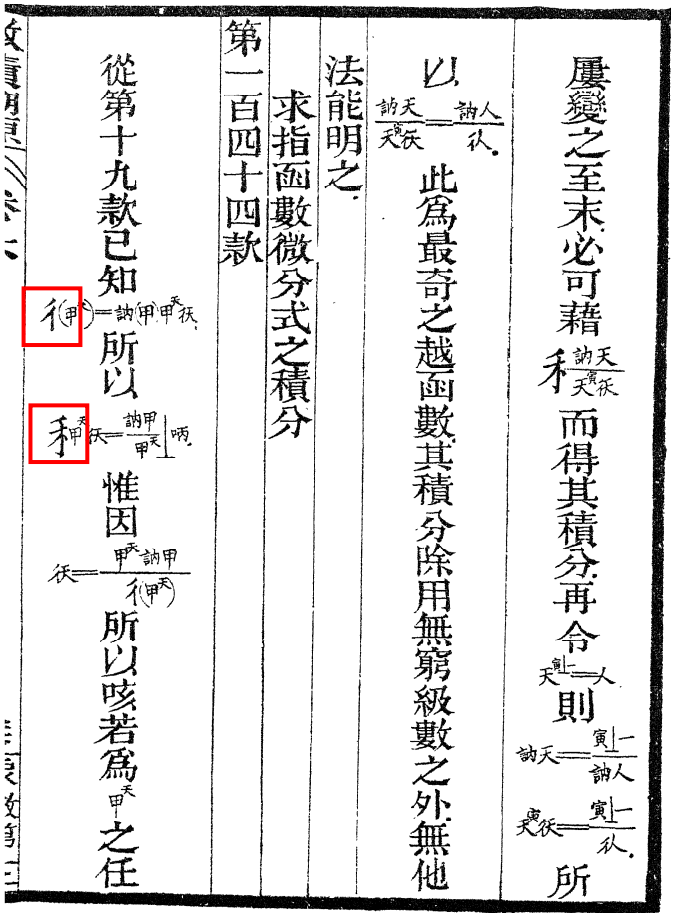
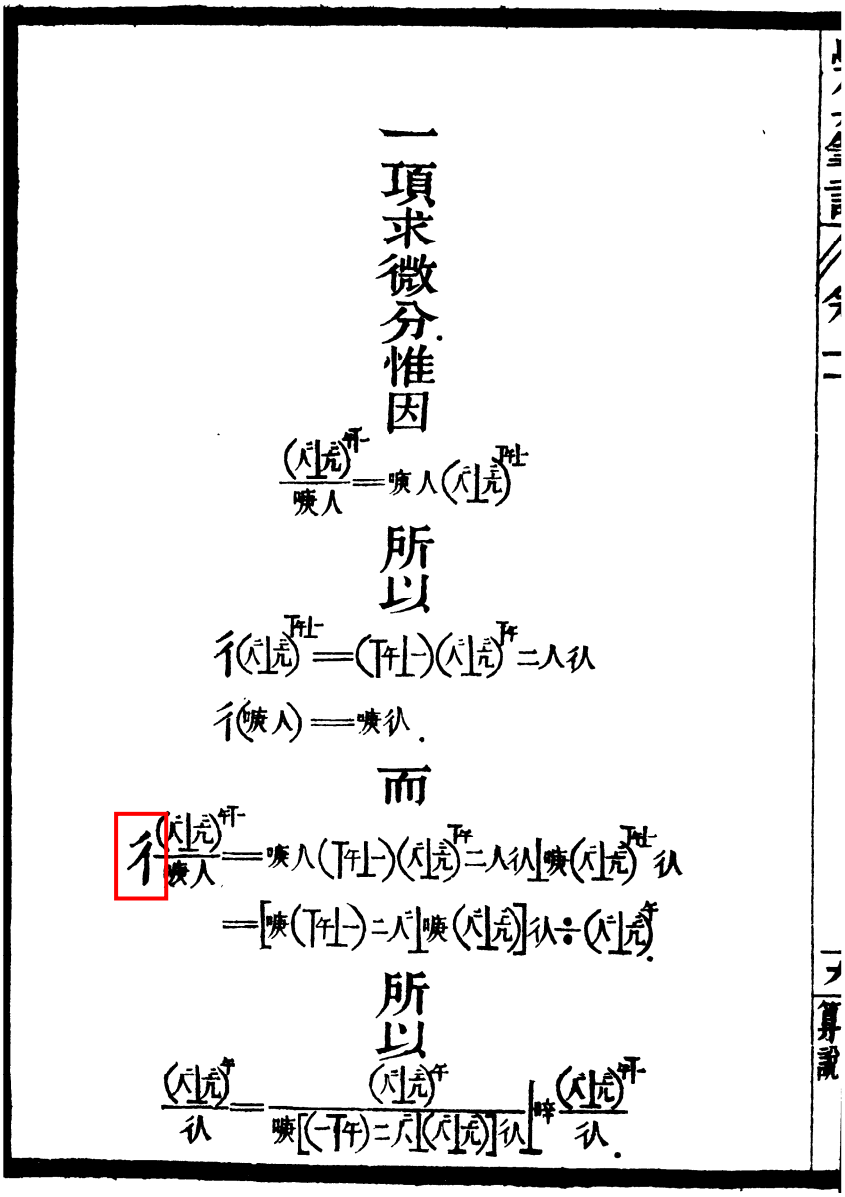


Figure 28. Example from Hua Hengfang's (華蘅芳) book *Weiji Suyuan* (微積溯源) vol.6 p.36 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing 亻 and 扌 in text.



一項求微分.惟因

$$\frac{(\text{入}+\text{元})^{\text{幷}-}}{\text{嗾}\,\text{人}}=\text{嗾}\,\text{人}(\text{入}+\text{元})^{\text{丅}\text{土}-}$$

所以

$$\text{彳}(\text{入}+\text{元})^{\text{丅}\text{土}-}=(\text{丅}\text{午}\text{土}-)(\text{入}+\text{元})^{\text{丅}\text{午}}\text{二人}\text{仌}$$

$$\text{彳}(\text{嗾}\,\text{人})=\text{嗾}\,\text{仌}.$$

而

$$\text{彳}\frac{(\text{入}+\text{元})^{\text{幷}-}}{\text{嗾}\,\text{人}}=\text{嗾}\,\text{人}(\text{丅}\text{午}\text{土}-)(\text{入}+\text{元})^{\text{丅}\text{午}}\text{二人}\text{仌}\text{土}\text{嗾}(\text{入}+\text{元})^{\text{丅}\text{土}-}\text{仌}$$

$$=[\text{嗾}(\text{丅}\text{午}\text{土}-)\text{二入}\text{土}\text{嗾}(\text{入}+\text{元})]\text{仌}\div(\text{入}+\text{元})^{\text{幸}}.$$

所以

$$\frac{(\text{入}+\text{元})^{\text{幸}}}{\text{仌}}=\frac{(\text{入}+\text{元})^{\text{午}}}{\text{嗾}[(-\text{丅}\text{午})\text{二入}\text{土}(\text{入}+\text{元})]\text{仌}\text{土}^{\text{時}}}\frac{(\text{入}+\text{元})^{\text{幷}-}}{\text{仌}}.$$

Figure 29. Example from Hua Hengfang's (華蘅芳) book *Xuesuan Bitan* (學算筆談) vol.11 p.19 (Shanghai: Arithmetic Publishing House (算學書局), 1898), showing 彳 in text.

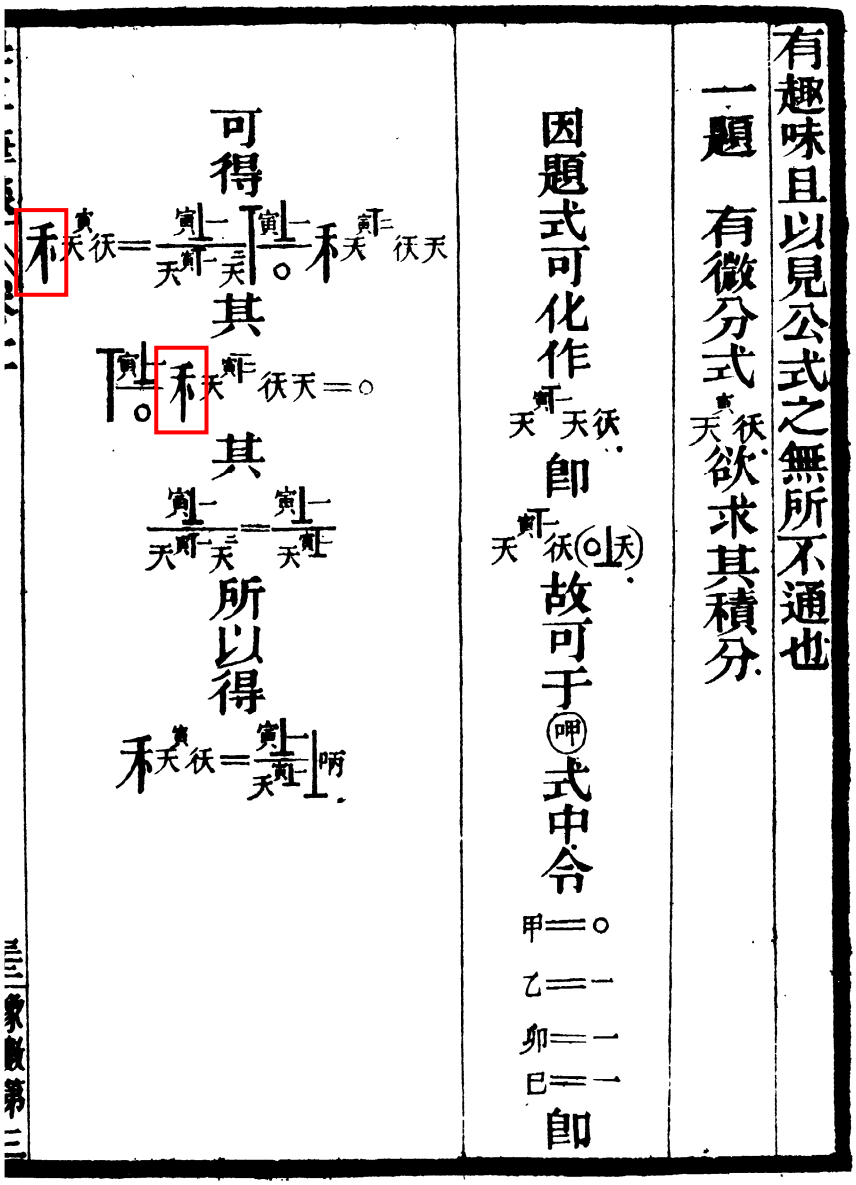


Figure 30. Example from Hua Hengfang's (華蘅芳) book *Xuesuan Bitan* (學算筆談) vol.11 p.32 (Shanghai: Arithmetic Publishing House (算學書局), showing 𠀋 in text.