Unicode request for Kaktovik numerals

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This document supersedes L2/20-070 by Eduardo Marín Silva.

The Kaktovik numerals are a set of base-20 digits with a sub-base of 5—that is, a *penta-vigesimal* system. Graphically, the sub-base forms the upper part of the digit and the remaining units the lower part, an iconic design that lends itself to graphical manipulation for arithmetic.

Kaktovik numerals are part of the curriculum in the North Slope Borough School District of Alaska. Though designed by speakers of Iñupiaq Eskimo (ISO code [esi]), they are equally suited to the penta-vigesimal systems of other Inuit and Yupik languages of Alaska, Canada and Russia, and they have the support of the Inuit Circumpolar Council.

Thanks to Deborah Anderson of the Universal Scripts Project for her assistance.

Design

Kaktovik numerals were made intentionally distinct from decimal Hindu-Arabic digits so that there could be no confusion between them. In speech as well, Kaktovik digits have been named in Iñupiaq and Hindu-Arabic digits in English in order to keep them distinct. There are 19 counting digits, composed of straight strokes joined at sharp angles, and a graphically distinct zero 𝋀.

The counting digits occupy the space of an upright golden rectangle, with a unit square at bottom and a smaller golden rectangle at top. The top rectangle is occupied by up to three horizontal strokes that tally the quinary sub-base (null, 𝋅, 𝋊, 𝋏). The bottom square is occupied by up to four vertical strokes that tally the remaining units (null, 𝋁, 𝋂, 𝋃, 𝋄). The strokes are joined into a single continuous line, up to a maximum of 𝋓 19 (𝋏 3×5 + 𝋄 4). This pattern, of combining fives and ones into scores, reflects the Inuit and Yupik languages much more closely than does the base-10 Hindu-Arabic system. It also lends itself directly to an abacus. Unlike the typical decimal abacus, which has one upper bead in each column, the Iñupiaq abacus has three upper beads (and four lower beads) in each column, corresponding to the three upper strokes and four lower strokes of the Kaktovik digits and to the sub-base numerals of the Iñupiaq language. See Figure 9.

Because the system is positional, the digit 𝋁, depending on its place, indicates 1, 20, 400, 8000, 16 000, 3 200 000, 64 000 000, etc., and the digit 𝋅 indicates five times those amounts. Thus, when a digit occurs in the units place, the vertical strokes indicate the number of units and the horizontal strokes the number of fives in that digit. When in the twenties place, they indicate the number of scores and of hundreds. That is, five 𝋁 (units) are 𝋅 (5), four 𝋅 (fives) are 𝋁𝋀 ($1\times20^1 = 20$), five 𝋁𝋀 (score) are 𝋅𝋀 ($5\times20^1 = 100$), four 𝋅𝋀 (hundreds) are 𝋁𝋀𝋀 ($1\times20^2 = 400$), etc., and similarly to the right of the radix mark.

Written notation is unbounded. Spoken Iñupiaq, through compounding and a series of suffixes, can accommodate numbers up to just under 20^{12}, or a bit over 4 quadrillion (MacLean 2014).

History

The invention of the numerals is recounted in Bartley (1997, 2002). Bartley reports that 'prior to the invention of the Kaktovik Iñupiaq numerals, the numbers of the Iñupiaq language were falling into disuse and, except for the lower numbers, were being forgotten.'

Bartley was a math teacher at Harold Kaveolook School in Kaktovik, the easternmost village of the North Slope of Alaska. He recounts that in September, 1994, during a math enrichment activity exploring base-2 numbers, students noted that Iñupiaq had a base-20 system, and tried to do the exercises in base-20. They added ten invented digits to the Hindu-Arabic system to accommodate Iñupiaq numerals, but found them difficult to remember. Eventually, using the pattern of counting in Iñupiaq as a guide, they came up with a prototype of the current system. Bartley says 'it is a system which is a direct reflection of the way one counts in Iñupiaq.'

The students soon found that arithmetical operations could be performed graphically and so were much easier in their new notation. For example, 𝋂 + 𝋂 = 𝋄 and 𝋒 ÷ 𝋃 = 𝋆 are visually obvious in a way that 2 + 2 = 4 and 18 ÷ 3 = 6 are not. For subtraction, one simply finds the shape of the subtrahend in the minuend; the answer is the strokes that remain, as in 𝋓 – 𝋌 = 𝋇 (19 – 12 = 7). For long division, the students discovered a method of chunking (partial quotients), by using colored pencils to match the strokes of the divisor in the dividend, that didn't require the sub-tables that they had learned for Hindu-Arabic numerals. Examples of graphical chunking of long-division problems using Kaktovik numerals are given in Figure 11. and Figure 12.

To switch to and from Hindu-Arabic, students needed to convert between base-20 and base-10. This was initially facilitated with counters that were assigned place values, and these in turn lead to the idea of a base-20 abacus, which the students built in the school shop. (See Figure 9.) Due to the one-to-one correspondence between the upper and lower strokes of the Kaktovik numerals and the upper and lower beads of the abacus, learning to use an abacus for arithmetic was straightforward, and the Iñupiaq abacus is now an integral part of math education.

In the spring of 1995, the North Slope Borough Board of Education invited the students from Kaveolook School to fly to Utqiaġvik (then Barrow) to present and explain their invention. In the fall of that year, Kaktovik numerals were added to the curricula of the Early Childhood Education immersion program in Utqiaġvik and of the Iñupiaq-language classes in the villages of Wainwright and Point Lay in the west of the district, then to other elementary and middle schools across the North Slope, as well as to the regional high school in Utqiaġvik. The Early Childhood Education program uses Kaktovik numerals to the exclusion of Hindu-Arabic, and an Iñupiaq-language textbook was written to teach math using Kaktovik numerals in the first-grade immersion classes. Iḷisaġvik College in Utqiaġvik started a Kaktovik mathematics course.

In 1996, the Commission on Inuit History, Language & Culture in Barrow gave their endorsement.

In 1997, scores on the California Achievement Test in mathematics at the Kaktovik middle school increased dramatically. Where the average score had previously been in the 20th percentile, after the introduction of the new numerals scores rose to above the national average.

In 1998, the Canadian chapter of the Inuit Circumpolar Council endorsed the numerals in Resolution 9, 'Regarding Kaktovik Numerals':

> WHEREAS there is no widely-accepted means of representing (with simple numeric symbols) the traditional base-twenty counting systems used in Inuit languages; and

> WHEREAS variations of a base-twenty counting system are part of our common Inuit cultural heritage, but these are being lost because fewer and fewer Inuit learn and use the traditional numbers; and
>
> WHEREAS students in the Inuit community of Kaktovik have developed a base-twenty counting system which they desire to further research as a part of their on-going math education; and
>
> WHEREAS the ICC recognizes the right of each community to its own numbering system;
>
> THEREFORE BE IT RESOLVED THAT the Inuit Circumpolar Conference endorse further research into the use and development of the Kaktovik Numbering System, as well as any other local Inuit numbering system; and
>
> BE IT FURTHER RESOLVED THAT the ICC encourage all member communities to try to make broader use especially in education of their own local base-twenty counting tradition in order to preserve and to revitalize the traditional Inuit counting systems.

Harold Kaveolook School burned nearly to the ground in February 2020. Though fortunately there was no loss of life, early records and many items attesting to the use of the system there were lost. We take this opportunity to express our condolences to the students and staff, and to the whole village that used the school as a community center, and hope for its swift reconstruction.

Name of the block

The Inuit Circumpolar Council (1998) calls these the *Kaktovik numerals*, after their place of origin. MacLean (2014) and teachers in the North Slope Borough School District do the same. There is a natural semantic distinction between ‘Kaktovik numerals’ – the graphic digits presented here – and ‘Iñupiaq numerals’ – the lexical numerals of the Iñupiaq language. Kaktovik and Iñupiaq numbers are shown side-by-side in Figure 1. If the digits were used instead in the medium of, say, Canadian Inuktitut, then the correlation would be between Kaktovik and Inuktitut numerals.

(In version L2/20-070 this proposal, it was incorrectly stated that there was no precedent of naming a Unicode block after a town, but that is exactly the case for the Elbasan script.)

Characters

The only requested characters are the twenty digits. Existing Unicode characters can be used for the radix mark, arithmetical symbols, parentheses, etc. The design of the characters seen here adheres to the golden-rectangle ideal, with the upper portion reserved for the fives count and otherwise left empty. It is copied from Bartley’s 1997 design, which is the basis of the *LaserIñupiaq* and *LaserYukon* fonts from Linguists’ Software used for example in MacLean (2014).

In the North Slope Borough School District, Kaktovik numerals are named in Iñupiaq while Hindu-Arabic numerals are named in English, but the names can be expected to vary according to the languages of the user (e.g. Inuktitut and French, or Chaplino and Russian). For the purposes of Unicode, English names are appropriate.

𝋀 1D2C0 KAKTOVIK NUMERAL ZERO

𝋁 1D2C1 KAKTOVIK NUMERAL ONE

𝋂 1D2C2 KAKTOVIK NUMERAL TWO

𝋃 1D2C3 KAKTOVIK NUMERAL THREE

𝋄 1D2C4 KAKTOVIK NUMERAL FOUR

𝋅 1D2C5 KAKTOVIK NUMERAL FIVE
𝋆 1D2C6 KAKTOVIK NUMERAL SIX
𝋇 1D2C7 KAKTOVIK NUMERAL SEVEN
𝋈 1D2C8 KAKTOVIK NUMERAL EIGHT
𝋉 1D2C9 KAKTOVIK NUMERAL NINE
𝋊 1D2CA KAKTOVIK NUMERAL TEN
𝋋 1D2CB KAKTOVIK NUMERAL ELEVEN
𝋌 1D2CC KAKTOVIK NUMERAL TWELVE
𝋍 1D2CD KAKTOVIK NUMERAL THIRTEEN
𝋎 1D2CE KAKTOVIK NUMERAL FOURTEEN
𝋏 1D2CF KAKTOVIK NUMERAL FIFTEEN
𝋐 1D2D0 KAKTOVIK NUMERAL SIXTEEN
𝋑 1D2D1 KAKTOVIK NUMERAL SEVENTEEN
𝋒 1D2D2 KAKTOVIK NUMERAL EIGHTEEN
𝋓 1D2D3 KAKTOVIK NUMERAL NINETEEN

Chart

	...0	...1	...2	...3	...4	...5	...6	...7	...8	...9	...A	...B	...C	...D	...E	...F
Kaktovik numerals																
U+1D2Cx	𝋀	𝋁	𝋂	𝋃	𝋄	𝋅	𝋆	𝋇	𝋈	𝋉	𝋊	𝋋	𝋌	𝋍	𝋎	𝋏
U+1D2Dx	𝋐	𝋑	𝋒	𝋓												

Properties

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1D2C0;KAKTOVIK NUMERAL ZERO;No;0;L;;;;0;N;;;;
1D2C1;KAKTOVIK NUMERAL ONE;No;0;L;;;;1;N;;;;
1D2C2;KAKTOVIK NUMERAL TWO;No;0;L;;;;2;N;;;;
1D2C3;KAKTOVIK NUMERAL THREE;No;0;L;;;;3;N;;;;
1D2C4;KAKTOVIK NUMERAL FOUR;No;0;L;;;;4;N;;;;
1D2C5;KAKTOVIK NUMERAL FIVE;No;0;L;;;;5;N;;;;
1D2C6;KAKTOVIK NUMERAL SIX;No;0;L;;;;6;N;;;;
1D2C7;KAKTOVIK NUMERAL SEVEN;No;0;L;;;;7;N;;;;
1D2C8;KAKTOVIK NUMERAL EIGHT;No;0;L;;;;8;N;;;;
1D2C9;KAKTOVIK NUMERAL NINE;No;0;L;;;;9;N;;;;
1D2CA;KAKTOVIK NUMERAL TEN;No;0;L;;;;10;N;;;;
1D2CB;KAKTOVIK NUMERAL ELEVEN;No;0;L;;;;11;N;;;;
1D2CC;KAKTOVIK NUMERAL TWELVE;No;0;L;;;;12;N;;;;
1D2CD;KAKTOVIK NUMERAL THIRTEEN;No;0;L;;;;13;N;;;;
1D2CE;KAKTOVIK NUMERAL FOURTEEN;No;0;L;;;;14;N;;;;
1D2CF;KAKTOVIK NUMERAL FIFTEEN;No;0;L;;;;15;N;;;;
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1D2D0;KAKTOVIK NUMERAL SIXTEEN;No;0;L;;;;16;N;;;;
1D2D1;KAKTOVIK NUMERAL SEVENTEEN;No;0;L;;;;17;N;;;;
1D2D2;KAKTOVIK NUMERAL EIGHTEEN;No;0;L;;;;18;N;;;;
1D2D3;KAKTOVIK NUMERAL NINETEEN;No;0;L;;;;19;N;;;;
```

Atomic encoding

We propose atomic encoding of twenty digits, parallel to the Mayan numeral block, where the digits are also composed graphically of fives and units but that is not reflected in their encoding. Other possibilities, per the discussion for Mende (L2/12-023 etc.), are (a) combining characters for the sub-bases of five and (b) ligatures, either of fives and strokes or just of strokes. Either would increase font overhead for no benefit other than reducing the number of characters. Ligatures would be further problematic because it would not be straightforward to distinguish strings of digits from the strings of strokes that form those digits, like encoding Hangul without syllabic blocks. (E.g. conflating 𝋊𝋂 202 with 𝋌 12, which would be a disaster in accounting.) Combining characters would have the complication of requiring two characters each for five, ten and fifteen, one standalone and one combining. Kaktovik digits are treated by their users as unitary symbols for most purposes, with decomposition only relevant for arithmetic – not the province of the font. Either ligatures or combining characters would complicate conversion with English and French vigesimal notation, such as conventional 0–9 + A–K; these notations would have a one-to-one correspondence with an atomic encoding of Kaktovik numerals.

References

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Figures

Iñupiaq Counting System
with Kaktovik Numerals (first column) and Arabic Numeral Equivalents (third column)

𝋀	suitchuq (*see note)	0	𝋁𝋀	iñuiññaq	20	𝋂𝋀	malġukipiak	40
𝋁	atausiq	1	𝋁𝋁	iñuiññaq atausiq	21	𝋂𝋁	malġukipiaq atausiq	41
𝋂	malġuk	2	𝋁𝋂	iñuiññaq malġuk	22	𝋂𝋂	malġukipiaq malġuk	42
𝋃	piŋasut	3	𝋁𝋃	iñuiññaq piŋasut	23	𝋂𝋃	malġukipiaq piŋasut	43
𝋄	sisamat	4	𝋁𝋄	iñuiññaq sisamat	24	𝋂𝋄	malġukipiaq sisamat	44
𝋅	tallimat	5	𝋁𝋅	iñuiññaq tallimat	25	𝋂𝋅	malġukipiaq tallimat	45
𝋆	itchaksrat	6	𝋁𝋆	iñuiññaq itchaksrat	26	𝋂𝋆	malġukipiaq itchaksrat	46
𝋇	tallimat malġuk	7	𝋁𝋇	iñuiññaq tallimat malġuk	27	𝋂𝋇	malġukipiaq tallimat malġuk	47
𝋈	tallimat piŋasut	8	𝋁𝋈	iñuiññaq tallimat piŋasut	28	𝋂𝋈	malġukipiaq tallimat piŋasut	48
𝋉	quliŋŋuġutaiḷaq	9	𝋁𝋉	iñuiññaq quliŋŋuġutaiḷaq	29	𝋂𝋉	malġukipiaq quliŋŋuġutaiḷaq	49
𝋊	qulit	10	𝋁𝋊	iñuiññaq qulit	30	𝋂𝋊	malġukipiaq qulit	50
𝋋	qulit atausiq	11	𝋁𝋋	iñuiññaq qulit atausiq	31	𝋂𝋋	malġukipiaq qulit atausiq	51
𝋌	qulit malġuk	12	𝋁𝋌	iñuiññaq qulit malġuk	32	𝋂𝋌	malġukipiaq qulit malġuk	52
𝋍	qulit piŋasut	13	𝋁𝋍	iñuiññaq qulit piŋasut	33	𝋂𝋍	malġukipiaq qulit piŋasut	53
𝋎	akimiaġutaiḷaq	14	𝋁𝋎	iñuiññaq akimiaġutaiḷaq	34	𝋂𝋎	malġukipiaq akimiaġutaiḷaq	54
𝋏	akimiaq	15	𝋁𝋏	iñuiññaq akimiaq	35	𝋂𝋏	malġukipiaq akimiaq	55
𝋐	akimiaq atausiq	16	𝋁𝋐	iñuiññaq akimiaq atausiq	36	𝋂𝋐	malġukipiaq akimiaq atausiq	56
𝋑	akimiaq malġuk	17	𝋁𝋑	iñuiññaq akimiaq malġuk	37	𝋂𝋑	malġukipiaq akimiaq malġuk	57
𝋒	akimiaq piŋasut	18	𝋁𝋒	iñuiññaq akimiaq piŋasut	38	𝋂𝋒	malġukipiaq akimiaq piŋasut	58
𝋓	iñuiññaġutaiḷaq	19	𝋁𝋓	malġukipiaġutaiḷaq	39	𝋂𝋓	piŋasukipiaġutaiḷaq	59

Figure 1. MacLean (2014: 836 *ff*). The lower numbers, 1 to 59. Note that the Kaktovik numbers in the tens' and fifteens' rows are graphically simpler than those immediately above and below them, and that the corresponding Iñupiaq numbers are lexically simpler than those above and below them. The word for twenty is *iñuiññaq*, but multiples of twenty (up to twenty score) use the suffix *-ipiak* 'score'. The word for 59 is *threescore-less-one*, with the subtractive suffix *-utaiḷaq*.

𝋊𝋀	qulikipiaq	200
𝋏𝋀	akimiakipiaq	300

Figure 2. *Ibid*. Simple combinations of Kaktovik digits generally correspond to single Iñupiaq words. Here the sub-base digits plus a zero, 𝋊𝋀 (2×5×20) and 𝋏𝋀 (3×5×20), are read *qulikipiaq* 'ten-score' and *akimiakipiaq* 'fifteen-score'.

𝋁𝋀𝋀	iñuiññakipiaq (traditional form) or, in reindeer herding and math, iḷagiññaq	400
𝋂𝋀𝋀	malġuagliaq	800
𝋃𝋀𝋀	piŋasuagliaq	1,200
𝋄𝋀𝋀	sisamaagliaq	1,600
𝋅𝋀𝋀	tallimaagliaq	2,000
𝋊𝋀𝋀	quliagliaq	4,000
𝋏𝋀𝋀	akimiagliaq	6,000

Figure 3. *Ibid*. Multiples of 400 (1×, 2×, 3×, 4×, 5×, 10×, 15×). The word for 400, *iñuiññakipiaq*, means *twenty-score*, but multiples of 400 use the suffix *-agliaq*.

𝋓𝋓𝋓	atausiqpautaiḷaq	7,999
𝋁.𝋀𝋀𝋀	atausiqpak	8,000
𝋂.𝋀𝋀𝋀	malġuqpak	16,000
𝋃.𝋀𝋀𝋀	piŋasuqpak	24,000
𝋄.𝋀𝋀𝋀	sisamaqpak	32,000
𝋅.𝋀𝋀𝋀	tallimaqpak	40,000
𝋆.𝋀𝋀𝋀	tallimaqpak atausiqpak	48,000
𝋇.𝋀𝋀𝋀	tallimaqpak malġuqpak	56,000
𝋈.𝋀𝋀𝋀	tallimatqak piŋasuqpak	64,000
𝋉.𝋀𝋀𝋀	tallimaqpak sisamaqpak	72,000
𝋊.𝋀𝋀𝋀	quliqpak	80,000
𝋋.𝋀𝋀𝋀	quliqpak atausiqpak	88,000

Figure 4. *Ibid*. Multiples of 8000 (1×, 2×, 3×, 4×, 5×, 6×, 7×, 8×, 9×, 10×, 11×), using the suffix -*pak*. The number word at top is *8000-less-one*, with the subtractive suffix -*utaiḷaq*. That suffix has no correspondence in the written number.

Table 4. The Decimal System and the Iñupiaq System.

97,531 in the Decimal System		𝋌,𝋃𝋐𝋋 = 97,531 in the Iñupiaq System			
		𝋌	𝋊 (𝋂)	2	forty-thousands
			𝋂	2	eight-thousands
		𝋃	(𝋀)	0	two-thousands
9	ten-thousands		𝋃	3	four-hundreds
7	thousands	𝋐	𝋏 (𝋃)	3	hundreds
5	hundreds		𝋁	1	score
3	tens	𝋋	𝋊 (𝋂)	2	fives
1	ones		𝋁	1	ones

Figure 5. *Ibid*. p. 834. The digits of the number 𝋌,𝋃𝋐𝋋 (97,531) divided into upper and lower portions, corresponding to the upper and lower beads of an Iñupiaq abacus. The last digit, for example, 𝋋 (11), is composed of 𝋊 (10, equivalent to 𝋂 (2) fives) and 𝋁 (1). It is input as 𝋂 (2) upper beads and 𝋁 (1) lower bead.

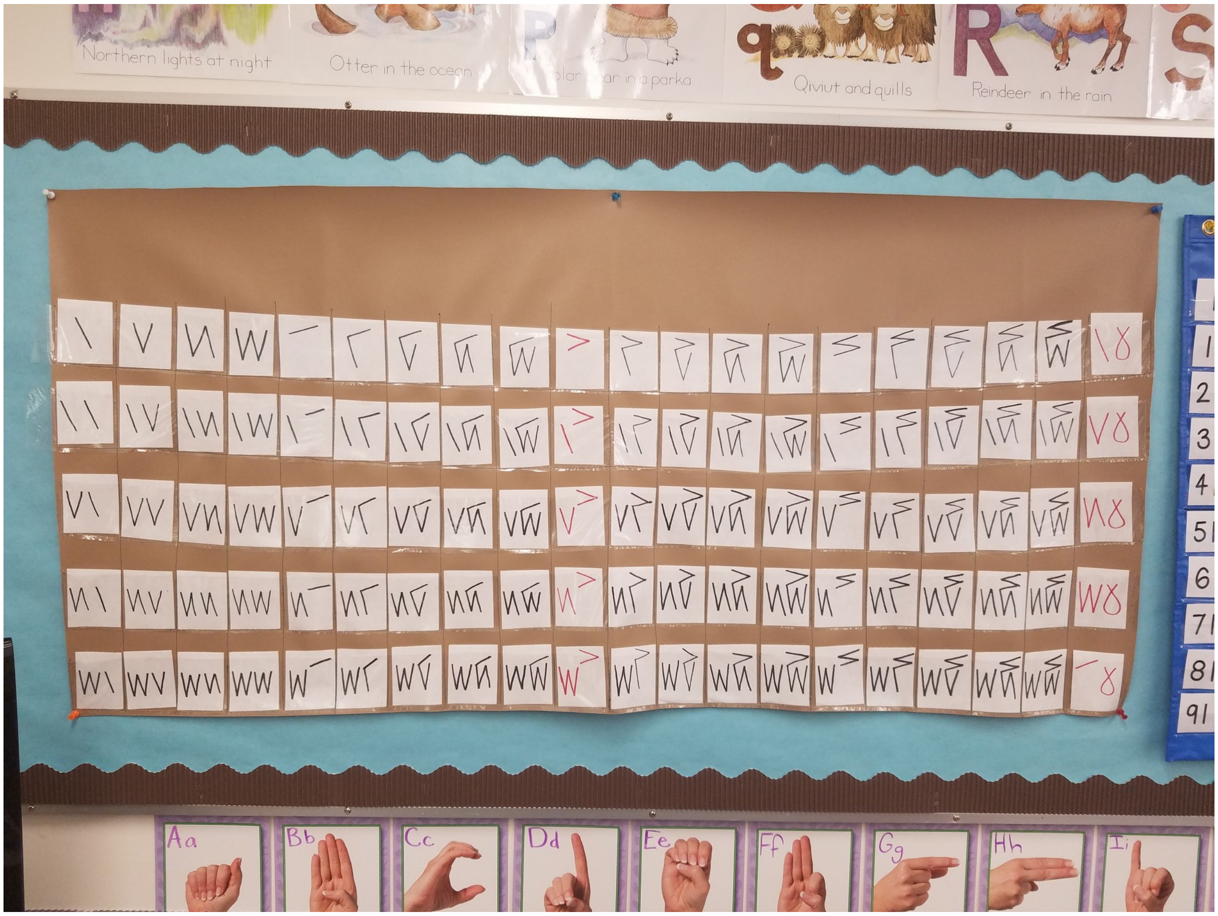


Figure 6. Handwritten forms on the front board in one of our (Catherine Strand's) Early Childhood Education language-immersion class in Kaktovik, 2017–2018 school year. Note that the digits without a sub-base of 5 are not as tall as the digits with a sub-base: the lower strokes do not expand into the upper region of the glyph even when nothing occupies it. Good font design will reflect this stroke arrangement.

Figure 7. Some of Strand's kindergartners with Kaktovik number blocks. The upward sides show 𝋄 4 and 𝋀 0, with counters to match.

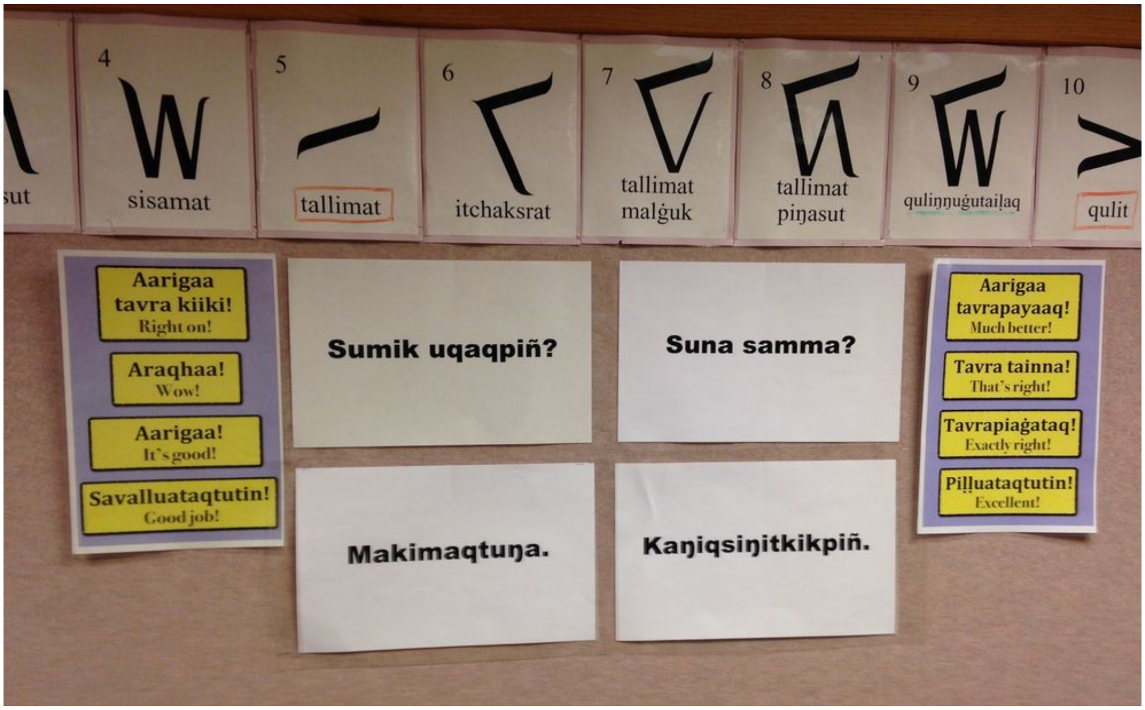


Figure 8. Wohlforth (2016). Front board of a classroom in Utqiaġvik (Barrow), displaying Kaktovik numbers and their Iñupiaq readings.

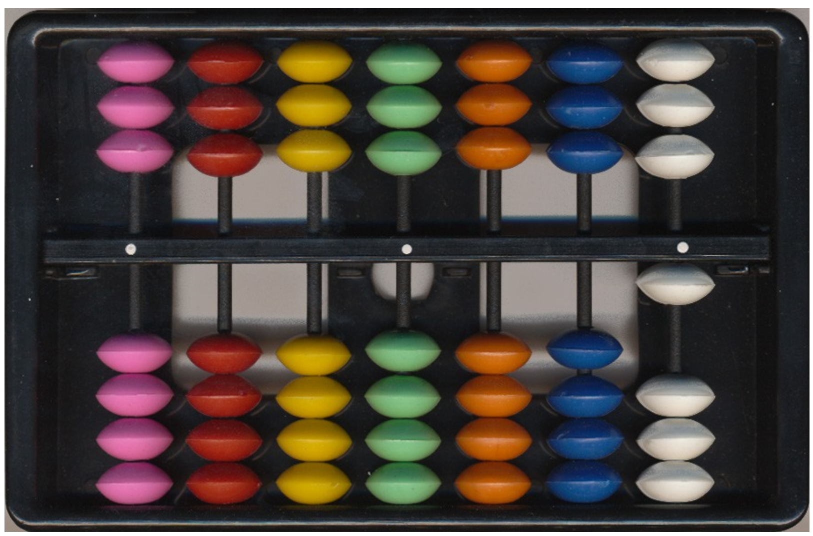


Figure 9. An Iñupiaq base-20 abacus.

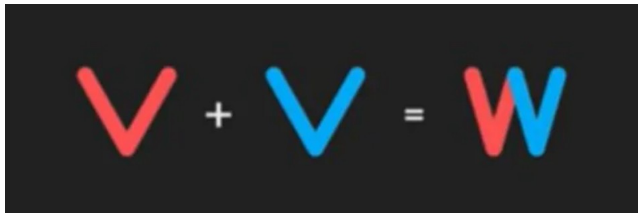


Figure 10. Visual addition (2 + 2 = 4).

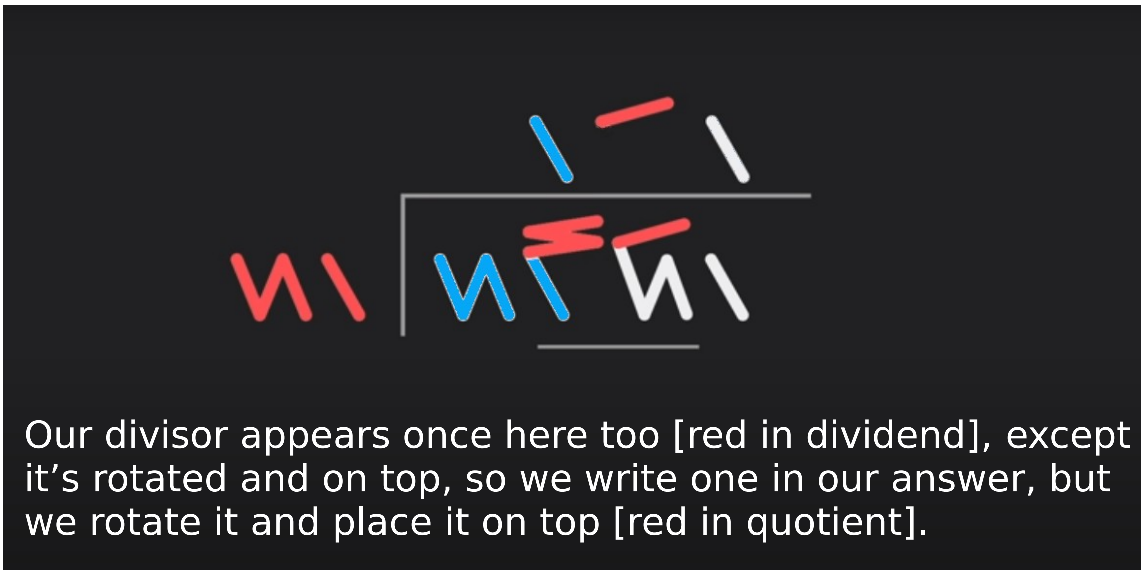


Figure 11. Grunewald (2019: 2m26s). Modified screenshot [some color added] of a subtitled video made by a language enthusiast, demonstrating how long division is easier in Kaktovik numerals than in Hindu-Arabic, due to one being able to see the divisor in the dividend. The problem here is 𝋃𝋐𝋈𝋁 (30,561) ÷ 𝋃𝋁 (61) = 𝋁𝋅𝋁 (501). This is a simple problem, appropriate for introducing the concept to children.

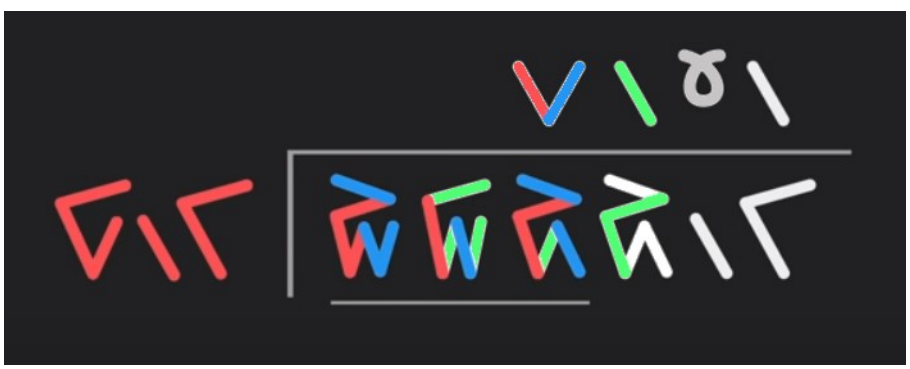
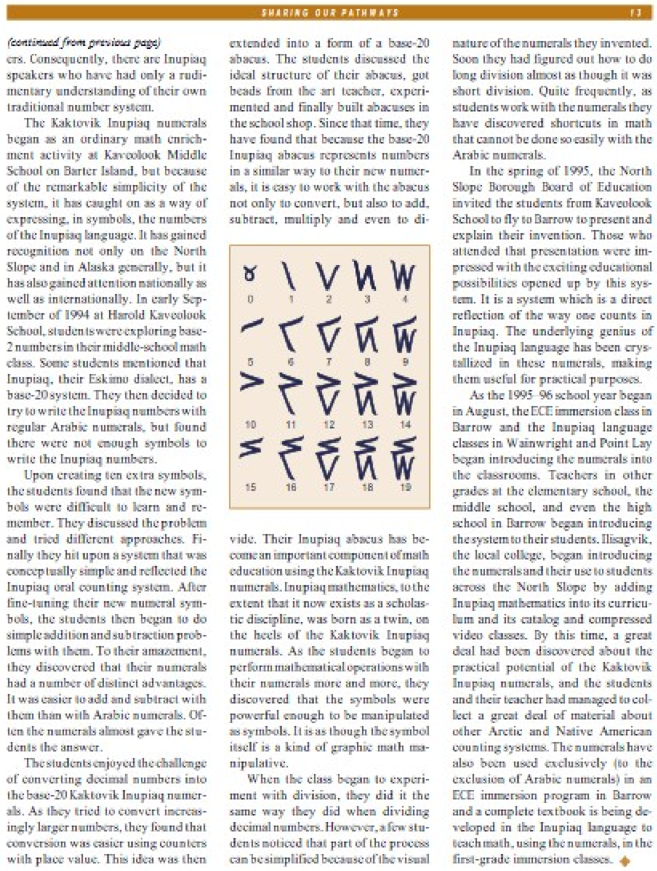


Figure 12. Grunewald (2019: 2m56s). A second modified screenshot, showing that the first digit in the quotient is 𝋂 (2), because the divisor fits into the first three digits of the dividend twice (seen in the red and blue chunking of the underlined digits). The second digit of the quotient is 𝋁 (1), because when we shift over one place to the right, the divisor can be traced in the dividend once (green). The third digit is 𝋀 (0), because when we shift one more place to the right, no strokes remain for chunking. The divisor matches the remaining digits of the dividend (white), for a final digit in the quotient of one.

The problem is 𝋎𝋉𝋍𝋍𝋁𝋆 (46,349,226) ÷ 𝋇𝋁𝋆 (2,826) = 𝋂𝋁𝋀𝋁 (16,401).



(continued from previous page)

ers. Consequently, there are Inupiaq speakers who have had only a rudimentary understanding of their own traditional number system.

The Kaktovik Inupiaq numerals began as an ordinary math enrichment activity at Kaveolook Middle School on Barter Island, but because of the remarkable simplicity of the system, it has caught on as a way of expressing, in symbols, the numbers of the Inupiaq language. It has gained recognition not only on the North Slope and in Alaska generally, but it has also gained attention nationally as well as internationally. In early September of 1994 at Harold Kaveolook School, students were exploring base-2 numbers in their middle-school math class. Some students mentioned that Inupiaq, their Eskimo dialect, has a base-20 system. They then decided to try to write the Inupiaq numbers with regular Arabic numerals, but found there were not enough symbols to write the Inupiaq numbers.

Upon creating ten extra symbols, the students found that the new symbols were difficult to learn and remember. They discussed the problem and tried different approaches. Finally they hit upon a system that was conceptually simple and reflected the Inupiaq oral counting system. After fine-tuning their new numeral symbols, the students then began to do simple addition and subtraction problems with them. To their amazement, they discovered that their numerals had a number of distinct advantages. It was easier to add and subtract with them than with Arabic numerals. Often the numerals almost gave the students the answer.

The students enjoyed the challenge of converting decimal numbers into the base-20 Kaktovik Inupiaq numerals. As they tried to convert increasingly larger numbers, they found that conversion was easier using counters with place value. This idea was then extended into a form of a base-20 abacus. The students discussed the ideal structure of their abacus, got beads from the art teacher, experimented and finally built abacuses in the school shop. Since that time, they have found that because the base-20 Inupiaq abacus represents numbers in a similar way to their new numerals, it is easy to work with the abacus not only to convert, but also to add, subtract, multiply and even to divide. Their Inupiaq abacus has become an important component of math education using the Kaktovik Inupiaq numerals. Inupiaq mathematics, to the extent that it now exists as a scholastic discipline, was born as a twin, on the heels of the Kaktovik Inupiaq numerals. As the students began to perform mathematical operations with their numerals more and more, they discovered that the symbols were powerful enough to be manipulated as symbols. It is as though the symbol itself is a kind of graphic math manipulative.

When the class began to experiment with division, they did it the same way they did when dividing decimal numbers. However, a few students noticed that part of the process can be simplified because of the visual nature of the numerals they invented. Soon they had figured out how to do long division almost as though it was short division. Quite frequently, as students work with the numerals they have discovered shortcuts in math that cannot be done so easily with the Arabic numerals.

In the spring of 1995, the North Slope Borough Board of Education invited the students from Kaveolook School to fly to Barrow to present and explain their invention. Those who attended that presentation were impressed with the exciting educational possibilities opened up by this system. It is a system which is a direct reflection of the way one counts in Inupiaq. The underlying genius of the Inupiaq language has been crystallized in these numerals, making them useful for practical purposes.

As the 1995–96 school year began in August, the ECE immersion class in Barrow and the Inupiaq language classes in Wainwright and Point Lay began introducing the numerals into the classrooms. Teachers in other grades at the elementary school, the middle school, and even the high school in Barrow began introducing the system to their students. Ilisagvik, the local college, began introducing the numerals and their use to students across the North Slope by adding Inupiaq mathematics into its curriculum and its catalog and compressed video classes. By this time, a great deal had been discovered about the practical potential of the Kaktovik Inupiaq numerals, and the students and their teacher had managed to collect a great deal of material about other Arctic and Native American counting systems. The numerals have also been used exclusively (to the exclusion of Arabic numerals) in an ECE immersion program in Barrow and a complete textbook is being developed in the Inupiaq language to teach math, using the numerals, in the first-grade immersion classes.

Figure 13. Bartley (1997: 13). An early account of the system.

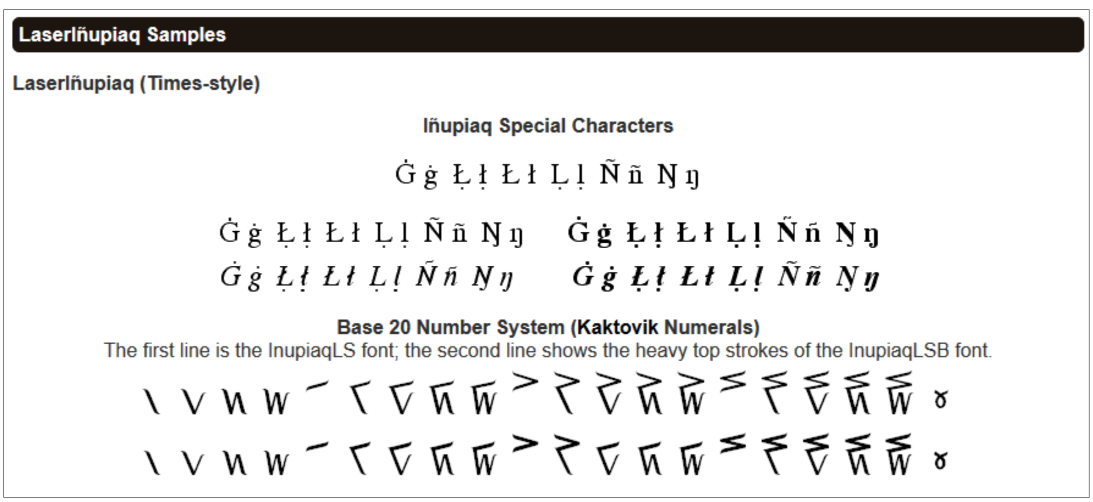


Figure 14. Kaktovik digits in Linguist's Software's *LaserIñupiaq* font (1997, redesigned 2010). This is the font used for MacLean (2014). *LaserYukon* uses the same design. The glyphs were (re)designed to Bartley's specifications, and closely follow Bartley (1997).

ISO/IEC JTC 1/SC 2/WG 2
PROPOSAL SUMMARY FORM TO ACCOMPANY SUBMISSIONS
FOR ADDITIONS TO THE REPERTOIRE OF ISO/IEC 10646[1]
Please fill all the sections A, B and C below.
Please read Principles and Procedures Document (P & P) from std.dkuug.dk/JTC1/SC2/WG2/docs/principles.html
for guidelines and details before filling this form.
Please ensure you are using the latest Form from std.dkuug.dk/JTC1/SC2/WG2/docs/summaryform.html.
See also std.dkuug.dk/JTC1/SC2/WG2/docs/roadmaps.html **for latest *Roadmaps*.**

A. Administrative

1. **Title:** ***Kaktovik numerals***
2. Requester's name: *Eduardo Marín Silva, Kirk Miller & Catherine Strand*
3. Requester type (Member body/Liaison/Individual contribution): *individual*
4. Submission date: *2021 April 29*
5. Requester's reference (if applicable):
6. Choose one of the following:
   This is a complete proposal: *yes*
   (or) More information will be provided later:

B. Technical – General

1. Choose one of the following:
   a. This proposal is for a new script (set of characters):
      Proposed name of script:
   b. The proposal is for addition of character(s) to an existing block: *(in Roadmap)*
      Name of the existing block: *Kaktovik numerals*
2. Number of characters in proposal: *20*
3. Proposed category (select one from below - see section 2.2 of P&P document):

A-Contemporary	x	B.1-Specialized (small collection)		B.2-Specialized (large collection)	
C-Major extinct		D-Attested extinct		E-Minor extinct	
F-Archaic Hieroglyphic or Ideographic		G-Obscure or questionable usage symbols			

4. Is a repertoire including character names provided? *yes*
   a. If YES, are the names in accordance with the “character naming guidelines” in Annex L of P&P document? *yes*
   b. Are the character shapes attached in a legible form suitable for review? *yes*
5. Fonts related:
   a. Who will provide the appropriate computerized font to the Project Editor of 10646 for publishing the standard?
      Kirk Miller or Fred Brennan
   b. Identify the party granting a license for use of the font by the editors (include address, e-mail, ftp-site, etc.):
      (the same)
6. References:
   a. Are references (to other character sets, dictionaries, descriptive texts etc.) provided? *yes*
   b. Are published examples of use (such as samples from newspapers, magazines, or other sources) of proposed characters attached? *yes*
7. Special encoding issues:
   Does the proposal address other aspects of character data processing (if applicable) such as input, presentation, sorting, searching, indexing, transliteration etc. (if yes please enclose information)? *no*

8. Additional Information:

Submitters are invited to provide any additional information about Properties of the proposed Character(s) or Script that will assist in correct understanding of and correct linguistic processing of the proposed character(s) or script. Examples of such properties are: Casing information, Numeric information, Currency information, Display behaviour information such as line breaks, widths etc., Combining behaviour, Spacing behaviour, Directional behaviour, Default Collation behaviour, relevance in Mark Up contexts, Compatibility equivalence and other Unicode normalization related information. See the Unicode standard at http://www.unicode.org for such information on other scripts. Also see Unicode Character Database (http://www.unicode.org/reports/tr44/) and associated Unicode Technical Reports for information needed for consideration by the Unicode Technical Committee for inclusion in the Unicode Standard.

[1] Form number: N4502-F (Original 1994-10-14; Revised 1995-01, 1995-04, 1996-04, 1996-08, 1999-03, 2001-05, 2001-09, 2003-11, 2005-01, 2005-09, 2005-10, 2007-03, 2008-05, 2009-11, 2011-03, 2012-01)

C. Technical - Justification

1. Has this proposal for addition of character(s) been submitted before? *no*
   If YES explain
2. Has contact been made to members of the user community (for example: National Body, user groups of the script or characters, other experts, etc.)? *yes*
   If YES, with whom? *Ronald H Brower Sr., Edna Ahgeak MacLean*
   If YES, available relevant documents:
3. Information on the user community for the proposed characters (for example: size, demographics, information technology use, or publishing use) is included?
   Reference:
4. The context of use for the proposed characters (type of use; common or rare) *numeric*
   Reference:
5. Are the proposed characters in current use by the user community? *yes*
   If YES, where? Reference: *Inupiat community, Alaska*
6. After giving due considerations to the principles in the P&P document must the proposed characters be entirely in the BMP? *no*
   If YES, is a rationale provided?
   If YES, reference:
7. Should the proposed characters be kept together in a contiguous range (rather than being scattered)? *yes*
8. Can any of the proposed characters be considered a presentation form of an existing character or character sequence? *no*
   If YES, is a rationale for its inclusion provided?
   If YES, reference:
9. Can any of the proposed characters be encoded using a composed character sequence of either existing characters or other proposed characters? *no*
   If YES, is a rationale for its inclusion provided?
   If YES, reference:
10. Can any of the proposed character(s) be considered to be similar (in appearance or function) to, or could be confused with, an existing character? *no*
    If YES, is a rationale for its inclusion provided?
    If YES, reference:
11. Does the proposal include use of combining characters and/or use of composite sequences? *no*
    If YES, is a rationale for such use provided?
    If YES, reference:
    Is a list of composite sequences and their corresponding glyph images (graphic symbols) provided?
    If YES, reference:
12. Does the proposal contain characters with any special properties such as control function or similar semantics? *no*
    If YES, describe in detail (include attachment if necessary)
13. Does the proposal contain any Ideographic compatibility characters? *no*
    If YES, are the equivalent corresponding unified ideographic characters identified?
    If YES, reference: