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Title: Proposal to encode Leibnizian ambiguous signs

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Background

For background information about the Philiumm project (headed by Prof. David Rabouin, Paris) and the related research work, please visit the Philiumm website.

Leibnizian ambiguous signs

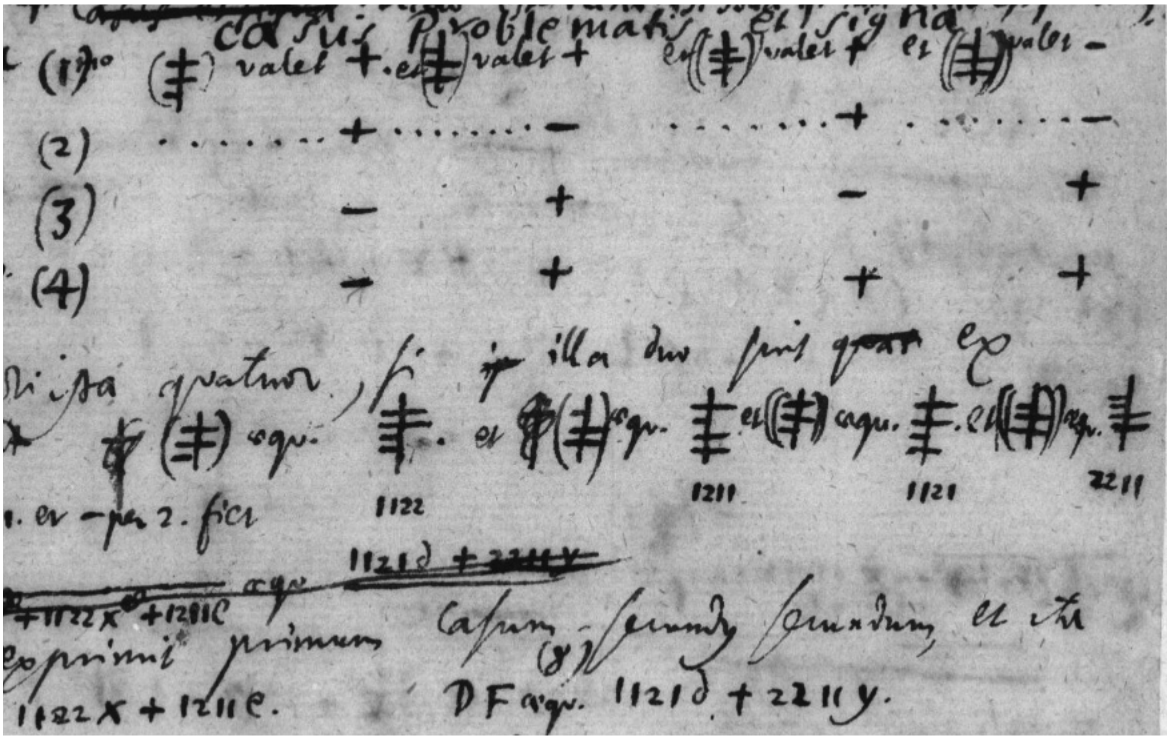
This proposal requests the encoding of 57 ambiguous operator signs (testified in works of Gottfried Wilhelm Leibniz) of which 6 characters count as variation sequences.
This document includes an in-depth explanation about the systematics and semantics of these signs. This explanation (author: Elisabeth Rinner) has been issued previously as doc. L-2404 (February 15, 2024).
The ambiguous characters are related to the well-known ± and ∓ characters (00B1, 2213), both by their graphical structure and historically. For editorial work the ambiguous signs are important for e.g. ascribing dates to manuscript sources which lack an original *datum*. The signs also inform about Leibniz's way of systematic thinking about how to notate certain logical concepts.
We propose an encoding scheme of complete sets of ambiguous signs because incomplete sets would be useless for editorial purposes.

1st Revision of proposal

Upon the first version N5329 (L-2504) we received the recommendation from the UTC, to introduce a different naming scheme for the ambiguous signs. Following that advice, we now propose character names which identify each character's semantical content with possible accuracy.
The initial nomenclature with A-xx, B-yy, C-zz numbering is, however, retained as *GWLB-ID* numbers, for sake of brevity and practical reasons. This nomenclature has been established at Gottfried-Wilhelm-Leibniz-Bibliothek Hannover (GWLB), for research and editorial usage.

2nd Revision of proposal

Subgroup B has seen a few changes. The wording “component” in the character names has been replaced by the more appropriate term “partial”. Some characters formerly named “variant” are now proposed as variation sequences. Despite these changes we kept the order of listing as it was, for ease of orientation.



Example of ambiguous signs in one of Leibniz’s manuscripts. The signs are based on the + and – symbols. – (LH 35 XII 1, 217v)

1. Introductory remarks on Leibnizian Ambiguity signs

Early in his career, Leibniz wrote several texts in which he designed and systematically examined systems of symbols for analytical calculations. Complex systems of ambiguity signs, with which more than two cases are distinguished, represent an elementary and novel component of this *Méthode de l’universalité*. As part of the *Ars Characteristica*, the treatment of this method belongs to that branch of philosophy that is “the art of forming and arranging characters so that they agree with thoughts” (Mugnai 2018, abstract).

However, Leibniz’s interest is not only theoretical. Rather, the design of higher ambiguity signs is closely linked to his occupation with the mathematics of conic sections. There he has to consider sub-cases of cases, but would like to write only one equation to treat them all together, since often the equations do not differ except for the signs of the terms. The use of double signs, which allows to represent two cases simultaneously, is already part of common practice in mathematics. The characters ± and ∓, which are still in use today, are used for this purpose.

According to current knowledge, Leibniz designed six different systems over the course of time—as long as transitional forms and preliminary considerations are ignored.

One reason why a system of ambiguity signs is abandoned by Leibniz is the consideration that a large number of specific new printing types are required if a system does not rely on the traditional set of printing types. Leibniz’s further penetration of the topic also led to improved, simpler or, in some cases, even more complex characters. The draft of the first system, for example, provides for special characters to express the product of the two double signs A-01 ∓ and A-02 ≢. For these, Leibniz envisages the ligature A-07 ⩱ and A-08 ⩲ of these two symbols with the LEIBNIZIAN PRODUCT SIGN he typically uses. Only later does he take advantage of the fact that the mathematical meaning can also be expressed using existing symbols.

Some systems also take into account the relationship between several ambiguity signs in the same expression. Ambiguity signs can be *homogeneous* or *corresponding* and therefore dependent on one another, as well as *heterogeneous* and therefore independent of one another.

Likewise, it was only in his 5th system that Leibniz gave up structuring ambiguity signs according to the distinction between cases and sub-cases as they arise in the calculation process. Even though, from the perspective of modern mathematics, it makes no difference with regard to calculations whether the ambiguity sign (*mp*)*m* (i. e. a sign which has the sub-cases *mp* in the first case and *m* in the second case, with *p* as abbreviation for plus and *m* for minus) or *m*(*pm*) (i. e. a sign which has *m* in the first case and the sub-cases *pm* in the second case) is used, they do refer to two fundamentally different conceptions of the mathematical situation.

Design questions also play a role when considering the layout of systems of ambiguity signs, which lead Leibniz to the discussion of different positioning of lines and thus to variants that are compared to the systems ultimately favored.

Particularly in Leibniz's drafts, the ambiguity signs that occur can contribute to the dating of the texts, as a sequence of systems can be observed.

2. Overview of systems and character names for the UCS

Some systems of ambiguity signs are designed in such a way that they can be extended to distinguish any number of cases. In systems that use specific new characters and do not use the existing character set of a typesetting box, the surviving texts only contain characters that distinguish a maximum of four different cases. Usually, not all possible combinations of *p* and *m* occur in the texts. However, the systematics described or reconstructed on the basis of the surviving texts often allows to reconstruct the full set of ambiguity signs that belong to a system.

The overview in the appendix therefore contains only systems that use special new characters. For them, a list of all possible cases is provided. A representation of their glyphs is given, provided their use is documented in the texts written by Leibniz. In the overview, the meaning of the ambiguity signs is also stated in an abbreviated form.

For the encoding of Leibniz's ambiguity signs in the Unicode standard, we propose a name consisting of the components "AMBIGUITY SIGN", an identifier for the system ("A" for system 1, "B" for system 2, and "C" for system 5) which is followed by a hyphen, and a sequential number, with a leading zero being added to single-digit numbers. The character-specific parts of the proposed names are also included in the overview.

The characteristics of the systems are briefly described below with references to the overview.

2. 1 System 1

System 1 is based on the signs + for *p* and – for *m* that are still in use today, with A-01 ∓ being understood as a combination of these signs. The additional bar in A-02 ≢ represents negation, so the sign stands for *mp*.

When looking at the layout of the triple signs A-03 +∓, A-04 +≢, A-05 −∓ and A-06 −≢, it can be seen that Leibniz takes the structure of possible distinctions of cases and sub-cases into account. In the texts, only signs for which a distinction between two sub-cases arises in the second case are described and documented. This second case with both sub-cases is represented in the right part of the sign in analogy to the associated double signs A-01 and A-02: the two upper crossbars suspended from the vertical bar again refer to the combination of *p* and *m*, while the third, lower bar appearing in A-04 and A-06 represents the negation of this part of the ambiguity sign. The value of the first case is on the left side (*p* or *m*). This part is connected to the second crossbar from the top in the right part of the ambiguity sign.

A special feature of the 1st system are the ambiguity signs A-07 and A-08 that stand for products of the double signs of the system.

2. 2 System 2

System 2 develops from the same combination of + and – that forms the sign A-01. Unlike in the first system, the negation of the sign is not expressed by a third crossbar with the same width, but by a longer, horizontal bar that is placed at the bottom of the vertical bar (e. g. in B-01). Such negations of the complete ambiguity sign can be applied to all of them, with B-05, B-06, B-07, and B-10 being examples.

As before, triple signs are composed of signs and double signs, with the first case on the left (subdivided or not) and the second case (not subdivided or subdivided) on the right. During this transition from double sign to triple sign, the negation bar of B-01 slides upwards, so to speak, so that the vertical bar in the partial sign that is given in one of Leibniz's texts, which rather coincidentally has the same design as A-02, now protrudes at the bottom, and the width of the crossbar is adjusted to that of others in the sign. In contrast to the first system, both parts of the triple sign are composed by connecting the horizontal bar of the single sign part to the top bar of the double sign part. The ambiguity signs B-02, B-03, B-04, B-05, B-06, B-07, B-08, B-09 and B-10 are of this kind.

The principle of composition is meant to be continued for distinguishing further ambiguities (i. e. the type of combination of cases that are distinguished). B-11 and B-12 which represent the negation of *p*(*mp*) and *m*(*mp*) make it clear that the negation bar is "moved up" again in these partial signs, and that its size corresponds to the size of all other crossbars.

As Leibniz discussed questions about the suitability of different positions of the crossbars when designing this system, the six ambiguity signs B-13, B-14, B-15, B-16, B-17, and B-18 have come down to us. They represent variants of ambiguity signs of the standard form.

2. 3 System 3

Leibniz builds system 3 from the ambiguity signs A-01 and B-01, which are used for *pm* and *mp* in the 2nd system. This means a reduction of the number of characters required, while still any complex ambiguity as well as all dependencies between ambiguity signs (i. e. whether they are *homogeneous*, *corresponding* or *heterogeneous*) can be expressed. To do this, numbers are added to the left and right of A-01 and B-01 according to certain given rules. Levels of case distinctions can also be expressed by building nested expressions according to rules. The entire expression is marked by a bracket with *vinculum* (i. e. they are connected by an overline).

As a ligature of the brackets "(" and ")" with the *vinculum* is needed, LEFT VIRGULA PARANTHESIS and RIGHT VIRGULA PARANTHESIS are included in the proposal to encode these expressions. The overview at the end of the document does not contain any characters that are specifically assigned to this system since the glyphs of the ambiguity signs used in this system match with those of signs from the first two systems.

2. 4 System 4

System 4 has no new characters at all and instead uses lowercase letters of the Greek alphabet. Ambiguity is expressed by strings of certain pairs of letters such as α and ω, β and ψ, or γ and χ, where the two letters are equidistant from the beginning or end of the alphabet, and the letter from the beginning stands for *p* and the other for *m*. As in the 3rd system, the letters are written one after the other, following the order of the cases, and marked by brackets and a *vinculum*.

By using several pairs of letters it is possible to express the relationship between the ambiguity signs that occur in an expression, because the same pair of letters is used for interdependent ambiguity signs and different pairs of letters for independent ones. The level of cases can be represented by structuring these sequences with commas.

2. 5 System 5

The principles of the standard form of system 5 and all its predecessors were reconstructed on the basis of ambiguity signs that can be found in Leibniz's manuscripts.

In its final version (see 3. 5 subgroup "Standard" in the overview), the 5th system probably has the simplest structure in the design of the glyphs. The set of *n*-fold ambiguity signs consists of almost all possible *n*-combinations of *p* and *m*. All the signs with the meaning *pp…p* (string with *n* characters) and *mm…m* (string with *n* characters) are omitted as they have the same meaning as + and –. The level of cases is not represented in this system. Likewise, dependencies that occur between several ambiguity signs of the same expression are not represented.

The double signs C-16 and C-17, the triple signs C-18, C-19, C-20, C-21, C-22, and C-23 as well as the quadruple signs C-24, C-25, C-26, C-27, C-28, C-29, C-30, and C-31 belong to this group.

Their design follows a uniform principle. On a vertical bar, horizontal bars of equal length are positioned at equal distances depending on the cases distinguished in the sign. If a bar represents *p*, it is bisected by the vertical bar. If it stands for *m*, it starts on the left at the same distance from the vertical bar as the *p*-bars, but already ends at the vertical bar.

This approach was derived from the 2nd system, and there is a total of four stages in the development of the standard version of system 5 of which only few examples of ambiguity signs have been preserved. The representation of the level of case distinction is a common feature of all these systems.

- The "Transition Form 2 → 5" (subgroup 3. 1) preserves the division into left and right part from the 2nd system as a means to represent two cases on the top level of case distinction. What is new, however, is the reduction of the representation of the cases on the second level, where, among other things, a crossbar shortened to the half width appears for the first time. The ambiguity sign C-01 has been handed down in this group.
- In the ambiguity signs of the group "prae-pro-proto-5" (subgroup 3. 2), instead of being divided into left and right halves to distinguish the two cases, there is a subdivision of the sign into an upper and lower section which is arranged along a vertical bar. In the case of the two known ambiguity signs C-02 and C-03, a subordinate case distinction occurs in the first (upper) case, where *p* and *m* are expressed by a long and a short horizontal bar, respectively, which are positioned in the middle of the vertical bar and to its left, respectively. A connection of these two

horizontal bars by a short vertical bar at their left which ends above the upper horizontal bar illustrates that they belong to the same group of sub-cases.

- From the group “pro-proto-5” (subgroup 3. 3), only the ambiguity signs C-04 ‡ and C-05 ‡ have come down to us. For both, signs that represent the same type of ambiguity in the previous group “prae-pro-proto-5” (subgroup 3. 2) are also known. Compared to them, the upper halves of the short vertical bars which illustrate in the previous variant that the horizontal bars that are connected by them belong to the same group of sub-cases are omitted, so that the resulting glyphs are further reduced compared to their predecessors.
- The group “proto-5” (subgroup 3. 4) already shows very close similarities to the standard form. However, Leibniz continues to distinguish the levels of case distinction in the triple signs, with the first case or its two sub-cases being shown in the upper part of the signs, the second case or its sub-cases in the lower section. C-09 ‡ and C-10 ‡, which stand for *p*(*mp*) and (*pm*)*p* respectively, differ only in the positions of the short crossbar that represents *m*. They have the same meaning as long as the levels of case distinction are ignored. The triple signs C-08 ‡ and C-11 ‡ also belong to this group, as well as the double signs C-06 † and C-07 †.
  In this system there are also composed forms: C-12 ∓ represents a composition of *mp* and *p*(*mp*) according to a rule, the negation is C-13 ∓. There are also combinations based on this: in C-14 ‡, C-13 ∓ occurs as the second case of an ambiguity sign whose other case is *p*, while in C-15 ‡ it is the first case, again in combination with *p*.

Thus, it is giving up the representation of the level of case distinctions and the choice of equal distances that ultimately constitute the final step towards the standard form. At the same time, this reduces the number of ambiguity signs to be taken into account. While in its standard form the continuation of the system for distinguishing more cases is known, it is not clear from the surviving texts for the transitional and preliminary forms.

2. 6 System 6

System 6 which can be derived from several manuscripts shares its basic idea with system 4. In all examples known from these texts, *p* is expressed by 1. For *m*, Leibniz uses 3 in one text and 2 in all others.

By relying entirely on character types which are included in the usual typesetting box, this system does not have to be taken into account in the proposal.

References and additional literature

Mugnai 2018 Massimo Mugnai, *Ars Characteristica, Logical Calculus,and Natural Languages*, in: Maria Rosa Antognazza (ed.): *The Oxford Handbook of Leibniz*, Oxford University Press. Oxford 2018, p. 177-207 (published online as https://doi.org/10.1093/oxfordhb/9780199744725.013.20).

Probst & Trunk 2019 Siegmund Probst and Achim Trunk: *Einleitung*, in: Gottfried Wilhelm Leibniz: *Sämtliche Schriften und Briefe*, Vol. VII, 7.

Trunk 2016-2017 Achim Trunk, *Sechs Systeme: Leibniz und seine signa ambigua*, in: Wenchao Li (ed.): *Für unser Glück oder das Glück anderer, Vorträge des X. Internationalen Leibniz-Kongresses*. Hildesheim 2016-2017, vol. 4.

Overview of Leibniz's systems of ambiguity signs

The columns in this overview contain the following information:

1 number within the system
2 mathematical meaning
3 specific part of proposed character name (if it is part of the proposal)
4 representative glyph
5 ID in the proto standard as defined by the Leibniz-Edition (LE)
6 number and glyph in the current font of the Leibniz-Edition (if existing). The current glyphs can deviate from their actual layout.

The following signs are used to express the meaning of the signs:

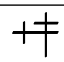
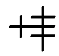
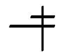
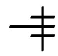
p plus
m minus
(…) group of sub-cases
non[…] negation
· multiplication of signs
∘ composition of signs

1 System 1

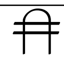
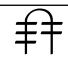
1. 1 Double Signs

nr.	*meaning*	*character name ID*	*repr. glyph*	*ID in proto standard of LE*	*glyphs used in the Leibniz Edition (LE)*
1	Sys. 1 pm (= sys. 2 pm)	A-01		T-01 = T-19	12: 
2	Sys. 1 mp (= sys. 2 (parts) mp)	A-02		T-20 = T-02	13: 

1. 2 Triple Signs

1	Sys. 1 p(pm)	A-03		T-03	—
2	Sys. 1 p(mp)	A-04		T-04	—
3	Sys. 1 m(pm)	A-05		T-05	—
4	Sys. 1 m(mp)	A-06		T-06	—

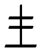
1. 3 Multiplication Forms

1	Sys. 1 pm · pm	A-07		T-07	—
2	Sys. 1 mp · pm	A-08		T-08	—

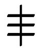
2 System 2

2. 1 Double Signs

2. 1. 1 Standard Forms

1	Sys. 2 pm (= sys. 1 pm)	(A-01)		T-01 = T-19	6: 
2	Sys. 2 mp	B-01		T-12	7: 

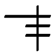
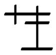
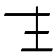
2. 1. 2 Standard Forms (Parts)

1	Sys. 2 (parts) pm	—	—	—	—
2	Sys. 2 (parts) mp (= sys. 1 mp)	(A-02)		T-20 = T-02	230: 

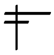
2. 2 Triple Signs

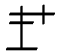
2. 2. 1 Standard Forms

2. 2. 1 a) Type _ (_ _)

1	Sys. 2 p(pm)	B-02		R-118	118: 
2	Sys. 2 p(mp)	B-03		T-10	120: 
3	Sys. 2 m(pm)	—	—	—	—
4	Sys. 2 m(mp)	B-04		T-09	—
5	Sys. 2 *non*[p(pm)]	B-05		T-15	222: 
6	Sys. 2 *non*[p(mp)]	B-06		R-119	119: 
7	Sys. 2 *non*[m(pm)]	B-07		R-84	84: 
8	Sys. 2 *non*[m(mp)]	—	—	—	—

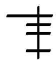
2. 2. 1 b) Type (_ _) _

9	Sys. 2 (pm)p	B-08		T-17	231: 
10	Sys. 2 (mp)p	—	—	—	—
11	Sys. 2 (pm)m	B-09		R-233	233: 

12	Sys. 2 (mp)m	—	—	—	—
13	Sys. 2 *non*[(pm)p]	B-10		R-234	234: 
14	Sys. 2 *non*[(mp)p]	—	—	—	—
15	Sys. 2 *non*[(pm)m]	—	—	—	—
16	Sys. 2 *non*[(mp)m]	—	—	—	—

2. 2. 2 Standard Forms (Parts)

2. 2. 2 a) Type _ (_ _)

1	Sys. 2 (parts) p(pm)	—	—	—	—
2	Sys. 2 (parts) p(mp)	—	—	—	—
3	Sys. 2 (parts) m(pm)	—	—	—	—
4	Sys. 2 (parts) m(mp)	—	—	—	—
5	Sys. 2 (parts) *non*[p(pm)]	—	—	—	—
6	Sys. 2 (parts) *non*[p(mp)]	B-11		T-25	224: 
7	Sys. 2 (parts) *non*[m(pm)]	—	—	—	—
8	Sys. 2 (parts) *non*[m(mp)]	B-12		R-226	226: 

2. 2. 2 b) Type (_ _) _

9	Sys. 2 (parts) (pm)p	—	—	—	—
10	Sys. 2 (parts) (mp)p	—	—	—	—
11	Sys. 2 (parts) (pm)m	—	—	—	—
12	Sys. 2 (parts) (mp)m	—	—	—	—
13	Sys. 2 (parts) *non*[(pm)p]	—	—	—	—
14	Sys. 2 (parts) *non*[(mp)p]	—	—	—	—
15	Sys. 2 (parts) *non*[(pm)m]	—	—	—	—
16	Sys. 2 (parts) *non*[(mp)m]	—	—	—	—

2. 2. 3 Variants

1	Sys. 2 p(pm)	B-13		T-13	228:
2	Sys. 2 p(mp)	B-14		T-14	223:
6	Sys. 2 *non*[p(mp)]	B-15		T-16	225:
10	Sys. 2 (mp)p	B-16		T-18	229:
6	Sys. 2 (part) *non*[p(mp)]	B-17		R-232	232:
6	Sys. 2 (part) *non*[p(mp)]	B-18		R-227	227:

3 System 5

3. 1 Subgroup “Transition Form 2 → 5”

—	Sys. Transition Form 2 to 5 p(mp)	C-01		T-75	—

3. 2 Subgroup “prae-pro-proto-5”

—	Sys. prae-pro-proto-5 (pm)p	C-02		T-72	—
—	Sys. prae-pro-proto-5(mp)p	C-03		T-73	—

3. 3 Subgroup “pro-proto-5”

—	Sys. pro-proto-5 (pm)p	C-04		R-220	220:
—	Sys. pro-proto-5 (mp)p	C-05		T-74	221:

3. 4 Subgroup “proto-5”

3. 4. 1 Double Signs

1	Sys. proto-5 pm	C-06		T-46	—
2	Sys. proto-5 mp	C-07		T-71	—

3. 4. 2 Triple Signs

1	Sys. proto-5 p(pm)	C-08		T-69	—
2	Sys. proto-5 p(mp)	C-09		T-47	—
3	Sys. proto-5 m(pm)	—	—	—	—

4	Sys. proto-5 m(mp)	—	—	—	—
5	Sys. proto-5 (pm)p	C-10		T-42	—
6	Sys. proto-5 (mp)p	C-11		T-70	—
7	Sys. proto-5 (pm)m	—	—	—	—
8	Sys. proto-5 (mp)m	—	—	—	—

3. 4. 3 Composed Forms

—	Sys. proto-5 mp ∘ p(mp)	C-12		T-41	—
—	Sys. proto-5 *non*[mp ∘ p(mp)]	C-13		T-48	—
—	Sys. proto-5 p*non*[mp ∘ p(mp)]	C-14		T-44	—
—	Sys. proto-5 *non*[mp ∘ p(mp)]p	C-15		T-43	—

3. 5 Subgroup “Standard”

3. 5. 1 Double Signs

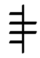
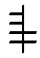
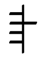
1	Sys. 5 pm	C-16		T-55	8:
2	Sys. 5 mp	C-17		T-56	38:

3. 5. 2 Triple Signs

1	Sys. 5 ppm	C-18		T-57	99:
2	Sys. 5 pmp	C-19		T-58	39:
3	Sys. 5 mpp	C-20		T-45	85:
4	Sys. 5 pmm	C-21		T-59	40:
5	Sys. 5 mpm	C-22		T-60	42:
6	Sys. 5 mmp	C-23		T-61	41:

3. 5. 3 Quadruple Signs

1	Sys. 5 pppm	C-24		T-66	46:
2	Sys. 5 ppmp	C-25		T-65	45:
3	Sys. 5 pmpp	C-26		T-64	44:
4	Sys. 5 mppp	C-27		T-63	43:
5	Sys. 5 ppmm	C-28		T-67	47:

6	Sys. 5 pmpm	—	—	—	—
7	Sys. 5 mppm	C-29		R-49	—
8	Sys. 5 pmmp	—	—	—	—
9	Sys. 5 mpmp	—	—	—	—
10	Sys. 5 mmpp	C-30		T-68	—
11	Sys. 5 pmmm	—	—	—	—
12	Sys. 5 mpmm	C-31		T-62	48: 
13	Sys. 5 mmpm	—	—	—	—
14	Sys. 5 mmmp	—	—	—	—

Introduction and tables by Elisabeth Rinner

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online

Leibniz-Akademie-Ausgabe (LAA, general edition of Leibniz's writings)
LAA series VII (mathematical manuscripts, volumes 3 to 7 available online)
LAA series VII volume 3
LAA series VII volume 5
LAA series VII volume 7

3. Characters proposed for encoding

If this proposal gets accepted, the following characters will exist:

Glyph	**Nº**	**GWLB -ID**	**Unicode Character Names**
	1	A-01	LEIBNIZIAN SYSTEM 1 AND 2 PLUS-MINUS AMBIGUOUS SIGN
	2	A-02	LEIBNIZIAN SYSTEM 1 MINUS-PLUS AMBIGUOUS SIGN
	3	A-03	LEIBNIZIAN SYSTEM 1 PLUS OR PLUS-MINUS AMBIGUOUS SIGN
	4	A-04	LEIBNIZIAN SYSTEM 1 PLUS OR MINUS-PLUS AMBIGUOUS SIGN
	5	A-05	LEIBNIZIAN SYSTEM 1 MINUS OR PLUS-MINUS AMBIGUOUS SIGN
	6	A-06	LEIBNIZIAN SYSTEM 1 MINUS OR MINUS-PLUS AMBIGUOUS SIGN
	7	A-07	LEIBNIZIAN SYSTEM 1 PLUS-MINUS TIMES PLUS-MINUS AMBIGUOUS SIGN
	8	A-08	LEIBNIZIAN SYSTEM 1 MINUS-PLUS TIMES PLUS-MINUS AMBIGUOUS SIGN
	9	B-01	LEIBNIZIAN SYSTEM 2 MINUS-PLUS AMBIGUOUS SIGN
	10	B-02	LEIBNIZIAN SYSTEM 2 PLUS OR PLUS-MINUS AMBIGUOUS SIGN
	11	B-03	LEIBNIZIAN SYSTEM 2 PLUS OR MINUS-PLUS AMBIGUOUS SIGN
	12	B-04	LEIBNIZIAN SYSTEM 2 MINUS OR MINUS-PLUS AMBIGUOUS SIGN
	13	B-05	LEIBNIZIAN SYSTEM 2 NEGATED PLUS OR PLUS-MINUS AMBIGUOUS SIGN
	14	B-06	LEIBNIZIAN SYSTEM 2 NEGATED PLUS OR MINUS-PLUS AMBIGUOUS SIGN
	15	B-07	LEIBNIZIAN SYSTEM 2 NEGATED MINUS OR PLUS-MINUS AMBIGUOUS SIGN
	16	B-08	LEIBNIZIAN SYSTEM 2 PLUS-MINUS OR PLUS AMBIGUOUS SIGN
	17	B-09	LEIBNIZIAN SYSTEM 2 PLUS-MINUS OR MINUS AMBIGUOUS SIGN
	18	B-10	LEIBNIZIAN SYSTEM 2 NEGATED PLUS-MINUS OR PLUS AMBIGUOUS SIGN
	19	B-11	LEIBNIZIAN SYSTEM 2 NEGATED PLUS OR MINUS-PLUS PARTIAL AMBIGUOUS SIGN
	20	B-12	LEIBNIZIAN SYSTEM 2 NEGATED MINUS OR MINUS-PLUS PARTIAL AMBIGUOUS SIGN
	21	*B-13*	*variation sequence to B-02*
	22	*B-14*	*variation sequence to B-03*
	23	*B-15*	*variation sequence to B-06*
	24	B-16	LEIBNIZIAN SYSTEM 2 NEGATED PLUS-MINUS OR PLUS PARTIAL AMBIGUOUS SIGN
	25	*B-17*	*variation sequence to B-11*
	26	*B-18*	*variation sequence to B-11*
	27	C-01	LEIBNIZIAN SYSTEM 5-1 PLUS OR PLUS-MINUS AMBIGUOUS SIGN
	28	C-02	LEIBNIZIAN SYSTEM 5-2 PLUS-MINUS OR PLUS AMBIGUOUS SIGN
	29	C-03	LEIBNIZIAN SYSTEM 5-2 MINUS-PLUS OR PLUS AMBIGUOUS SIGN
	30	C-04	LEIBNIZIAN SYSTEM 5-3 PLUS-MINUS OR PLUS AMBIGUOUS SIGN
	31	C-05	LEIBNIZIAN SYSTEM 5-3 MINUS-PLUS OR PLUS AMBIGUOUS SIGN
	32	C-06	LEIBNIZIAN SYSTEM 5-4 PLUS OR MINUS AMBIGUOUS SIGN
	33	C-07	LEIBNIZIAN SYSTEM 5-4 MINUS OR PLUS AMBIGUOUS SIGN
	34	C-08	LEIBNIZIAN SYSTEM 5-4 PLUS OR PLUS-MINUS AMBIGUOUS SIGN
	35	C-09	LEIBNIZIAN SYSTEM 5-4 PLUS OR MINUS-PLUS AMBIGUOUS SIGN
	36	C-10	LEIBNIZIAN SYSTEM 5-4 PLUS-MINUS OR PLUS AMBIGUOUS SIGN
	37	C-11	LEIBNIZIAN SYSTEM 5-4 MINUS-PLUS OR PLUS AMBIGUOUS SIGN
	38	C-12	LEIBNIZIAN SYSTEM 5-5 INVERTED MINUS-PLUS AMBIGUOUS SIGN
	39	C-13	LEIBNIZIAN SYSTEM 5-5 INVERTED PLUS-MINUS AMBIGUOUS SIGN
	40	C-14	LEIBNIZIAN SYSTEM 5-5 INVERTED PLUS OR PLUS-MINUS AMBIGUOUS SIGN
	41	C-15	LEIBNIZIAN SYSTEM 5-5 INVERTED PLUS-MINUS OR PLUS AMBIGUOUS SIGN
	42	C-16	LEIBNIZIAN SYSTEM 5 PLUS-MINUS AMBIGUOUS SIGN
	43	C-17	LEIBNIZIAN SYSTEM 5 MINUS-PLUS AMBIGUOUS SIGN
	44	C-18	LEIBNIZIAN SYSTEM 5 PLUS-PLUS-MINUS AMBIGUOUS SIGN
	45	C-19	LEIBNIZIAN SYSTEM 5 PLUS-MINUS-PLUS AMBIGUOUS SIGN
	46	C-20	LEIBNIZIAN SYSTEM 5 MINUS-PLUS-PLUS AMBIGUOUS SIGN
	47	C-21	LEIBNIZIAN SYSTEM 5 PLUS-MINUS-MINUS AMBIGUOUS SIGN
	48	C-22	LEIBNIZIAN SYSTEM 5 MINUS-PLUS-MINUS AMBIGUOUS SIGN
	49	C-23	LEIBNIZIAN SYSTEM 5 MINUS-MINUS-PLUS AMBIGUOUS SIGN
	50	C-24	LEIBNIZIAN SYSTEM 5 PLUS-PLUS-PLUS-MINUS AMBIGUOUS SIGN
	51	C-25	LEIBNIZIAN SYSTEM 5 PLUS-PLUS-MINUS-PLUS AMBIGUOUS SIGN
	52	C-26	LEIBNIZIAN SYSTEM 5 PLUS-MINUS-PLUS-PLUS AMBIGUOUS SIGN
	53	C-27	LEIBNIZIAN SYSTEM 5 MINUS-PLUS-PLUS-PLUS AMBIGUOUS SIGN
	54	C-28	LEIBNIZIAN SYSTEM 5 PLUS-PLUS-MINUS-MINUS AMBIGUOUS SIGN
	55	C-29	LEIBNIZIAN SYSTEM 5 MINUS-PLUS-PLUS-MINUS AMBIGUOUS SIGN
	56	C-30	LEIBNIZIAN SYSTEM 5 MINUS-MINUS-PLUS-PLUS AMBIGUOUS SIGN
	57	C-31	LEIBNIZIAN SYSTEM 5 MINUS-PLUS-MINUS-MINUS AMBIGUOUS SIGN

We propose the following encoding scheme:

	1xx1	1xx2	1xx3	1xx4
0		B-07	C-09	C-25
1	A-01	B-08	C-10	C-26
2	A-02	B-09	C-11	C-27
3	A-03	B-10	C-12	C-28
4	A-04	B-11	C-13	C-29
5	A-05	B-12	C-14	C-30
6	A-06	B-16	C-15	C-31
7	A-07		C-16	
8	A-08	C-01	C-17	
9		C-02	C-18	
A	B-01	C-03	C-19	
B	B-02	C-04	C-20	
C	B-03	C-05	C-21	
D	B-04	C-06	C-22	
E	B-05	C-07	C-23	
F	B-06	C-08	C-24	

Standardized Variation Sequences

1xx1B
B-02

1xx1B — LEIBNIZIAN SYSTEM 2 PLUS OR PLUS-MINUS AMBIGUOUS SIGN

1xx1B FE00 — variant form

1xx1C
B-03

1xx1C — LEIBNIZIAN SYSTEM 2 PLUS OR MINUS-PLUS AMBIGUOUS SIGN

1xx1C FE00 — variant form

1xx1F
B-06

1xx1F — LEIBNIZIAN SYSTEM 2 NEGATED PLUS OR MINUS-PLUS AMBIGUOUS SIGN

1xx1F FE00 — variant form

1xx24
B-11

1xx24 — LEIBNIZIAN SYSTEM 2 NEGATED PLUS OR MINUS-PLUS PARTIAL AMBIGUOUS SIGN

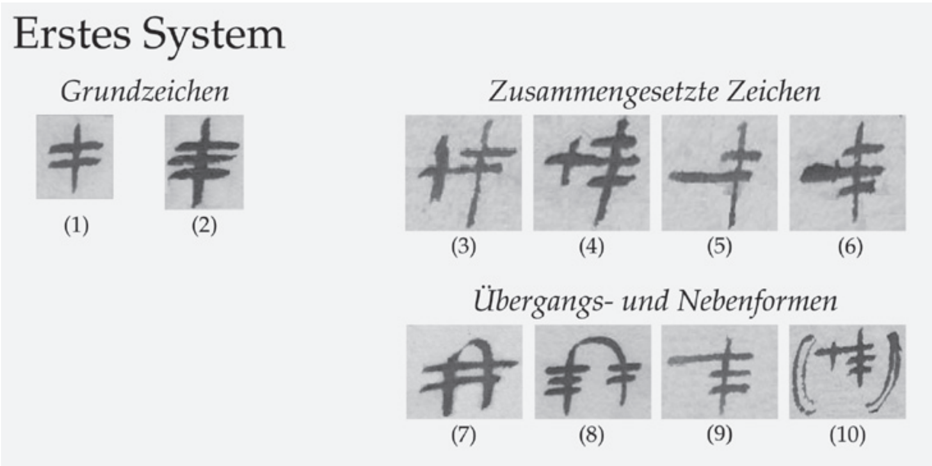
1xx24 FE00 — variant form 1

1xx24 FE01 — variant form 2

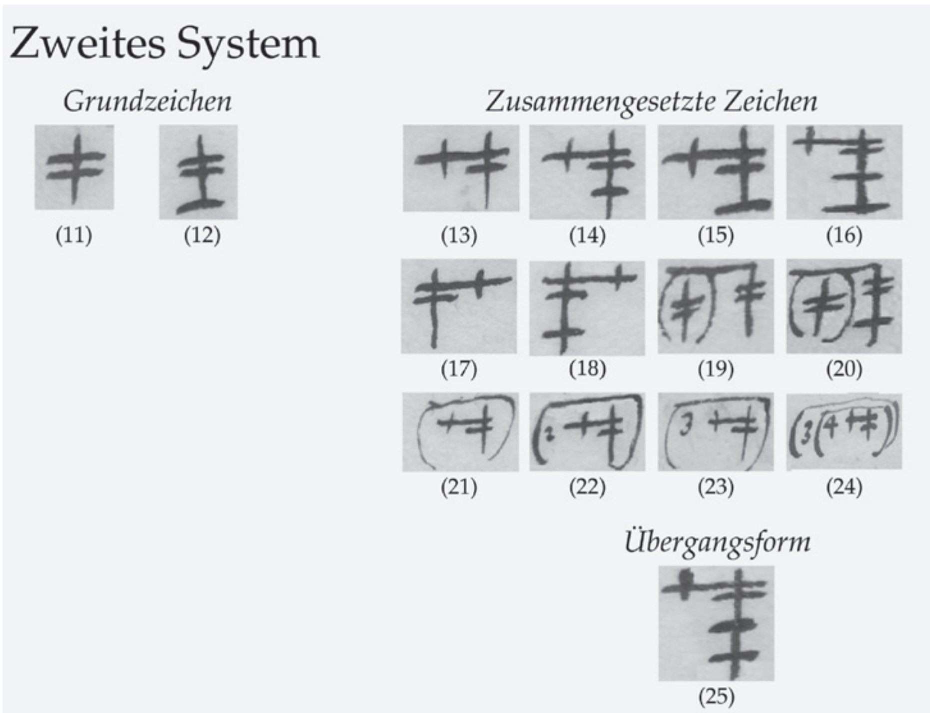
Although utmost care has been taken to document a complete picture of all ambiguous symbols known in MS and printed sources, it is possible that few more symbols will be found in the course of continuing research upon yet unpublished works.
We advise to keep one open slot at the beginning of each group (A, B, C) in order to possibly accommodate such future additions in a sensible way.

Achim Trunk (GWLB Hanover) described six different systems, invented by Leibniz. System 3 deploys the same characters as system 2, mostly. The fourth system employes Greek letters and the sixth system uses ordinary numbers, so basically three systems remain (1., 2. and 5.) which consist of special graphic symbols.

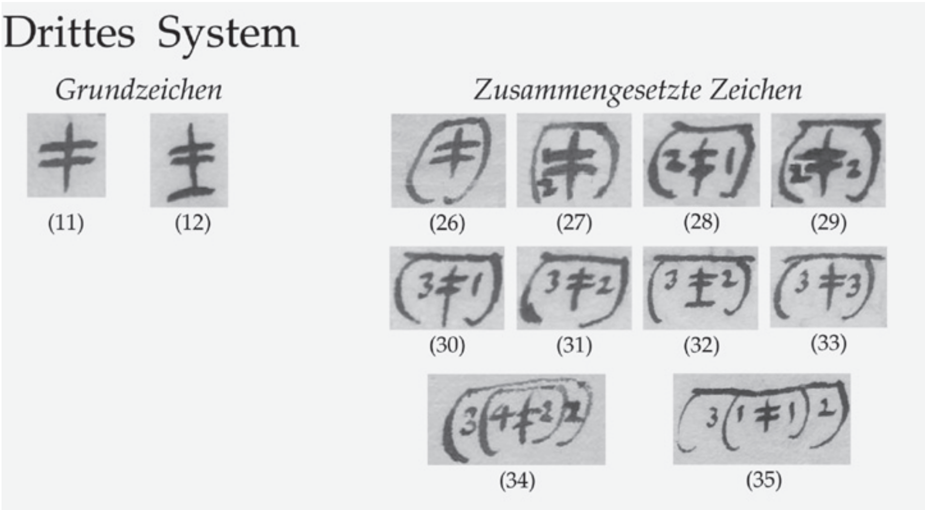
4. The ambiguous signs in samples from Leibniz's manuscripts



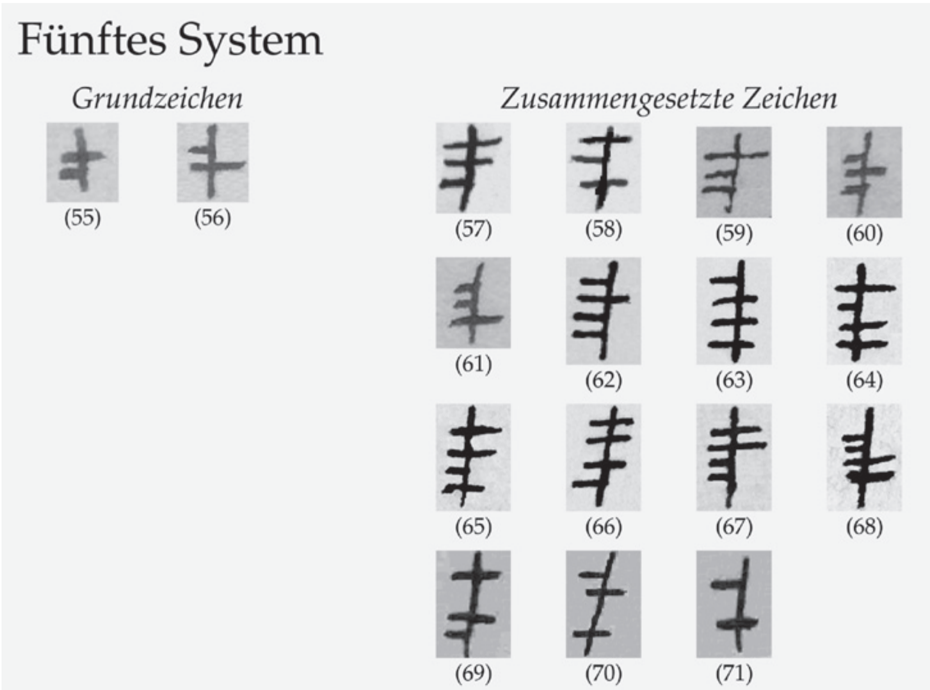
First system *(basic signs, compound signs, transitional and secondary forms)*



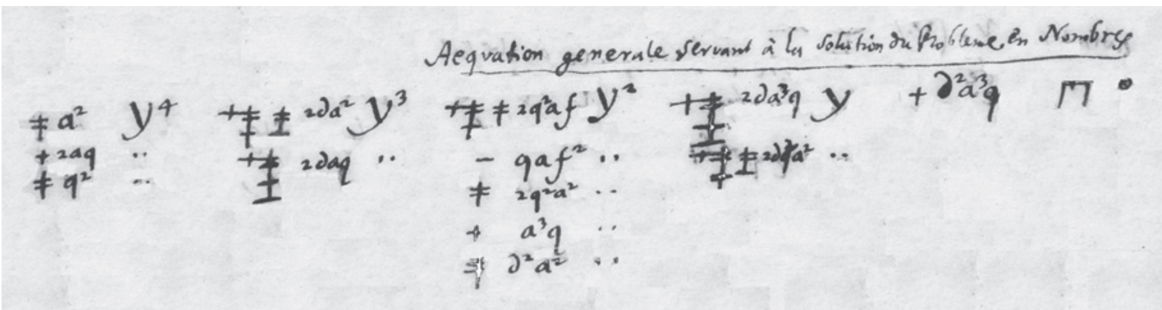
Second system *(basic signs, compound signs, transitional form)*



Third system *(basic signs, compound signs)*

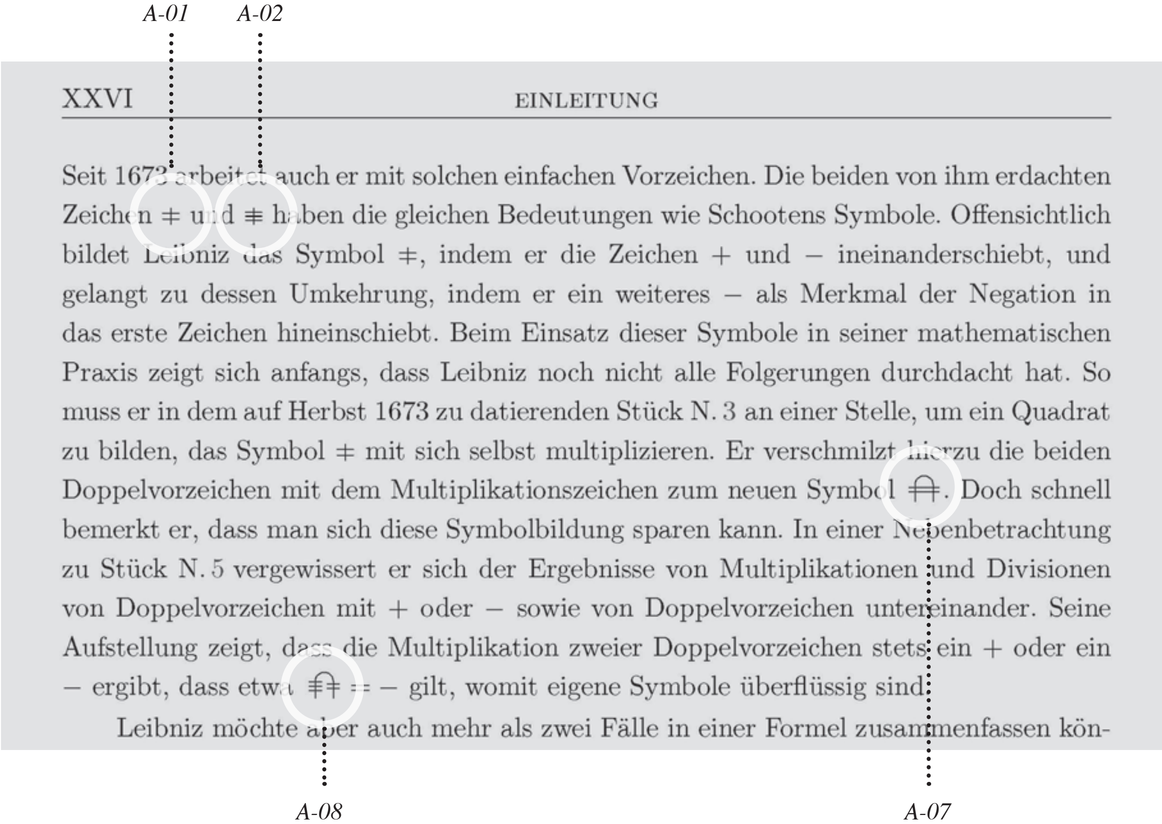


Fifth system *(basic signs, compound signs)*



A notation of an algebraic problem by Leibniz, using symbols of the 2nd system. (after A. Trunk)

5. Figures and explanations

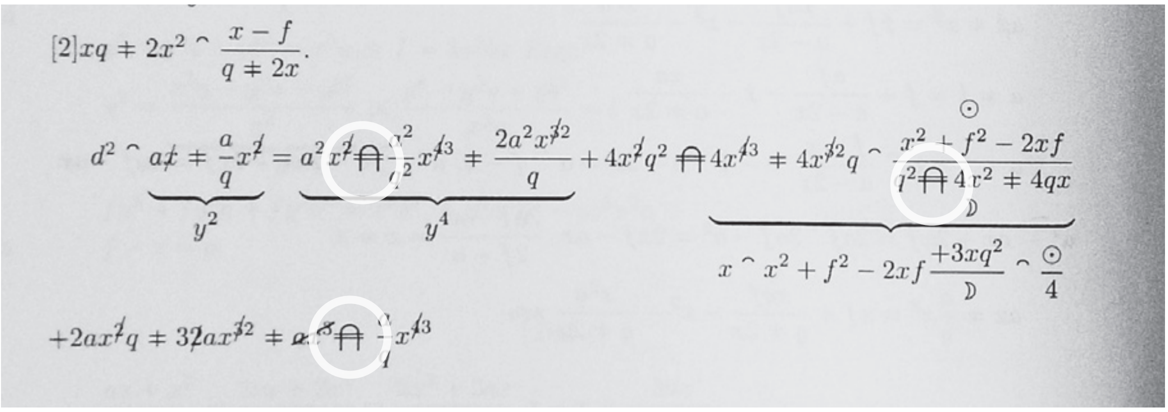


XXVI EINLEITUNG

Seit 1673 arbeitet auch er mit solchen einfachen Vorzeichen. Die beiden von ihm erdachten Zeichen ∓ und ⧧ haben die gleichen Bedeutungen wie Schootens Symbole. Offensichtlich bildet Leibniz das Symbol ∓, indem er die Zeichen + und − ineinanderschiebt, und gelangt zu dessen Umkehrung, indem er ein weiteres − als Merkmal der Negation in das erste Zeichen hineinschiebt. Beim Einsatz dieser Symbole in seiner mathematischen Praxis zeigt sich anfangs, dass Leibniz noch nicht alle Folgerungen durchdacht hat. So muss er in dem auf Herbst 1673 zu datierenden Stück N. 3 an einer Stelle, um ein Quadrat zu bilden, das Symbol ∓ mit sich selbst multiplizieren. Er verschmilzt hierzu die beiden Doppelvorzeichen mit dem Multiplikationszeichen zum neuen Symbol ⩚. Doch schnell bemerkt er, dass man sich diese Symbolbildung sparen kann. In einer Nebenbetrachtung zu Stück N. 5 vergewissert er sich der Ergebnisse von Multiplikationen und Divisionen von Doppelvorzeichen mit + oder − sowie von Doppelvorzeichen untereinander. Seine Aufstellung zeigt, dass die Multiplikation zweier Doppelvorzeichen stets ein + oder ein − ergibt, dass etwa ⧧ = − gilt, womit eigene Symbole überflüssig sind.

Leibniz möchte aber auch mehr als zwei Fälle in einer Formel zusammenfassen kön-

Example of ambiguous signs, 1st system. LAA VII-7, p. XXVI



⩚ AMBIGUITY SIGN A-07;
Example of ambiguous sign A-07, 1st system. LAA VII-7, p. 14

$$\left.\begin{array}{lllll} +4g & +8ag & +4ag^2 & -2c^2g^2 \\ & -2c^2 & -4ae^2 & -2c^2e^2 \\ & +6g^2 & -4gc^2 & +\ \ g^4 \\ & +2e^2 & +\ 4g^3 & +2g^2e^2 \\ & & +4ge^2 & +\ \ e^4 \end{array}\right\} = 0$$

Examinato ergo Canone, per exempla circuli, et parabolae, pergem⟨us⟩ ⟨cum⟩ Calculo generali. Habuimus paulo ante valorem ipsius g. indagemus eum adhuc semel ope terminorum tertiorum, collatorum, seu ope multiplicantium secundae dimensionis incognitos. Fiet

$$\frac{\mp 2\frac{a}{q}g^2\,(\mp\ ¬\mp)\,8a\,g\ \ \frac{+16a^{[2]}}{\mp 2\frac{a}{q}+6},, \frown a^2 \quad +6}{h^2} = \frac{\mp 2\frac{a}{q}e^2 \mp 2\frac{a}{q}c^2 - 2e^2 + 2c^2 - 4a^2, \frown \frac{a^2}{h^2},, \quad ((¬\mp))\ 2q^2f, -qf^2 \mp 2q^2a + a^2q \mp d^2a}{\mp a + 2q \mp \frac{q^2}{a}}$$

---

20 *Kontrollansatz zur quadratischen Ergänzung:* $\sqrt{\mp 2\frac{a}{q}+6}\ g \frown \frac{(\mp\ ¬\mp)\,4a}{\sqrt{\mp 2\frac{a}{q}+6}}$

15 f. $\} = 0$ *(1)* Ponendo jam $x^2 = z^2\frac{h}{a}$ *(2)* Examinato *L*

---

20 … = … : Die Koeffizienten, die Leibniz vergleicht, bezieht er wie oben aus den Gleichungen in N. 5 S. 35 sowie auf S. 48 Z. 3–12. Erneut vergisst er den Faktor $\frac{a^2}{q^2} \mp 2\frac{a}{q} + 1$. Zudem nimmt er die

¬∓ AMBIGUITY SIGN B-04; a character belonging to the 2nd system. LAA VII-7, p. 52

N. 7 AEQUATIO EX INTERSECTIONE ORIENS, Ende Dezember 1673 – Juni 1674 53

seu extracta utrobique Radice

$$\frac{g\sqrt{\mp 2\frac{a}{q}+6}\ \ (\mp\ ¬\mp)\,\frac{4a}{\sqrt{\mp 2\frac{a}{q}+6}}}{\frac{h}{a}} = \sqrt{\frac{\ldots\ldots}{\ldots\ldots}} \qquad \text{sive}$$

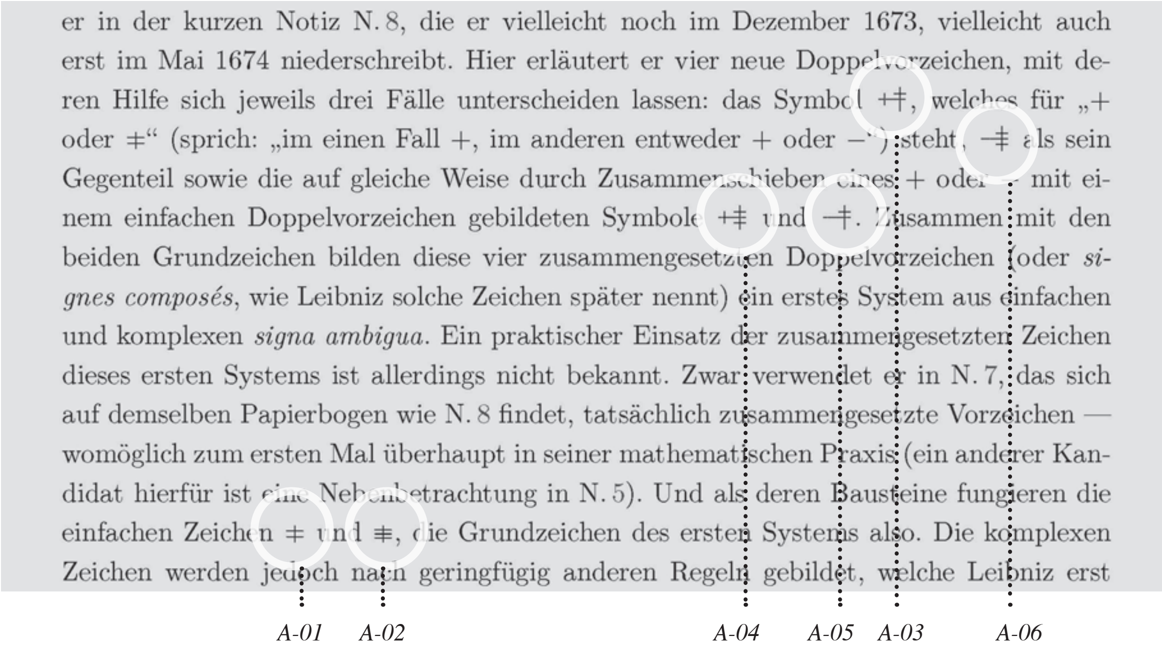
$$g = \sqrt{\frac{\mp 2\frac{a}{q}e^2 \mp 2\frac{a}{q}c^2 - 2e^2 + 2c^2 - 4a^2,, \ \text{א}\ \frac{((¬\mp))\ 2q^2f - qf^2 \mp 2q^2a + a^2q \mp d^2a}{\mp a + 2q \mp \frac{q^2}{a}\quad \gamma} \text{ב} \frown a^2 \frown \frac{h^2}{a^2}}{[\mp]\,2\frac{a}{q}+6 \quad \delta}}\ \ (\mp ¬\mp) - \frac{4 \frown h}{\mp 2\frac{a}{q}+6}\quad \delta$$

Unde evanescit incognita g. valore ejus jam aliter supra dato. Ubi erat:

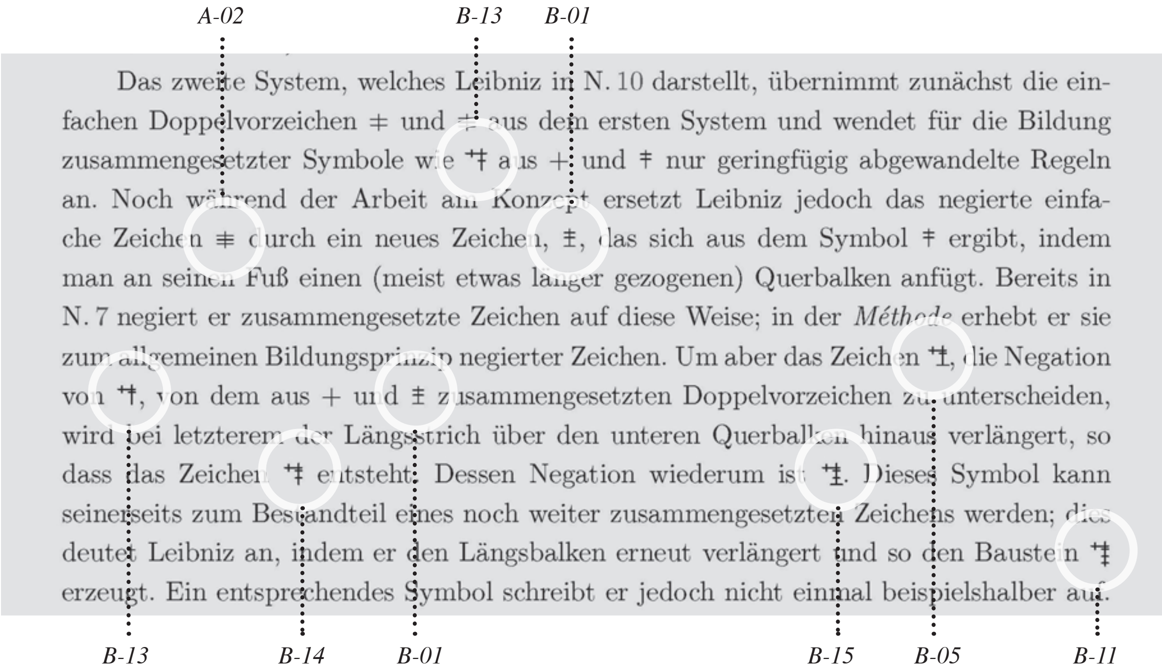
$$g = \frac{\frac{((¬\mp)) \mp a\,((¬\mp))\,q,}{\mp a + 2q \mp \frac{q^2}{a}\quad \gamma} \frown 2d \frown \frac{h^2}{a^2}(\theta) \frown h^2 \mp 4\frac{a^2}{q} - 4a \quad \text{ל}}{\mp(\mp ¬\mp)\,4\frac{a}{q}\,(\mp\ ¬\mp)\,4 \quad \text{ט}}$$

Atque ita novam habemus aequationem inter hos duos valores, cujus aequationis ope

¬∓ AMBIGUITY SIGN B-04; LAA VII-7, p. 53

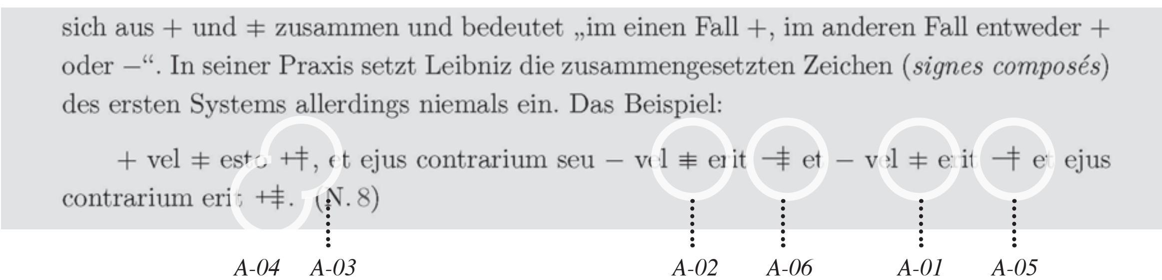

er in der kurzen Notiz N. 8, die er vielleicht noch im Dezember 1673, vielleicht auch erst im Mai 1674 niederschreibt. Hier erläutert er vier neue Doppelvorzeichen, mit deren Hilfe sich jeweils drei Fälle unterscheiden lassen: das Symbol +∓, welches für „+ oder ∓“ (sprich: „im einen Fall +, im anderen entweder + oder −“) steht, −∓ als sein Gegenteil sowie die auf gleiche Weise durch Zusammenschieben eines + oder − mit einem einfachen Doppelvorzeichen gebildeten Symbole +± und −±. Zusammen mit den beiden Grundzeichen bilden diese vier zusammengesetzten Doppelvorzeichen (oder *signes composés*, wie Leibniz solche Zeichen später nennt) ein erstes System aus einfachen und komplexen *signa ambigua*. Ein praktischer Einsatz der zusammengesetzten Zeichen dieses ersten Systems ist allerdings nicht bekannt. Zwar verwendet er in N. 7, das sich auf demselben Papierbogen wie N. 8 findet, tatsächlich zusammengesetzte Vorzeichen — womöglich zum ersten Mal überhaupt in seiner mathematischen Praxis (ein anderer Kandidat hierfür ist eine Nebenbetrachtung in N. 5). Und als deren Bausteine fungieren die einfachen Zeichen ∓ und ±, die Grundzeichen des ersten Systems also. Die komplexen Zeichen werden jedoch nach geringfügig anderen Regeln gebildet, welche Leibniz erst

Example of ambiguous signs, 1st system. Introduction to LAA series VII volume 7, p. XXVI



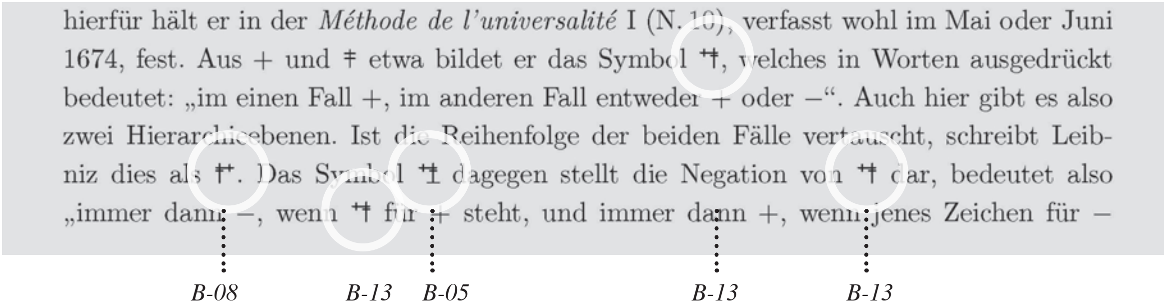
Das zweite System, welches Leibniz in N. 10 darstellt, übernimmt zunächst die einfachen Doppelvorzeichen ∓ und ± aus dem ersten System und wendet für die Bildung zusammengesetzter Symbole wie +∓ aus + und ∓ nur geringfügig abgewandelte Regeln an. Noch während der Arbeit am Konzept ersetzt Leibniz jedoch das negierte einfache Zeichen ± durch ein neues Zeichen, ±, das sich aus dem Symbol ∓ ergibt, indem man an seinen Fuß einen (meist etwas länger gezogenen) Querbalken anfügt. Bereits in N. 7 negiert er zusammengesetzte Zeichen auf diese Weise; in der *Méthode* erhebt er sie zum allgemeinen Bildungsprinzip negierter Zeichen. Um aber das Zeichen +±, die Negation von +∓, von dem aus + und ± zusammengesetzten Doppelvorzeichen zu unterscheiden, wird bei letzterem der Längsstrich über den unteren Querbalken hinaus verlängert, so dass das Zeichen +± entsteht. Dessen Negation wiederum ist +±. Dieses Symbol kann seinerseits zum Bestandteil eines noch weiter zusammengesetzten Zeichens werden; dies deutet Leibniz an, indem er den Längsbalken erneut verlängert und so den Baustein +± erzeugt. Ein entsprechendes Symbol schreibt er jedoch nicht einmal beispielshalber auf.

Example of ambiguous signs, 1st and 2nd system. LAA VII-7, p. XXVIII

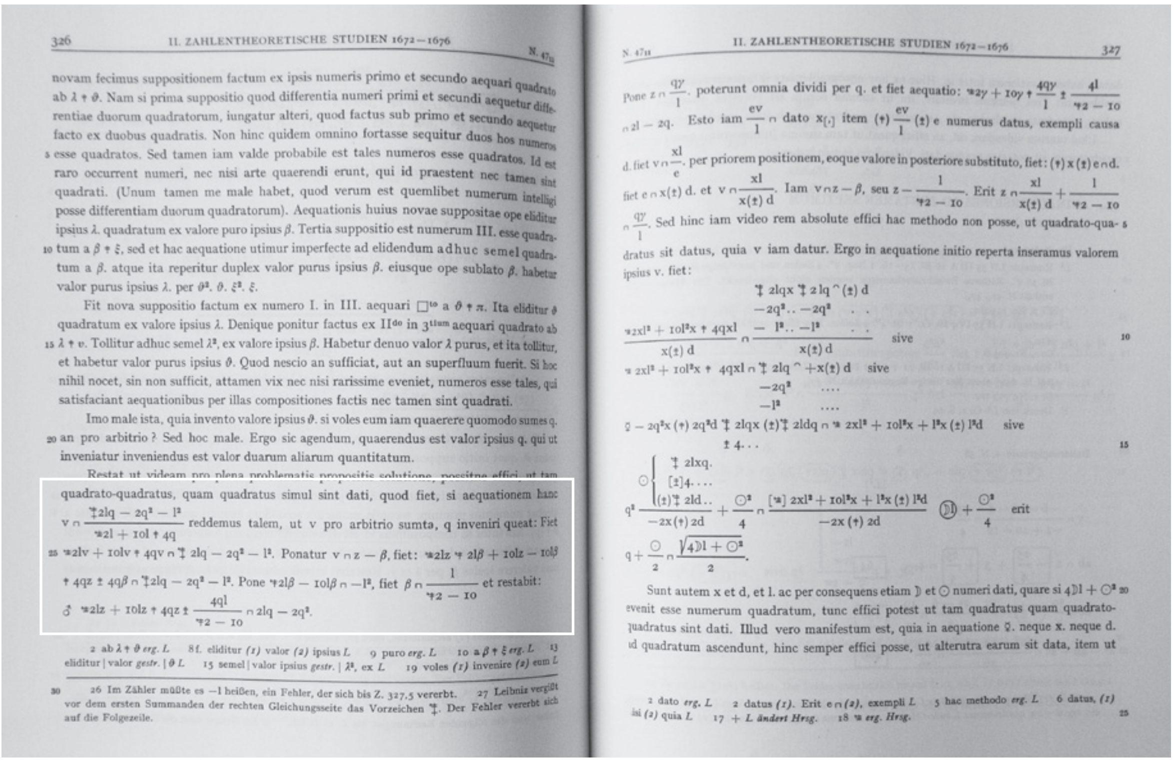

sich aus + und ∓ zusammen und bedeutet „im einen Fall +, im anderen Fall entweder + oder −“. In seiner Praxis setzt Leibniz die zusammengesetzten Zeichen (*signes composés*) des ersten Systems allerdings niemals ein. Das Beispiel:

+ vel ∓ esto +∓, et ejus contrarium seu − vel ± erit −∓ et − vel ∓ erit −± et ejus contrarium erit +±. (N. 8)

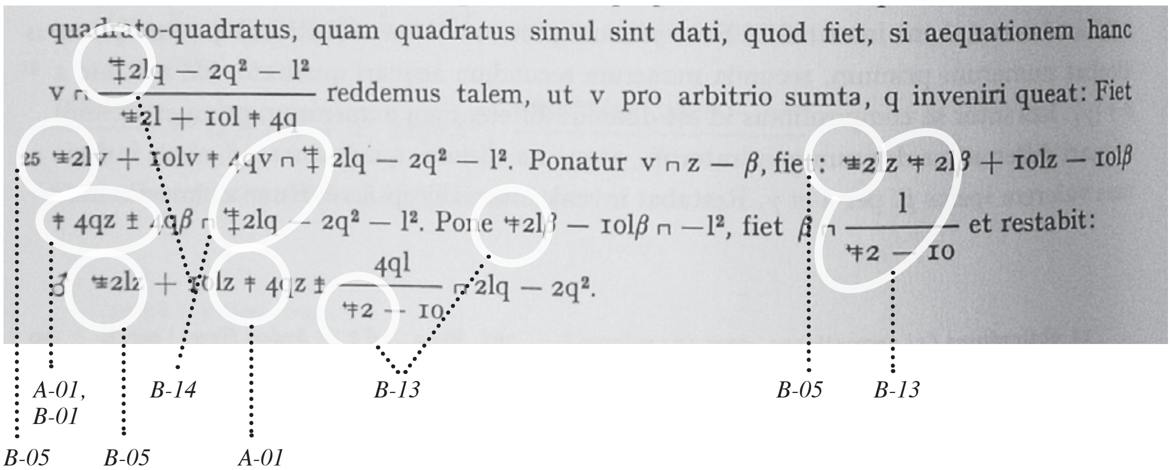
Example of ambiguous signs, 1st system. LAA VII-7, p. XLI

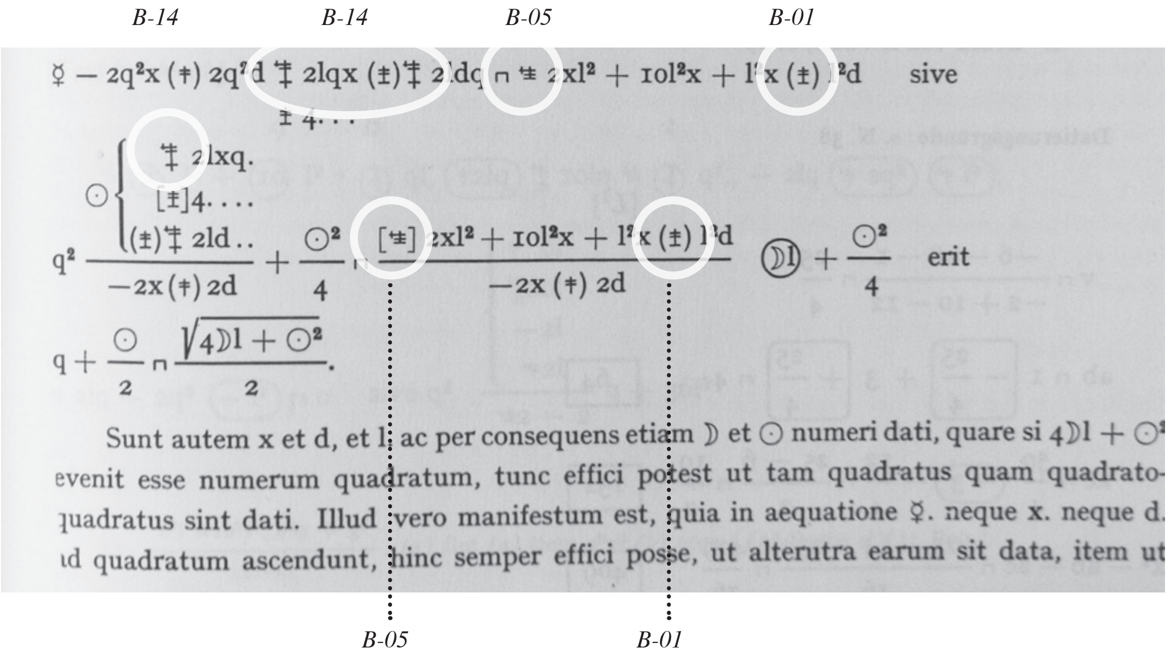

hierfür hält er in der *Méthode de l'universalité* I (N. 10), verfasst wohl im Mai oder Juni 1674, fest. Aus + und ∓ etwa bildet er das Symbol ⁺∓, welches in Worten ausgedrückt bedeutet: „im einen Fall +, im anderen Fall entweder + oder −“. Auch hier gibt es also zwei Hierarchieebenen. Ist die Reihenfolge der beiden Fälle vertauscht, schreibt Leibniz dies als ∓⁺. Das Symbol ⁺± dagegen stellt die Negation von ⁺∓ dar, bedeutet also „immer dann −, wenn ⁺∓ für + steht, und immer dann +, wenn jenes Zeichen für −

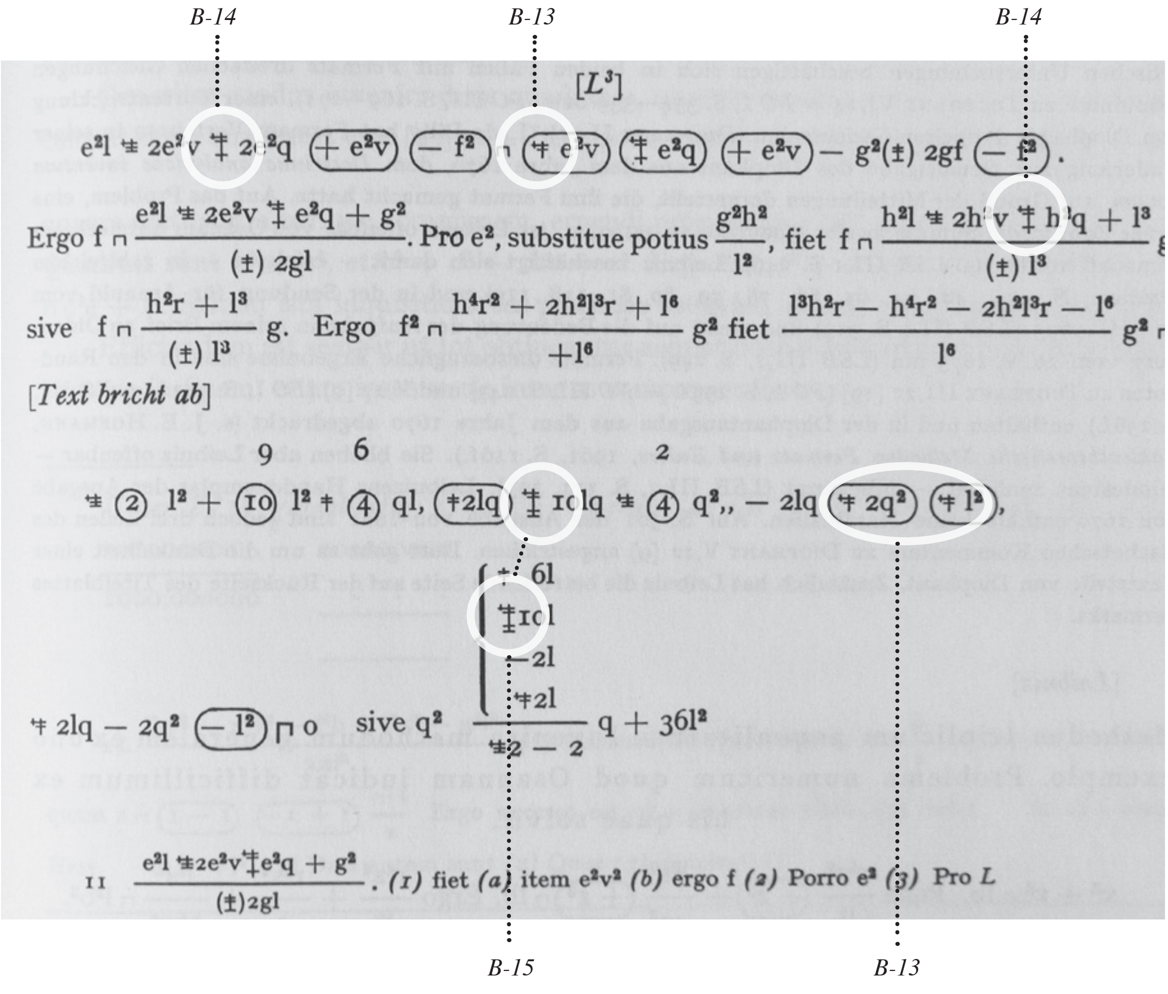
Example of ambiguous signs, 2nd system. LAA VII-7, p. XLI



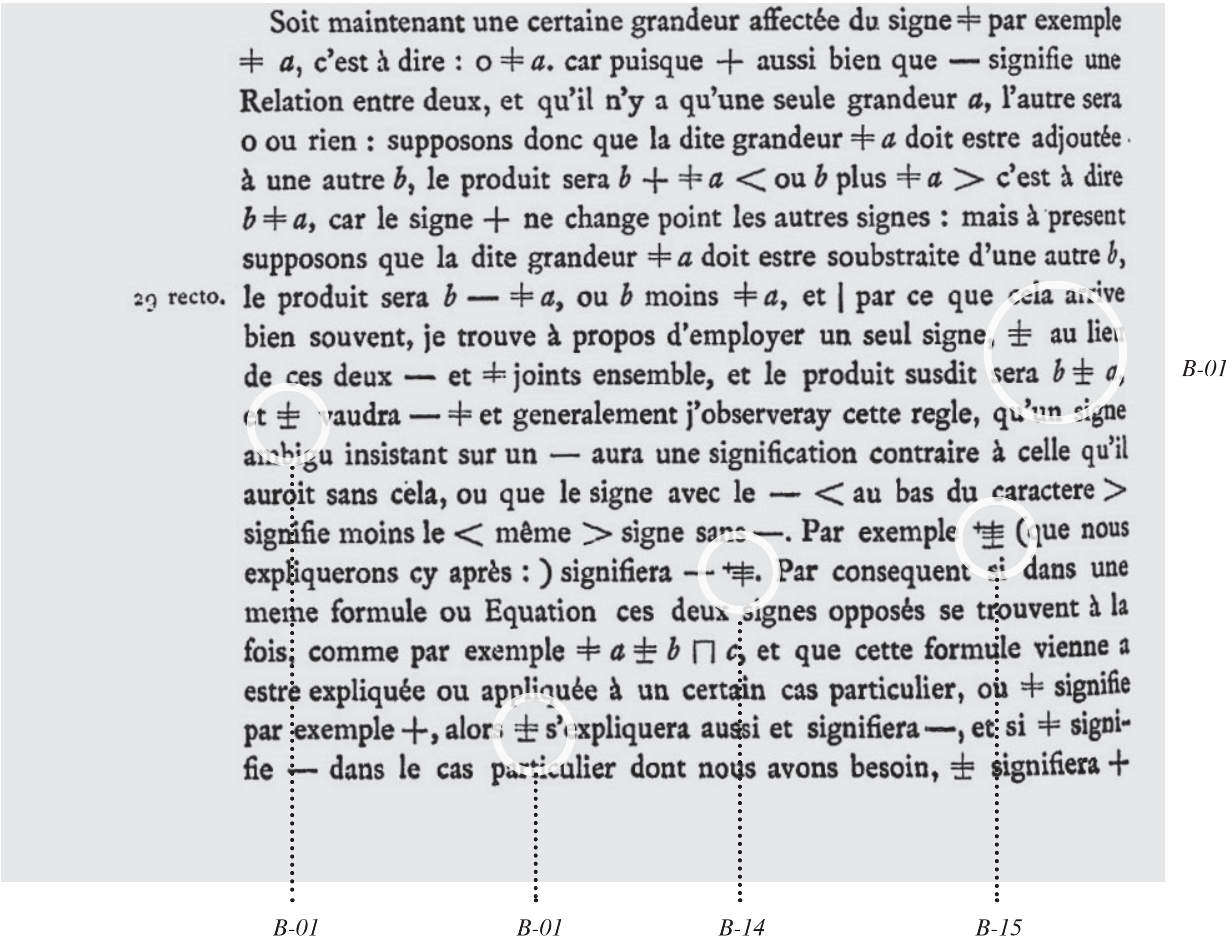
Example of ambiguous signs, 2nd system. LAA VII-1, p. 326–327. See following figure for details.


quadrato-quadratus, quam quadratus simul sint dati, quod fiet, si aequationem hanc v ⊓ (⁺±2lq − 2q² − l²)/(⁺∓2l + 10l ⁺∓ 4q) reddemus talem, ut v pro arbitrio sumta, q inveniri queat: Fiet ⁺∓2lv + 10lv ⁺∓ 4qv ⊓ ⁺± 2lq − 2q² − l². Ponatur v ⊓ z − β, fiet: ⁺∓2lz ⁺∓ 2lβ + 10lz − 10lβ ⁺∓ 4qz ± 4qβ ⊓ ⁺±2lq − 2q² − l². Pone ⁺∓2lβ − 10lβ ⊓ −l², fiet β ⊓ 1/(⁺∓2 − 10) et restabit: ⁺∓2lz + 10lz ⁺∓ 4qz ± 4ql/(⁺∓2 − 10) ⊓ 2lq − 2q².



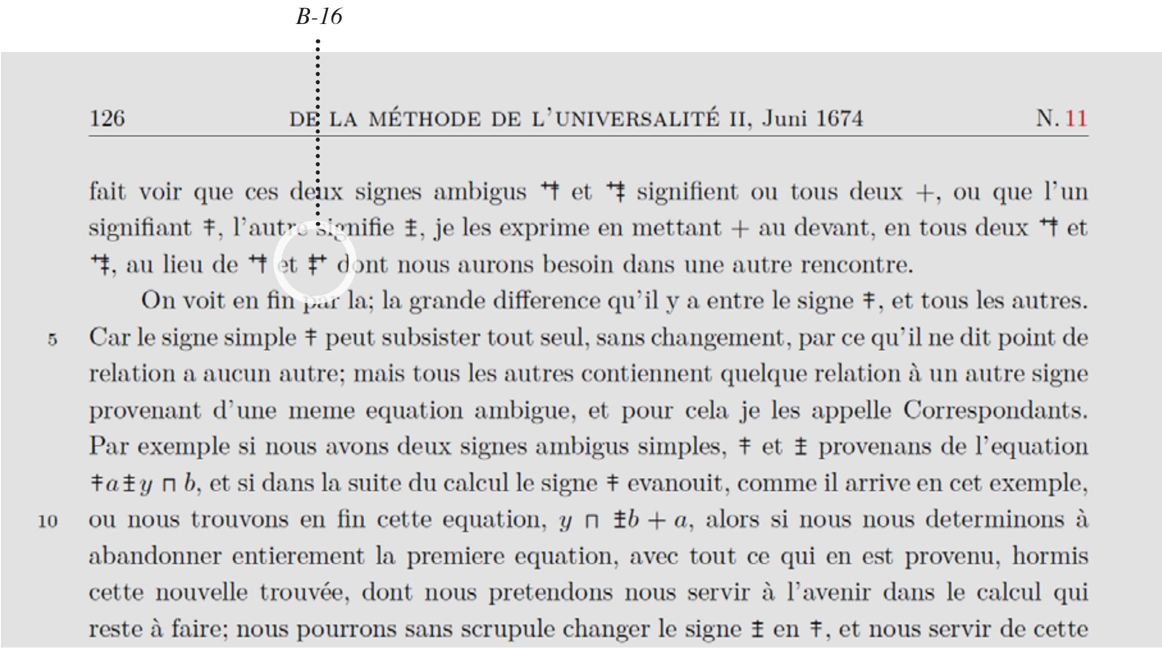


Example of ambiguous signs, 2nd system. LAA VII-1, p. 327 (top), 329


Soit maintenant une certaine grandeur affectée du signe ∓ par exemple ∓ *a*, c'est à dire : o ∓ *a*. car puisque + aussi bien que — signifie une Relation entre deux, et qu'il n'y a qu'une seule grandeur *a*, l'autre sera o ou rien : supposons donc que la dite grandeur ∓ *a* doit estre adjoutée à une autre *b*, le produit sera *b* + ∓ *a* < ou *b* plus ∓ *a* > c'est à dire *b* ∓ *a*, car le signe + ne change point les autres signes : mais à present supposons que la dite grandeur ∓ *a* doit estre soubstraite d'une autre *b*,

29 recto. le produit sera *b* — ∓ *a*, ou *b* moins ∓ *a*, et | par ce que cela arrive bien souvent, je trouve à propos d'employer un seul signe, ± au lieu de ces deux — et ∓ joints ensemble, et le produit susdit sera *b* ± *a*, et ± vaudra — ∓ et generalement j'observeray cette regle, qu'un signe ambigu insistant sur un — aura une signification contraire à celle qu'il auroit sans cela, ou que le signe avec le — < au bas du caractere > signifie moins le < même > signe sans —. Par exemple ⸗ (que nous expliquerons cy après :) signifiera — ⸗. Par consequent si dans une meme formule ou Equation ces deux signes opposés se trouvent à la fois, comme par exemple ∓ *a* ± *b* ⊓ *c*, et que cette formule vienne a estre expliquée ou appliquée à un certain cas particulier, où ∓ signifie par exemple +, alors ± s'expliquera aussi et signifiera —, et si ∓ signifie — dans le cas particulier dont nous avons besoin, ± signifiera +

Example of ambiguous signs, 2nd system.
Couturat 1903 (1961), p. 126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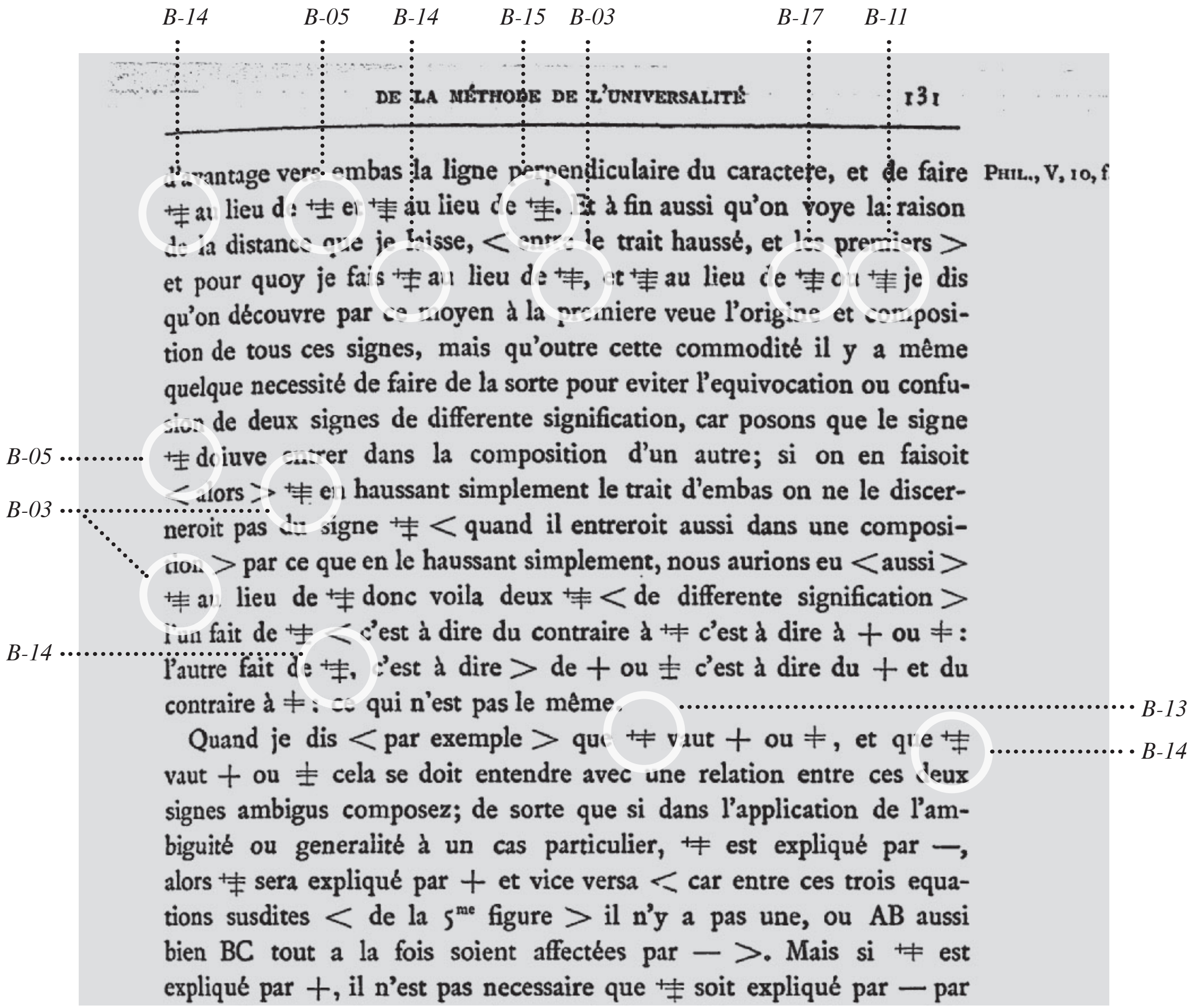
fait voir que ces deux signes ambigus ⁺‡ et ⁺‡ signifient ou tous deux +, ou que l'un signifiant ‡, l'autre signifie ‡, je les exprime en mettant + au devant, en tous deux ⁺‡ et ⁺‡, au lieu de ⁺‡ et ‡⁺ dont nous aurons besoin dans une autre rencontre.

On voit en fin par la; la grande difference qu'il y a entre le signe ‡, et tous les autres. Car le signe simple ‡ peut subsister tout seul, sans changement, par ce qu'il ne dit point de relation a aucun autre; mais tous les autres contiennent quelque relation à un autre signe provenant d'une meme equation ambigue, et pour cela je les appelle Correspondants. Par exemple si nous avons deux signes ambigus simples, ‡ et ‡ provenans de l'equation ‡*a*‡*y* ⊓ *b*, et si dans la suite du calcul le signe ‡ evanouit, comme il arrive en cet exemple, ou nous trouvons en fin cette equation, *y* ⊓ ‡*b* + *a*, alors si nous nous determinons à abandonner entierement la premiere equation, avec tout ce qui en est provenu, hormis cette nouvelle trouvée, dont nous pretendons nous servir à l'avenir dans le calcul qui reste à faire; nous pourrons sans scrupule changer le signe ‡ en ‡, et nous servir de cette

Example of ambiguous sign B-16, 2nd system. LAA VII-7, p. 126



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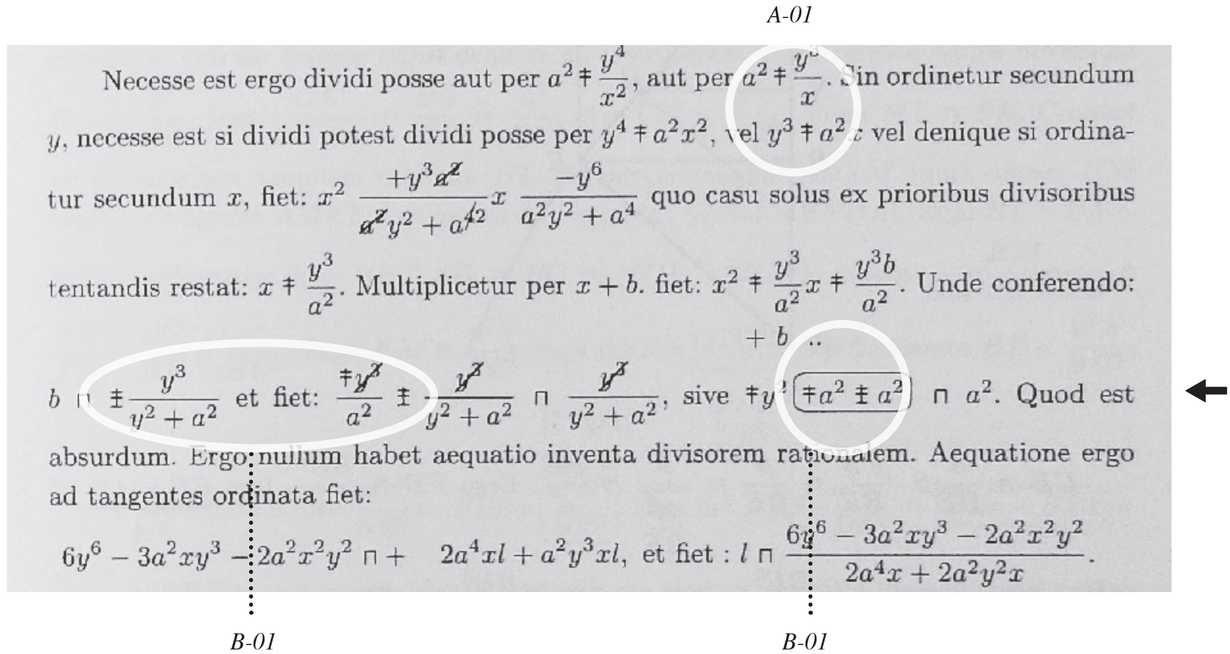
d'avantage vers embas la ligne perpendiculaire du caractere, et de faire ∓ au lieu de ∓ et ∓ au lieu de ∓. Et à fin aussi qu'on voye la raison de la distance que je laisse, < entre le trait haussé, et les premiers > et pour quoy je fais ∓ au lieu de ∓, et ∓ au lieu de ∓ ou ∓ je dis qu'on découvre par ce moyen à la premiere veue l'origine et composition de tous ces signes, mais qu'outre cette commodité il y a même quelque necessité de faire de la sorte pour eviter l'equivocation ou confusion de deux signes de differente signification, car posons que le signe ∓ deuve entrer dans la composition d'un autre; si on en faisoit < alors > ∓ en haussant simplement le trait d'embas on ne le discerneroit pas du signe ∓ < quand il entreroit aussi dans une composition > par ce que en le haussant simplement, nous aurions eu < aussi > ∓ au lieu de ∓ donc voila deux ∓ < de differente signification > l'un fait de ∓ < c'est à dire du contraire à ∓ c'est à dire à + ou ∓ : l'autre fait de ∓, c'est à dire > de + ou ± c'est à dire du + et du contraire à ∓ : ce qui n'est pas le même. PHIL., V, 10, f

Quand je dis < par exemple > que ∓ vaut + ou ∓, et que ∓ vaut + ou ± cela se doit entendre avec une relation entre ces deux signes ambigus composez; de sorte que si dans l'application de l'ambiguité ou generalité à un cas particulier, ∓ est expliqué par —, alors ∓ sera expliqué par + et vice versa < car entre ces trois equations susdites < de la 5^{me} figure > il n'y a pas une, ou AB aussi bien BC tout a la fois soient affectées par — >. Mais si ∓ est expliqué par +, il n'est pas necessaire que ∓ soit expliqué par — par

Example of ambiguous signs, 2nd system.
Couturat 1903 (1961), p. 131



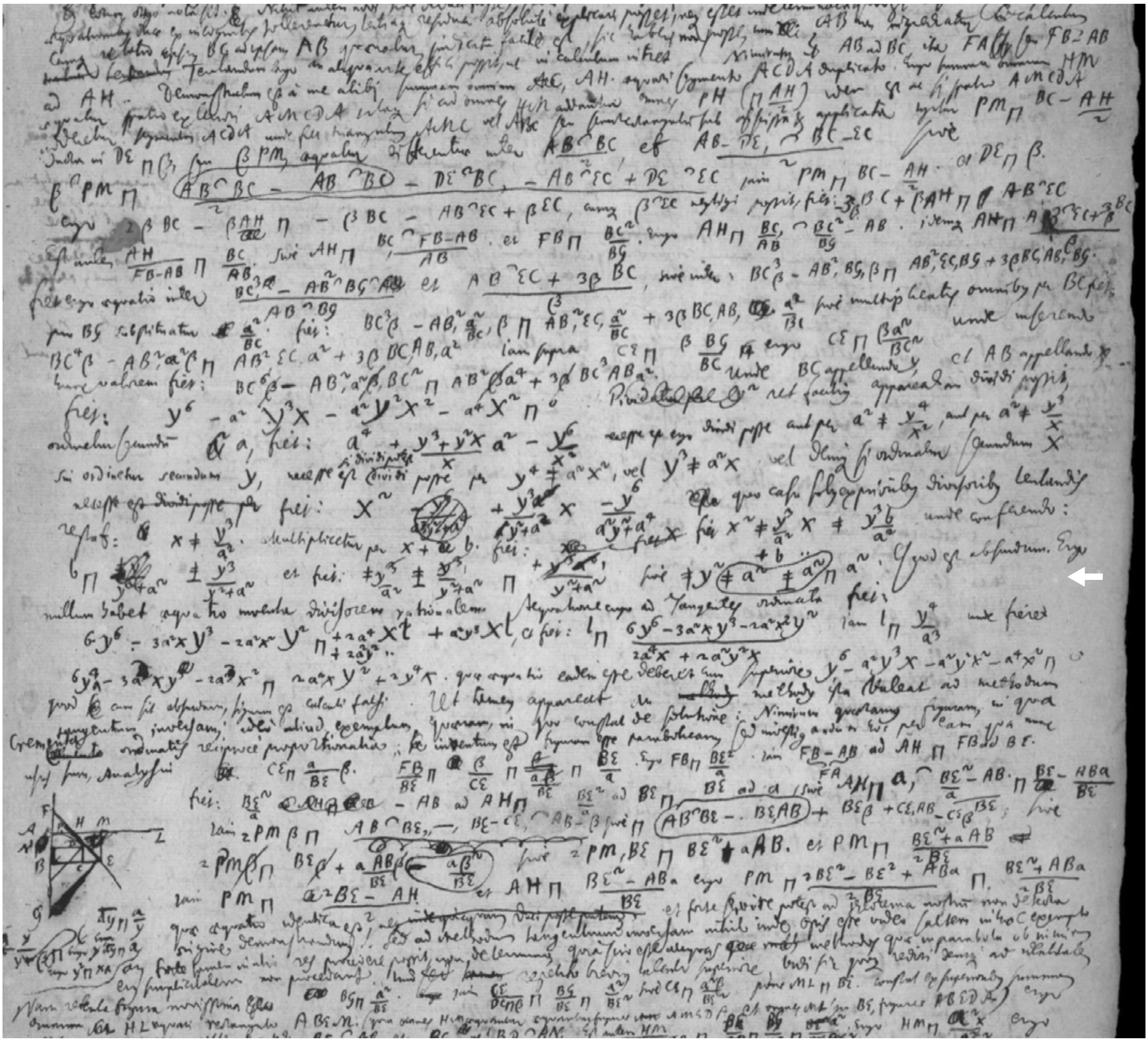
Necesse est ergo dividi posse aut per $a^2 \mp \frac{y^4}{x^2}$, aut per $a^2 \mp \frac{y^3}{x}$. Sin ordinetur secundum y, necesse est si dividi potest dividi posse per $y^4 \mp a^2x^2$, vel $y^3 \mp a^2 x$ vel denique si ordinatur secundum x, fiet: $x^2 \quad \frac{+y^3 a^2}{a^2y^2+a^4}x \quad \frac{-y^6}{a^2y^2+a^4}$ quo casu solus ex prioribus divisoribus tentandis restat: $x \mp \frac{y^3}{a^2}$. Multiplicetur per $x+b$. fiet: $x^2 \mp \frac{y^3}{a^2}x \mp \frac{y^3 b}{a^2}$. Unde conferendo:

$+b$ ..

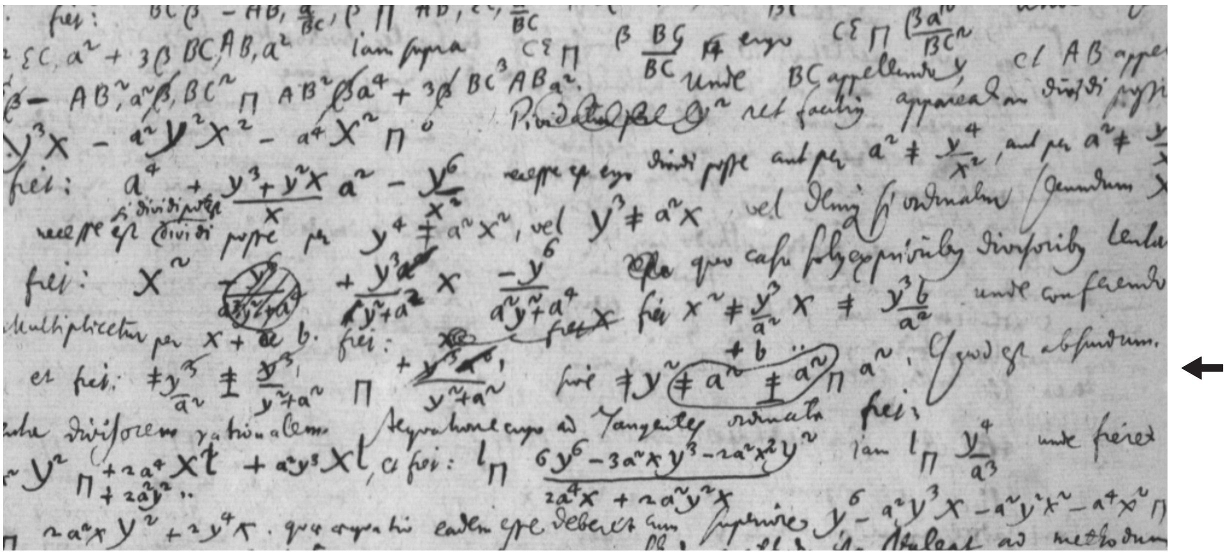
$b \sqcap \pm \frac{y^3}{y^2+a^2}$ et fiet: $\frac{\mp y^3}{a^2} \pm \frac{y^3}{y^2+a^2} \sqcap \frac{y^3}{y^2+a^2}$, sive $\mp y^2 \boxed{\mp a^2 \pm a^2} \sqcap a^2$. Quod est absurdum. Ergo nullum habet aequatio inventa divisorem rationalem. Aequatione ergo ad tangentes ordinata fiet:

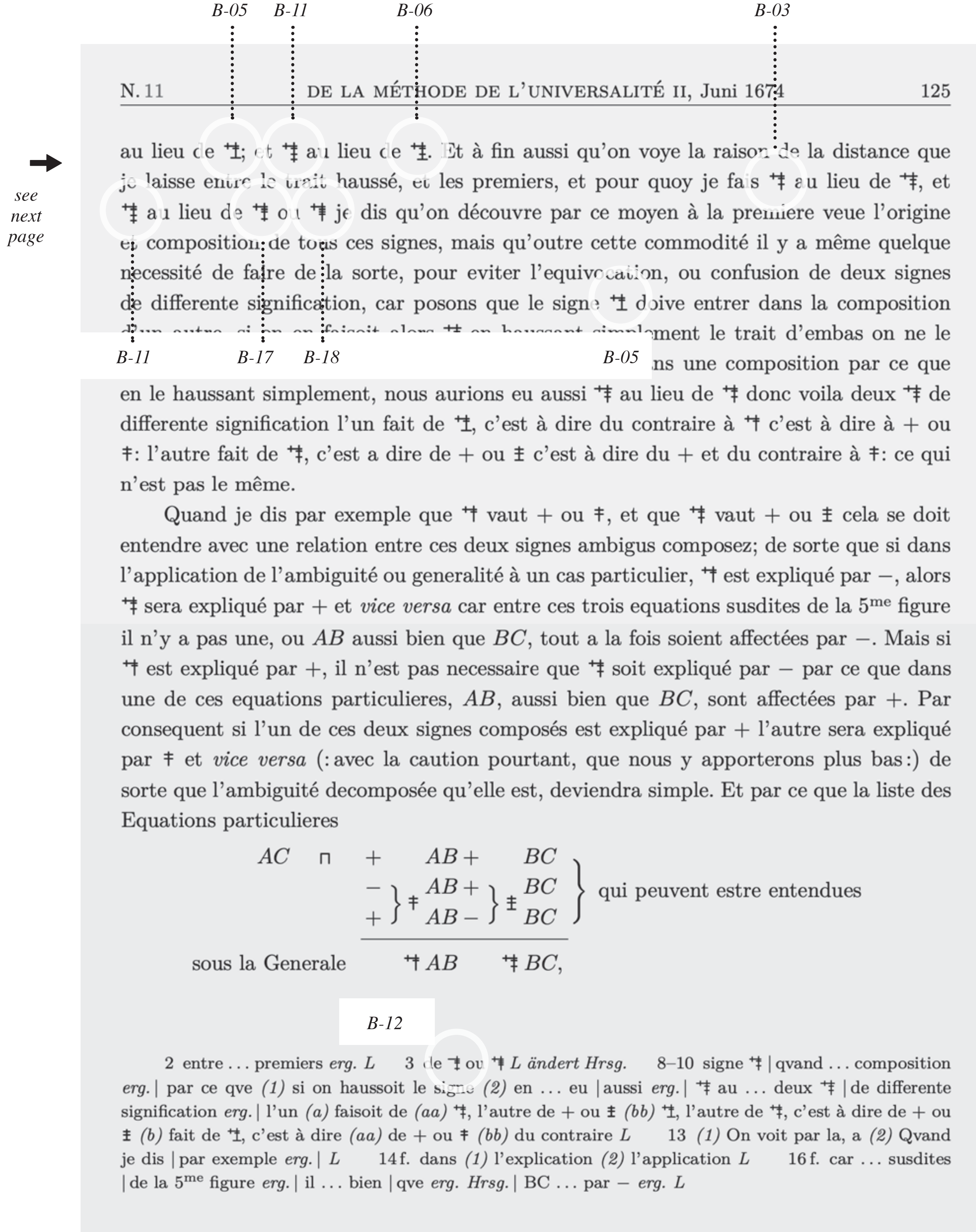
$$6y^6 - 3a^2xy^3 - 2a^2x^2y^2 \sqcap + \quad 2a^4xl + a^2y^3xl, \text{ et fiet : } l \sqcap \frac{6y^6 - 3a^2xy^3 - 2a^2x^2y^2}{2a^4x + 2a^2y^2x}.$$

Ambiguous signs, 2nd system. LAA VII-3, p. 567
see also LH 35 V 2, f. 4v (next page)



Manuscript by G.W.Leibniz, LH 35 V 2, f. 4v





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au lieu de ⁺⫠; et ⁺‡ au lieu de ⁺‡. Et à fin aussi qu'on voye la raison de la distance que je laisse entre le trait haussé, et les premiers, et pour quoy je fais ⁺‡ au lieu de ⁺‡, et ⁺‡ au lieu de ⁺‡ ou ⁺‡ je dis qu'on découvre par ce moyen à la premiere veue l'origine et composition de tous ces signes, mais qu'outre cette commodité il y a même quelque necessité de faire de la sorte, pour eviter l'equivocation, ou confusion de deux signes de differente signification, car posons que le signe ⁺⫠ doive entrer dans la composition d'un autre, si on en faisoit alors ⁺‡ en haussant simplement le trait d'embas on ne le … ns une composition par ce que en le haussant simplement, nous aurions eu aussi ⁺‡ au lieu de ⁺‡ donc voila deux ⁺‡ de differente signification l'un fait de ⁺⫠, c'est à dire du contraire à ⁺ǂ c'est à dire à + ou ∓: l'autre fait de ⁺‡, c'est a dire de + ou ± c'est à dire du + et du contraire à ∓: ce qui n'est pas le même.

Quand je dis par exemple que ⁺ǂ vaut + ou ∓, et que ⁺‡ vaut + ou ± cela se doit entendre avec une relation entre ces deux signes ambigus composez; de sorte que si dans l'application de l'ambiguité ou generalité à un cas particulier, ⁺ǂ est expliqué par −, alors ⁺‡ sera expliqué par + et *vice versa* car entre ces trois equations susdites de la 5^me^ figure il n'y a pas une, ou AB aussi bien que BC, tout a la fois soient affectées par −. Mais si ⁺ǂ est expliqué par +, il n'est pas necessaire que ⁺‡ soit expliqué par − par ce que dans une de ces equations particulieres, AB, aussi bien que BC, sont affectées par +. Par consequent si l'un de ces deux signes composés est expliqué par + l'autre sera expliqué par ∓ et *vice versa* (: avec la caution pourtant, que nous y apporterons plus bas :) de sorte que l'ambiguité decomposée qu'elle est, deviendra simple. Et par ce que la liste des Equations particulieres

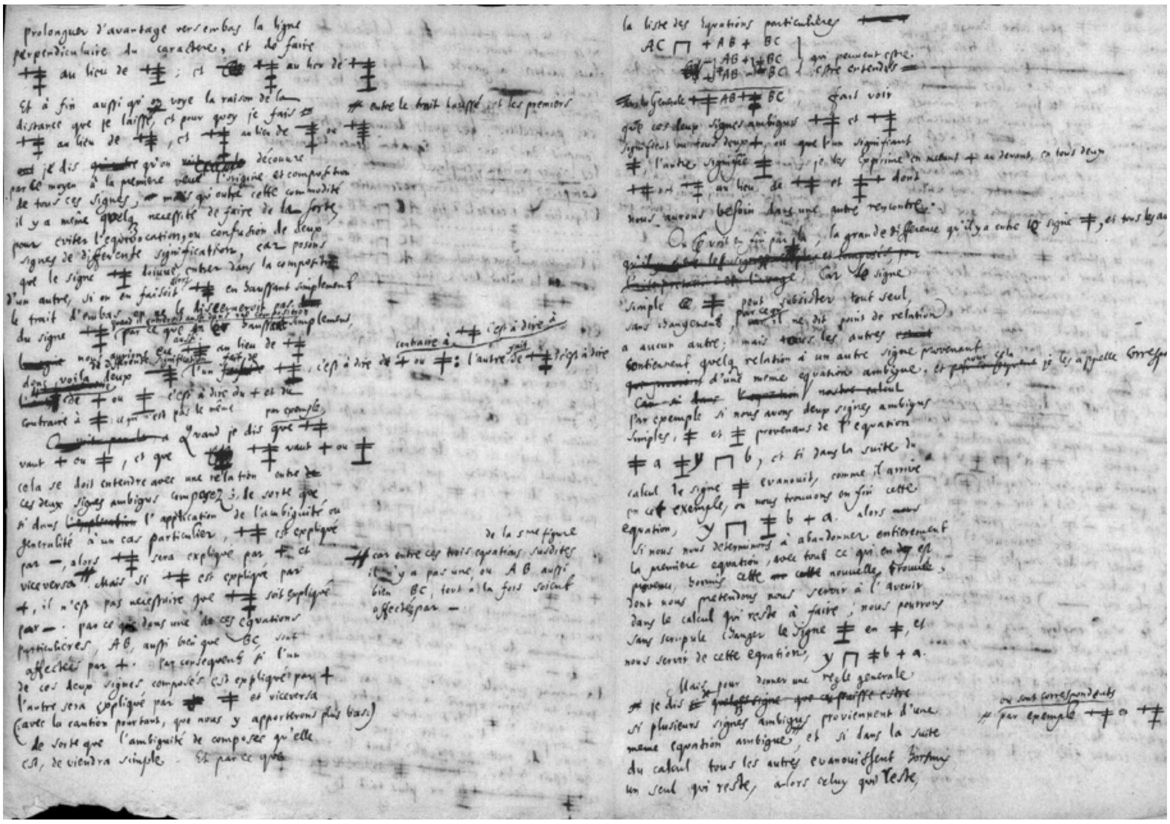
AC ⊓ + AB + BC
− } ∓ AB + } ± BC
+ } AB − } BC } qui peuvent estre entendues

sous la Generale ⁺ǂ AB ⁺‡ BC,

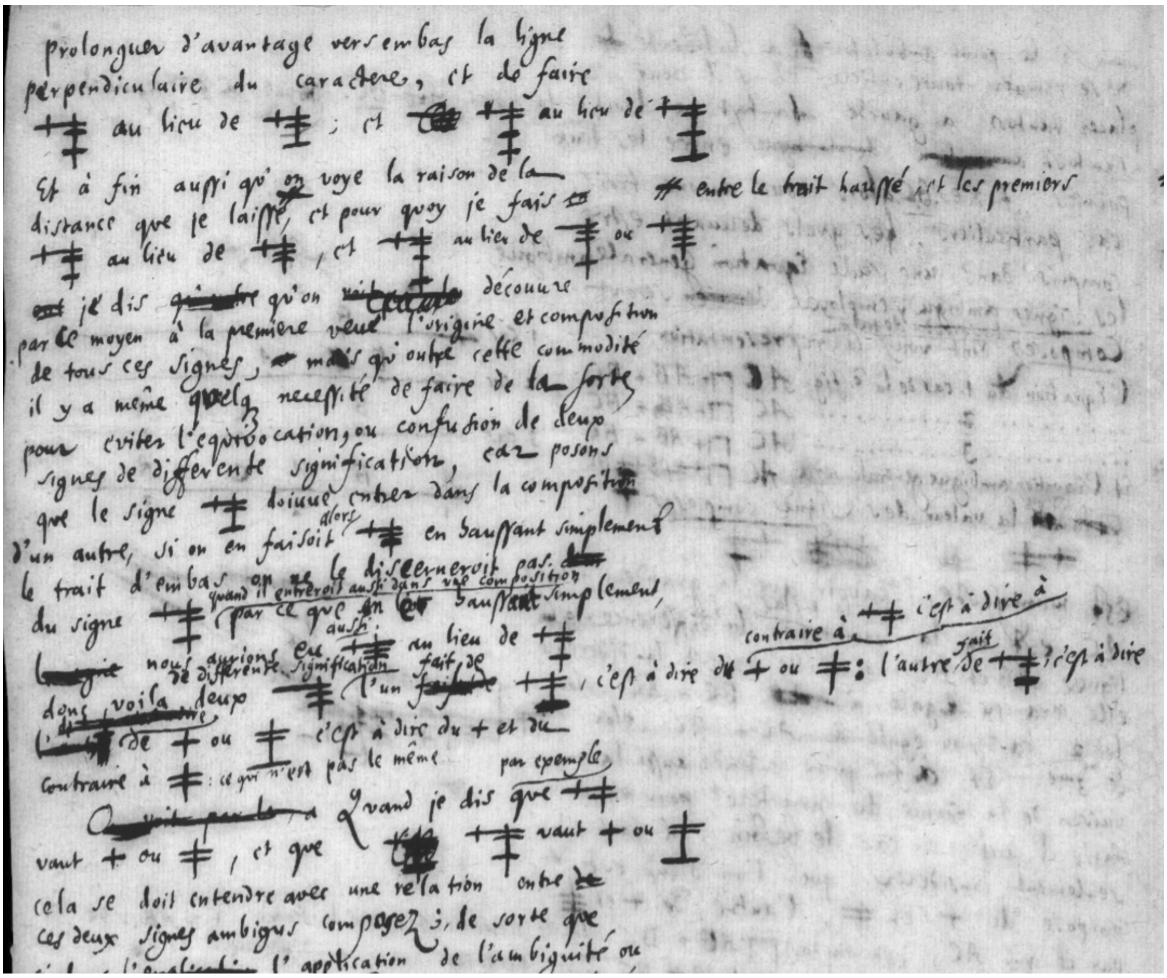
2 entre … premiers *erg. L* 3 de ⁺ǂ ou ⁺‡ *L ändert Hrsg.* 8–10 signe ⁺‡ | qvand … composition *erg.* | par ce qve *(1)* si on haussoit le signe *(2)* en … eu | aussi *erg.* | ⁺‡ au … deux ⁺‡ | de differente signification *erg.* | l'un *(a)* faisoit de *(aa)* ⁺‡, l'autre de + ou ± *(bb)* ⁺⫠, l'autre de ⁺‡, c'est à dire de + ou ± *(b)* fait de ⁺⫠, c'est à dire *(aa)* de + ou ∓ *(bb)* du contraire *L* 13 *(1)* On voit par la, a *(2)* Qvand je dis | par exemple *erg.* | *L* 14 f. dans *(1)* l'explication *(2)* l'application *L* 16 f. car … susdites | de la 5^me^ figure *erg.* | il … bien | qve *erg. Hrsg.* | BC … par − *erg. L*

Ambiguous signs, 2nd system. LAA VII-7, p. 125; *see also LH 4 V 10, f. 31 (next page)*

In this passage, Leibniz discusses how ambiguity signs of higher order should be derived from those of lower order. He suggests to slightly modify each ambiguity sign of order *n* (for example B-06) when it is used as a part of an ambiguity sign of order *n* + 1 (for example B-11 as a partial variant of B-06). Since the modification allows for freedom in design, Leibniz discusses different types as for example B-11, B-17, and B-18 for the partial ambiguity sign that corresponds to B-06. The lower part of the page shows the transition from ambiguity signs of degree 2 to those of degree 3.



LH 4 V 10, f. 31v-32r


prolonguer d'avantage vers embas la ligne
perpendiculaire du caractere, et de faire

IV. Ambig.

a ⊓ (3⁺ǂ) b (3⁺ǂ) c signifie a ⊓ $+b$ (3ǂ){ $+b$ / $-b$ } (3±){ $-c$ / $+c$ } $+c$, c'est à dire, a ⊓ $+b$ $+c$ ou (3ǂ) b (3±) c

a estant ou la somme, ou la difference de b. c. cela fait voir clairement la raison de la fabrique des signes, et il faut remarquer seulement que de $+$ ou (3±), on a fait tout expres (3⁺ǂ) au lieu de (3⁺±) par ce que (3⁺±) signifie le signe opposé à (3⁺ǂ).

Si nous eussions eu

f ⊓ $+b$ (3ǂ){ $+b$ / $-b$ } $+c$ (3±){ $-c$ / $+c$ } (3ǂ){ $+e$ / $-e$ / $-e$ }, cela auroit fait

f ⊓ (3⁺ǂ)b (3⁺ǂ)c (3ǂ⁻)e

LAA VII-7, p. 144

B-09

B-02 *B-05* *B-08* *B-14* *B-05* *B-10*

traits, ou –, horsmis un qui se pourra placer ou l'on voudra, par exemple ǂ (3⁺±) (2±⁺) c fait ǂ (3⁺ǂ) (2ǂ⁺) $-a$ ou ǂ (3⁺±) (2ǂ⁺) c et ǂ (3⁺±) a, fait ǂ (3⁺ǂ) a.

Si les signes qui se multiplient, ou qui se divisent sont correspondants seulement: leur nature particuliere qui se reconnoit par la forme du Caractere, fera juger du produit. Par exemple

(2⁺ǂ) b ⌒ (2ǂ) a, fait (2ǂ⁺) ab
(2⁺ǂ) a ⌒ (2ǂ⁺) b, fait (2ǂ) ab
(2⁺ǂ) a ⌒ (2ǂ⁺) b ⌒ (2ǂ) c, fait $+abc$
(3⁺ǂ) a ⌒ (3⁺ǂ) b, fait (3ǂ) ab

Exemple d'une extraction d'une Racine Quarrée

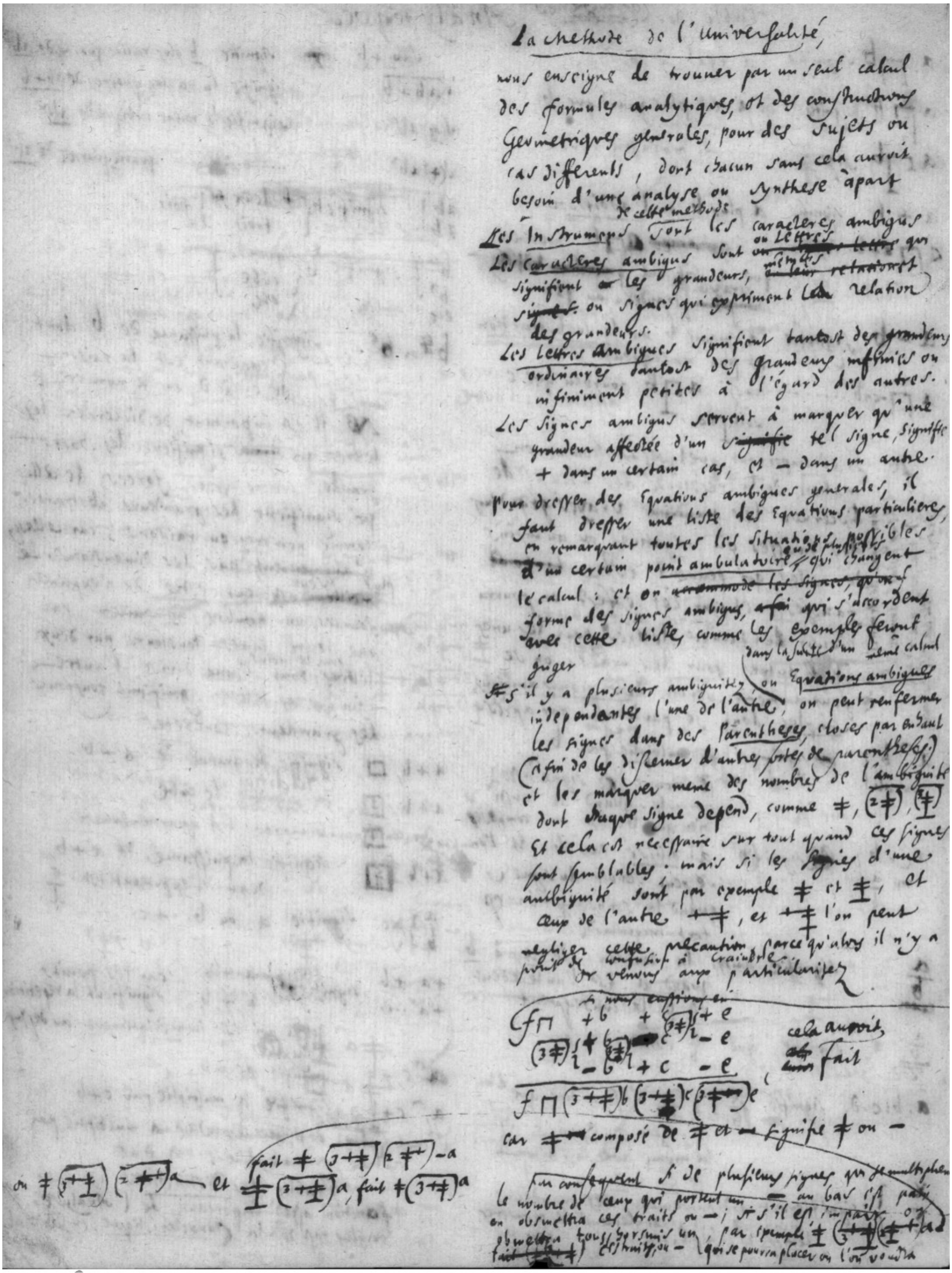
Soit une Equation $2ax$ ǂ $\frac{a}{q}x^2$, ⊓ y^2, et la question est comment il faut exprimer la valeur de x, conformement à cette Equation, or je dis, que

$$+x \sqcap ǂ\sqrt{\frac{aq^2 ǂ y^2q}{a}} \pm q$$

dont voicy la preuve. En transposant nous aurons $+x$ ǂ q ⊓ ǂ$\sqrt{\frac{aq^2 ǂ y^2q}{a}}$, dont le quarré sera, $+x^2$ ǂ $2xq + q^2$ ⊓ $\frac{aq^2 ǂ y^2q}{a}$. Multipliant tout par a, ce sera: $+ax^2$ ǂ $2aqx$ + ~~aq^2~~ ⊓ ~~aq^2~~ ǂ y^2q, ostons aq^2 de deux costez, et divisons tout, par ǂq, $\frac{+ax^2}{ǂq}$ $\frac{ǂ2aqx}{ǂq}$ ⊓ $\frac{ǂy^2q}{ǂq}$, et nous aurons: ǂ$\frac{a}{q}x^2 + 2ax$ ⊓ y^2 qui est l'equation donnée.

La consideration de cette operation peut servir d'exemple à la pluspart de nos preceptes.

LAA VII-7, p. 146



La Methode de l'Universalité,

nous enseigne de trouver par un seul calcul des formules analytiques, et des constructions Geometriques generales, pour des sujets ou cas differents, dont chacun sans cela auroit besoin d'une analyse ou synthese à part

Les Instrumens de cette methode sont les caracteres ambigus ou lettres

Les caracteres ambigus sont ou lettres qui signifient les grandeurs, ou signes qui expriment leur relation des grandeurs.

Les lettres ambigues signifient tantost des grandeurs ordinaires tantost des grandeurs infinies ou infiniment petites à l'egard des autres.

Les signes ambigus servent à marquer qu'une grandeur affectée d'un tel signe, signifie + dans un certain cas, et − dans un autre.

Pour dresser des Equations ambigues generales, il faut dresser une liste des Equations particulieres, en remarquant toutes les situations possibles d'un certain point ambulatoire qui changent le calcul : et on forme des signes ambigus, qui s'accordent avec cette liste, comme les exemples feront juger

S'il y a plusieurs ambiguités dans la suite d'un meme calcul ou Equations ambigues independantes l'une de l'autre, on peut renfermer les signes dans des Parentheses, closes par en haut (afin de les discerner d'autres sortes de parentheses) et les marquer meme des nombres de l'ambiguité dont chaque signe depend, comme ∓, (2∓), (3∓)

Et cela est necessaire sur tout quand ces signes sont semblables. mais si les signes d'une ambiguité sont par exemple ∓ et ±, et ceux de l'autre +∓, et +± l'on peut negliger cette precaution, parce qu'alors il n'y a point de confusion à craindre [illegible]

[illegible]

cela auroit fait

car ∓ composé de ∓ et − signifie ∓ ou −

[illegible]

LH 4 V 10, f. 39v

This manuscript by Leibniz shows the use of ambiguous signs of the 1st, 2nd and 3rd systems. (see also the 2 following pages)

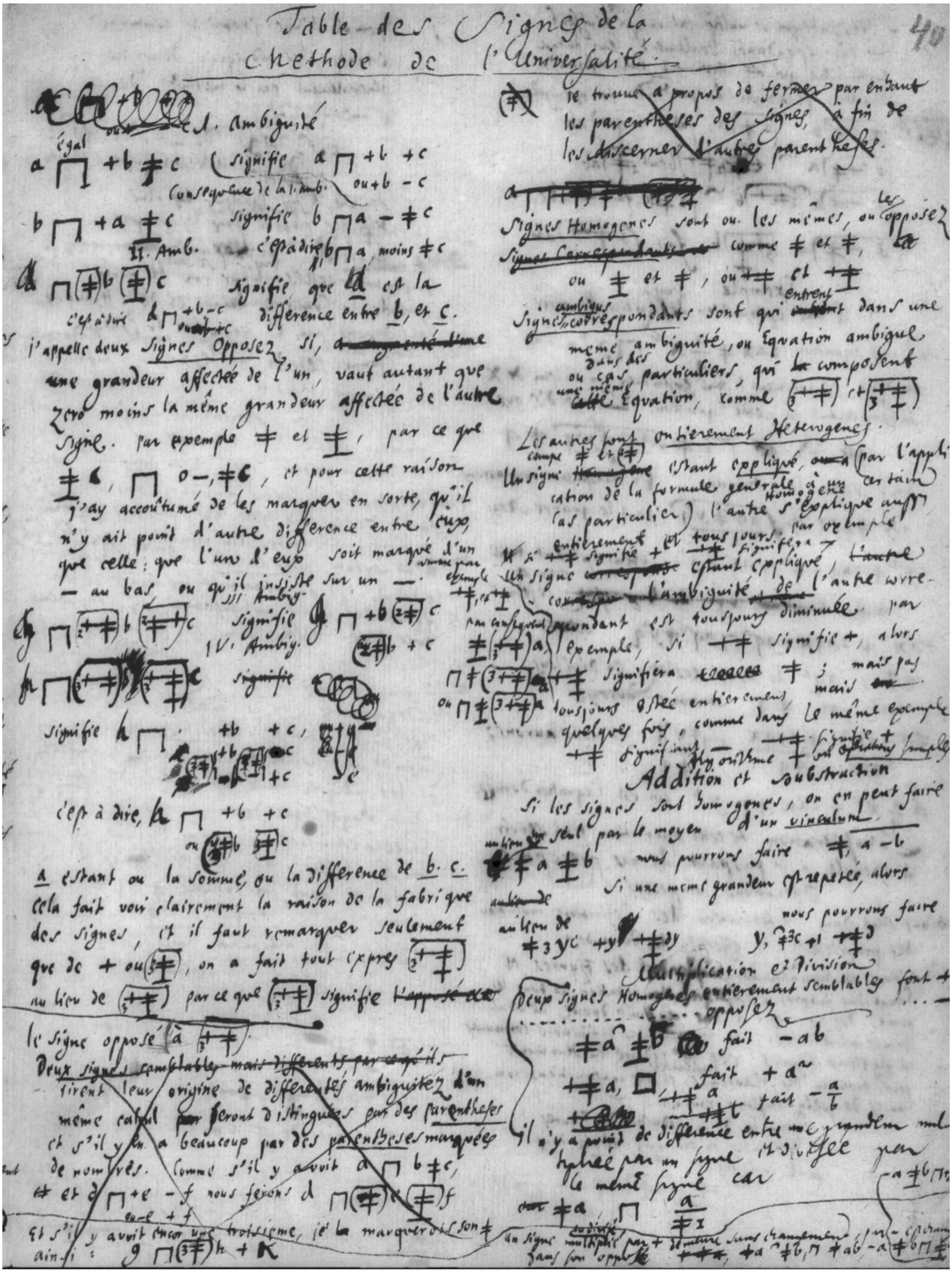


Table des Signes de la Methode de l'Universalité

1. Ambiguité

signifie a ⊓ +b +c ou +b −c

Consequence de la I. amb.

II. Amb. c'est à dire b ⊓ a, moins ∓c

signifie que d est la difference entre b, et c.

j'appelle deux Signes Opposez si, une grandeur affectée de l'un, vaut autant que zero moins la même grandeur affectée de l'autre signe. par exemple ∓ et ±, par ce que ... et pour cette raison j'ay accoutumé de les marquer en sorte, qu'il n'y ait point d'autre difference entre eux, que celle: que l'un d'eux soit marqué d'un — au bas ou qu'il insiste sur un —.

IV. Ambig.

signifie

c'est à dire

a estant ou la somme, ou la difference de b. c. cela fait voir clairement la raison de la fabrique des signes, et il faut remarquer seulement que de + ou ... on a fait tout exprés ... au lieu de ... par ce que ... signifie

le signe opposé à ...

Deux signes semblables mais differents par ce qu'ils tirent leur origine de differentes ambiguitez d'un même calcul seront distinguez par des parentheses et s'il y en a beaucoup par des parentheses marquées de nombres. Comme s'il y avoit a ⊓ b ∓ c, et d ⊓ +e −f nous ferons d ...

Et s'il y avoit encor une troisiéme, je la marquerois ainsi: g ⊓ ... h + k

Je trouve à propos de fermer par en haut les parentheses des signes, à fin de les discerner d'autres parentheses.

Signes Homogenes sont ou les mêmes, ou les opposez comme ∓ et ∓, ou ± et ∓, ou ...

Signes ambigus correspondants sont qui entrent dans une même ambiguité, ou Equation ambigue, ou cas particuliers, qui composent une même equation, comme ...

Les autres sont entierement Heterogenes.

Un signe estant expliqué par l'application de la formule generale à un certain cas particulier, l'autre s'explique aussi ...

Addition et soustraction

Si les signes sont homogenes, on en peut faire un seul par le moyen d'un vinculum ... nous pourrons faire ... Si une même grandeur est repetée, alors nous pourrons faire ...

Multiplication et Division

Deux signes Homogenes entierement semblables font + opposez ...

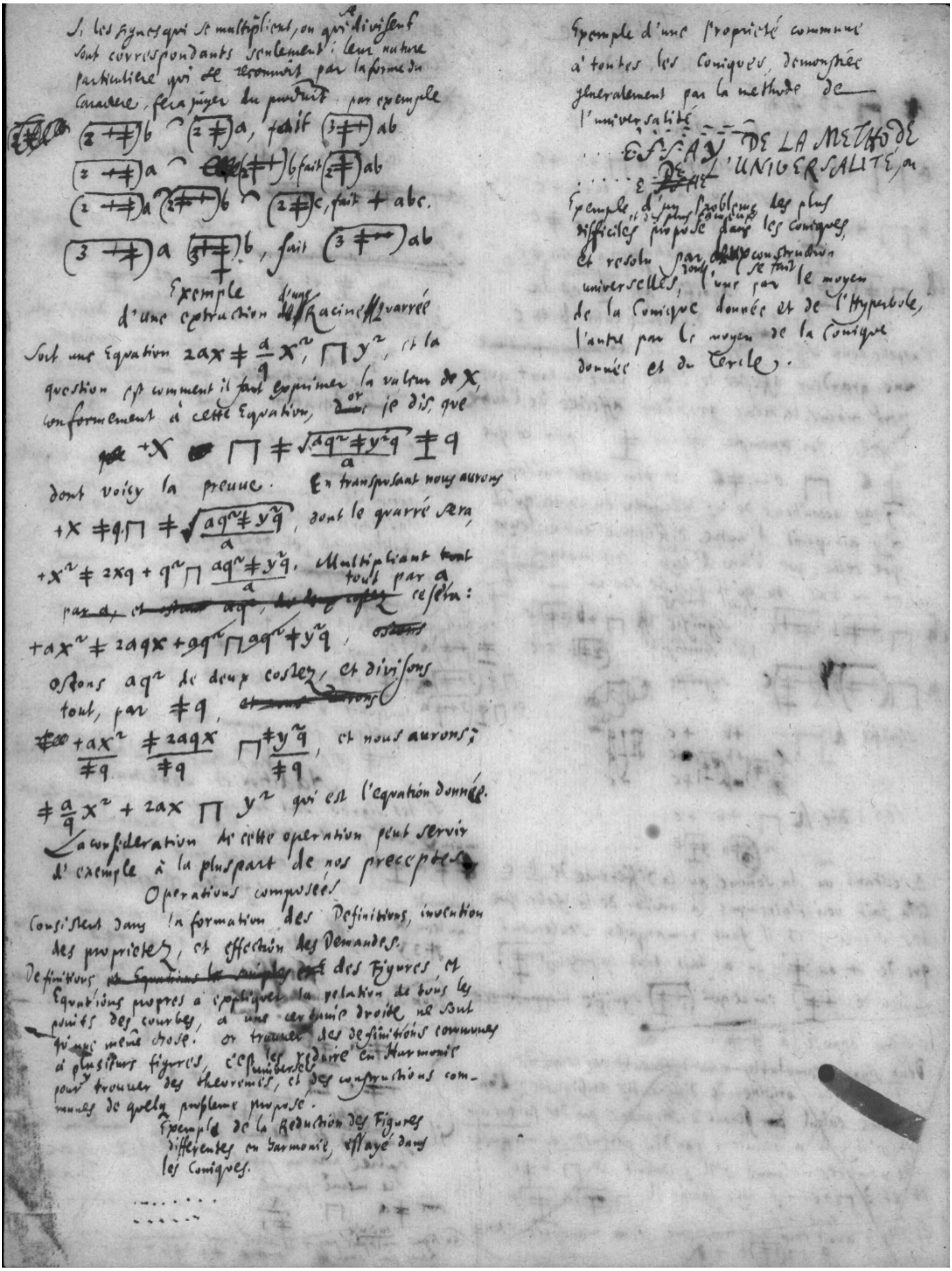
fait −ab

fait +a²

fait −a/b

il n'y a point de difference entre une grandeur multipliée par un signe et divisée par le même signe car

LH 4 V 10, f. 40r



Si les signes qui se multiplient, ou qui divisent sont correspondants seulement; leur nature particuliere qui se reconnoist par la forme du caractere, fera juger du produit. par exemple

[illegible]

Exemple d'une extraction d'une Racine quarrée

Soit une Equation $2ax \pm \frac{a}{q}x^2 \sqcap y^2$, et la question est comment il faut exprimer la valeur de x, conformement à cette Equation, or je dis, que

$+x \sqcap \pm \frac{\sqrt{aq^2 \pm y^2 q}}{a} \pm q$

dont voicy la preuve. En transposant nous aurons

$+x \pm q \sqcap \pm \sqrt{\frac{aq^2 \pm y^2 q}{a}}$, dont le quarré sera

$+x^2 \pm 2xq + q^2 \sqcap \frac{aq^2 \pm y^2 q}{a}$. Multipliant tous par a ce sera:

$+ax^2 \pm 2aqx + aq^2 \sqcap aq^2 \pm y^2 q$,

ostons aq^2 de deux costez, et divisons tout, par $\pm q$, [illegible] et nous aurons;

$\frac{+ax^2}{\pm q} \quad \frac{\pm 2aqx}{\pm q} \sqcap \frac{\pm y^2 q}{\pm q}$,

$\pm \frac{a}{q}x^2 + 2ax \sqcap y^2$ qui est l'equation donnée.

La consideration de cette operation peut servir d'exemple à la pluspart de nos preceptes.

Operations composées

Consistent dans la formation des Definitions, invention des proprietez, et effection des Demandes.

Definitions [illegible] des Figures et Equations propres à expliquer la relation de tous les points des courbes, à une certaine droite, ne sont qu'une même chose. Or trouver des definitions communes à plusieurs figures, c'est les reduire en Harmonie pour trouver des theoremes, et des constructions communes de quelque probleme proposé.

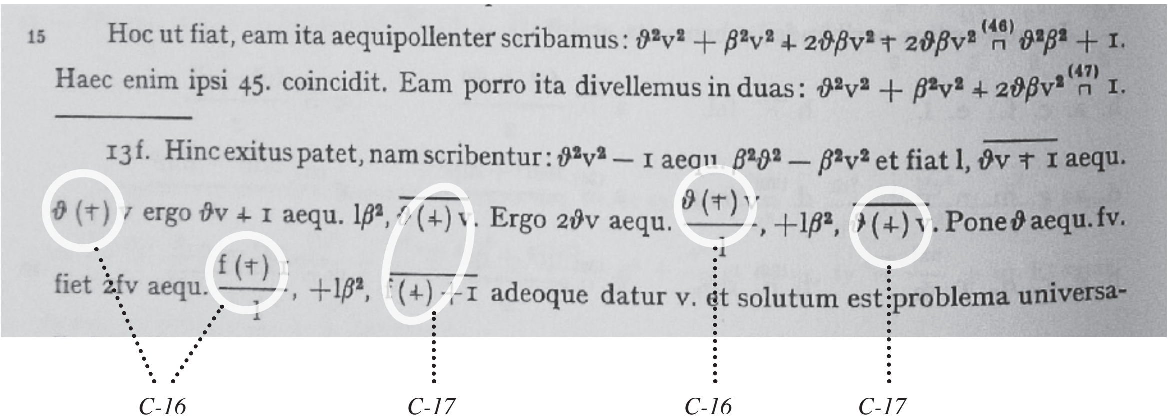
Exemple de la Reduction des Figures differentes en Harmonie, essayé dans les Coniques.

Exemple d'une propriété commune à toutes les Coniques, demonstrée generalement par la methode de l'universalité

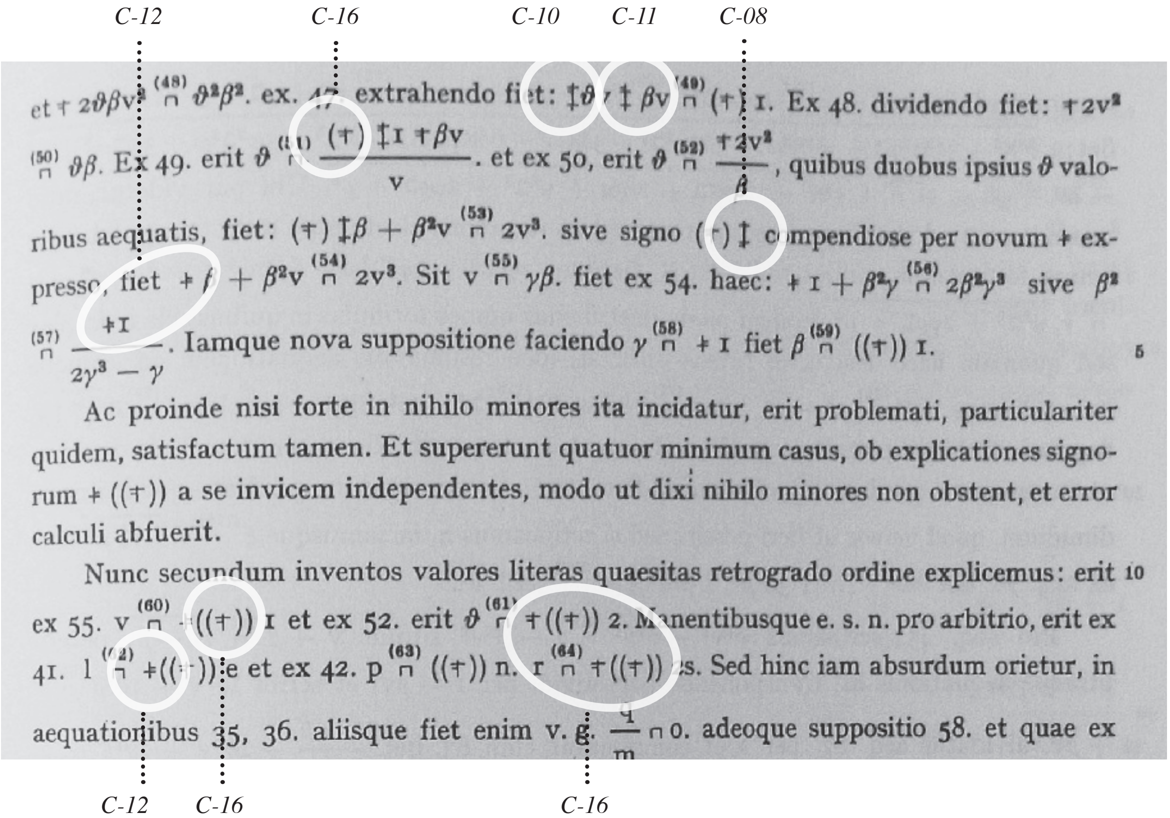
ESSAY DE LA METHODE DE L'UNIVERSALITÉ, ou

Exemple d'un des plus difficiles problemes proposé dans les Coniques, et resolu par deux constructions universelles, l'une se fait par le moyen de la Conique donnée et de l'Hyperbole, l'autre par le moyen de la Conique donnée et du Cercle.

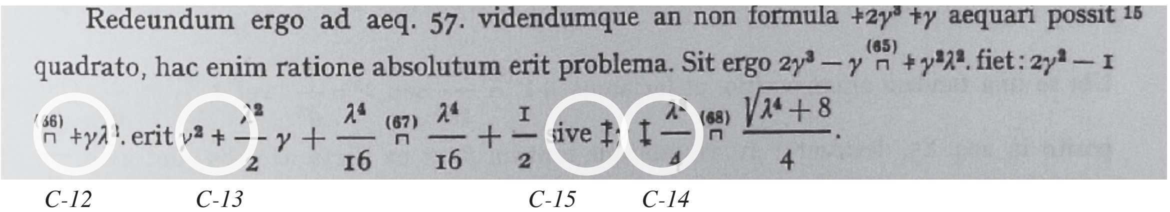
LH 4 V 10, f. 40v



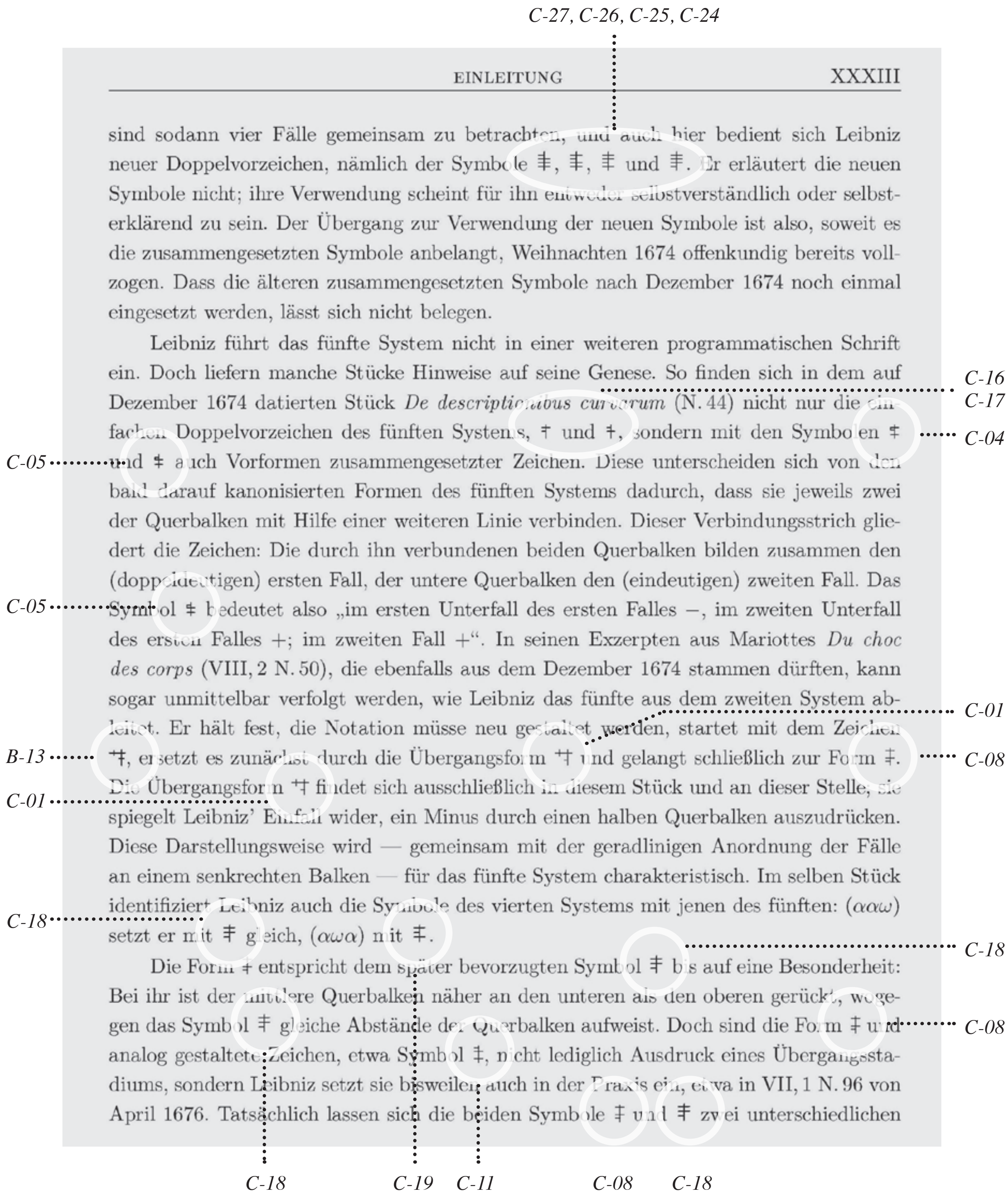
Ambiguous signs, 5th system. LAA VII-1, p. 618



Ambiguous signs, 5th system. LAA VII-1, p. 619



Ambiguous signs, 5th system. LAA VII-1, p. 619



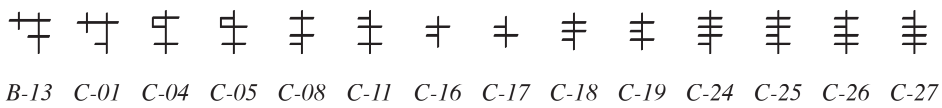
EINLEITUNG XXXIII

sind sodann vier Fälle gemeinsam zu betrachten, und auch hier bedient sich Leibniz neuer Doppelvorzeichen, nämlich der Symbole ‡, ‡, ‡ und ‡. Er erläutert die neuen Symbole nicht; ihre Verwendung scheint für ihn entweder selbstverständlich oder selbsterklärend zu sein. Der Übergang zur Verwendung der neuen Symbole ist also, soweit es die zusammengesetzten Symbole anbelangt, Weihnachten 1674 offenkundig bereits vollzogen. Dass die älteren zusammengesetzten Symbole nach Dezember 1674 noch einmal eingesetzt werden, lässt sich nicht belegen.

Leibniz führt das fünfte System nicht in einer weiteren programmatischen Schrift ein. Doch liefern manche Stücke Hinweise auf seine Genese. So finden sich in dem auf Dezember 1674 datierten Stück *De descriptionibus curvarum* (N. 44) nicht nur die einfachen Doppelvorzeichen des fünften Systems, † und ∔, sondern mit den Symbolen ‡ und ‡ auch Vorformen zusammengesetzter Zeichen. Diese unterscheiden sich von den bald darauf kanonisierten Formen des fünften Systems dadurch, dass sie jeweils zwei der Querbalken mit Hilfe einer weiteren Linie verbinden. Dieser Verbindungsstrich gliedert die Zeichen: Die durch ihn verbundenen beiden Querbalken bilden zusammen den (doppeldeutigen) ersten Fall, der untere Querbalken den (eindeutigen) zweiten Fall. Das Symbol ‡ bedeutet also „im ersten Unterfall des ersten Falles −, im zweiten Unterfall des ersten Falles +; im zweiten Fall +“. In seinen Exzerpten aus Mariottes *Du choc des corps* (VIII, 2 N. 50), die ebenfalls aus dem Dezember 1674 stammen dürften, kann sogar unmittelbar verfolgt werden, wie Leibniz das fünfte aus dem zweiten System ableitet. Er hält fest, die Notation müsse neu gestaltet werden, startet mit dem Zeichen †, ersetzt es zunächst durch die Übergangsform † und gelangt schließlich zur Form ‡. Die Übergangsform † findet sich ausschließlich in diesem Stück und an dieser Stelle, sie spiegelt Leibniz’ Einfall wider, ein Minus durch einen halben Querbalken auszudrücken. Diese Darstellungsweise wird — gemeinsam mit der geradlinigen Anordnung der Fälle an einem senkrechten Balken — für das fünfte System charakteristisch. Im selben Stück identifiziert Leibniz auch die Symbole des vierten Systems mit jenen des fünften: ($\alpha\alpha\omega$) setzt er mit ‡ gleich, ($\alpha\omega\alpha$) mit ‡.

Die Form ‡ entspricht dem später bevorzugten Symbol ‡ bis auf eine Besonderheit: Bei ihr ist der mittlere Querbalken näher an den unteren als den oberen gerückt, wogegen das Symbol ‡ gleiche Abstände der Querbalken aufweist. Doch sind die Form ‡ und analog gestaltete Zeichen, etwa Symbol ‡, nicht lediglich Ausdruck eines Übergangsstadiums, sondern Leibniz setzt sie bisweilen auch in der Praxis ein, etwa in VII, 1 N. 96 von April 1676. Tatsächlich lassen sich die beiden Symbole ‡ und ‡ zwei unterschiedlichen

Ambiguous signs, 2nd and 5th system. LAA VII-7, introduction, p. XXXIII



N. 22 INFINITESIMALMATHEMATIK 1674–1676 191

(31) Ponamus jam contra directricem esse non AD, sed AE, constantem WL, quam vocabimus λ. Crementum ordinatarum EG, esse GW; ipsam $EH \sqcap l$. primum investigemus hoc modo: $2ax \mp \frac{2a}{q}x^2 \sqcap 2yl$. sive $l \sqcap \frac{ax \mp \frac{a}{q}x^2}{y} \sqcap \frac{2ax \mp \frac{a}{q}x^2 - ax}{y}$ sive $\frac{y^2 - ax}{y}$. Jam ut x inveniatur, erit $x^2 \mp \frac{2q\not{a}}{\not{a}}x + q^2 \sqcap q^2 \mp y^2$, adeoque fiet $\mp x \mp q \sqcap \sqrt{,q^2 \mp y^2}$, et $x \sqcap \mp q \mp \sqrt{q^2 \mp y^2}$ adeoque $l \sqcap \frac{y^2 \mp qa \mp a\sqrt{q^2 \mp y^2}}{y} \sqcap EH$. Ergo GW erit $\sqcap \frac{\lambda,, \frown y^2 \mp qa \mp a\sqrt{q^2 \mp y^2}}{y,, \frown \mp q \mp \sqrt{q^2 \mp y^2}}$; et $\frac{GB \frown WL^2}{GW} \sqcap \frac{y \frown \lambda^{\not{2}}, \frown y, \frown \mp q \mp \sqrt{q^2 \mp y^2}}{\not{\lambda},, \frown y^2 \mp qa \mp a\sqrt{q^2 \mp y^2}}$, cujus seriei itidem habetur summa, ex datis omnibus $\sqrt{q^2 \mp y^2}$.

Quae theoremata vel ideo annotanda duxi, quod semel elapsa non facile rursus in mentem venirent, et non nisi per multas ambages deprehensa sint. Et haec quidem de Trianguli characteristici usu ad dimensiones curvilineorum nunc sufficiant.

Ambiguous sign C-21, 5th system. LAA VII-5, p. 191

C-25, C-24 *C-31* *C-26* *C-28* *C-24*

(4)D . . . $TD \sqcap +TA + AD$ $EL \sqcap +EC - CL$

Generaliter ergo TD ita exprimemus:

$$TD \sqcap \mp TA \mp AD, \quad EL \sqcap \mp EC \mp CL.$$

Ergo hoc modo $DE \sqcap \frac{\mp ax \mp af \mp xf}{a}$ et $EL \sqcap \mp \frac{f\sqrt{2ax - x^2}}{a} \mp \sqrt{2ax - x^2}$.

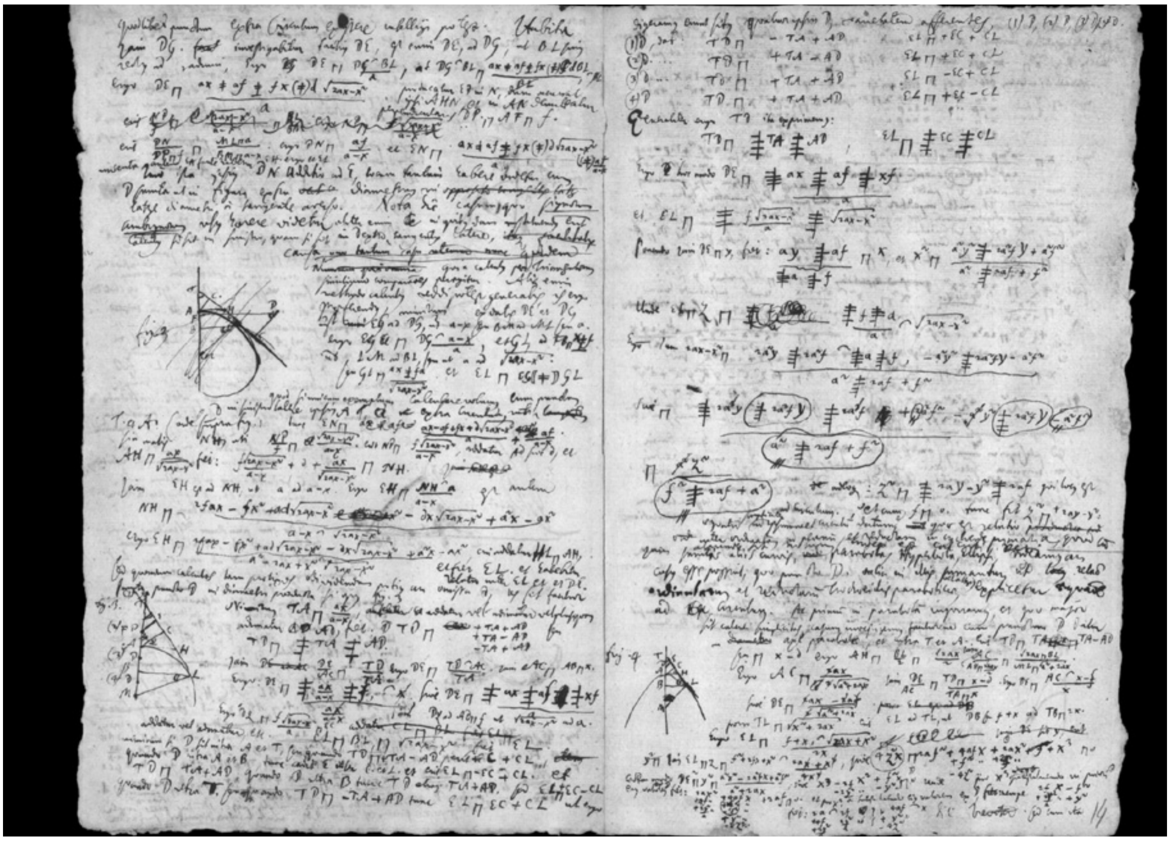
Ponendo jam $DE \sqcap y$, fiet: $\frac{ay \mp af}{\mp a \mp f} \sqcap x$, et $x^2 \sqcap \frac{a^2y^2 \mp 2a^2fy + a^2f^2}{a^2 \mp 2af + f^2}$.

Unde $EL \sqcap z \sqcap \frac{\mp f \mp a}{a} \frown \sqrt{2ax - x^2}$.

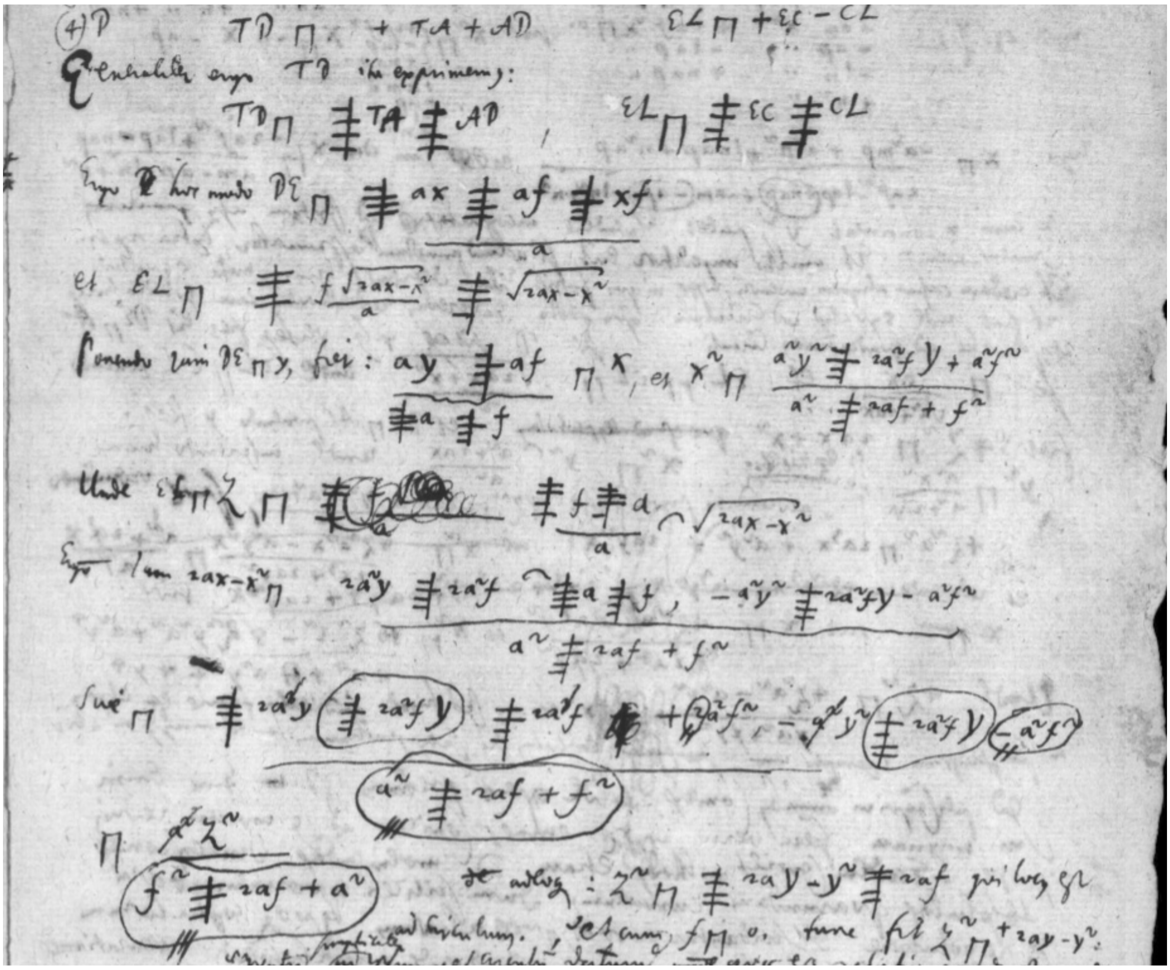
Jam $2ax - x^2 \sqcap \frac{2a^2y \mp 2a^2f \frown \mp a \mp f, -a^2y^2 \mp 2a^2fy - a^2f^2}{a^2 \mp 2af + f^2}$ sive

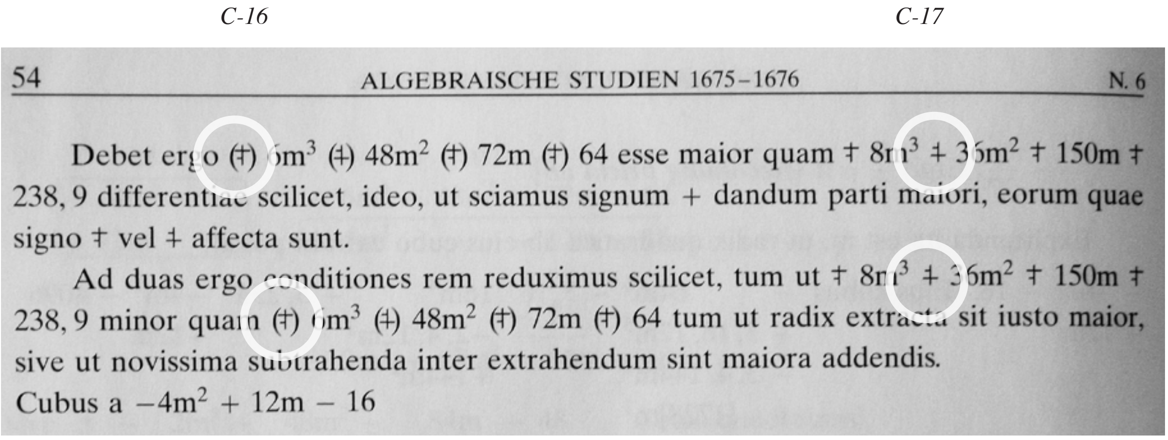
$$\sqcap \frac{\mp 2a^{\not{3}}y \boxed{\mp 2a^2fy} \mp 2a^{\not{3}}f + \boxed{2}a^2f^2, -\not{a}^{\not{2}}y^2 \boxed{\mp 2a^2fy} \boxed{-a^2f^2}}{\boxed{a^2 \mp 2af + f^2}} \sqcap \frac{\not{a}^{\not{2}}z^2}{\boxed{f^2 \mp 2af + a^2}}.$$

Ambiguous signs, 5th system. LAA VII-5, p. 164
see also LH 35 V 3 f. 13v (next page)

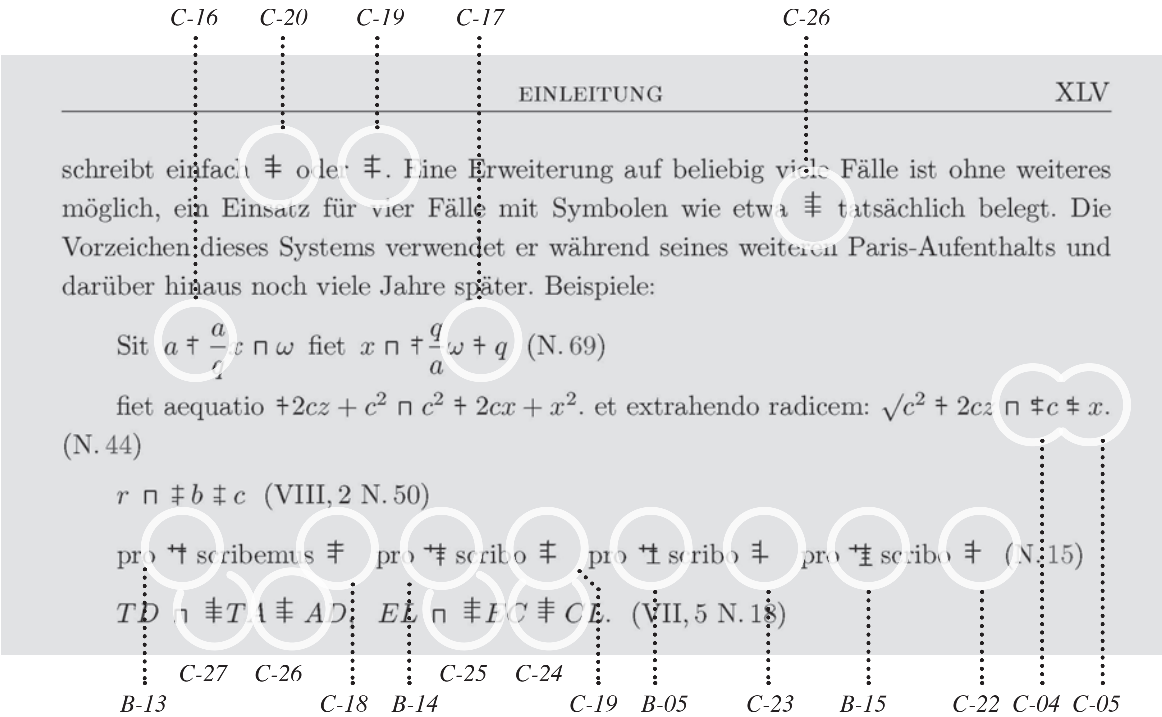


LH 35 V 3, f. 13v

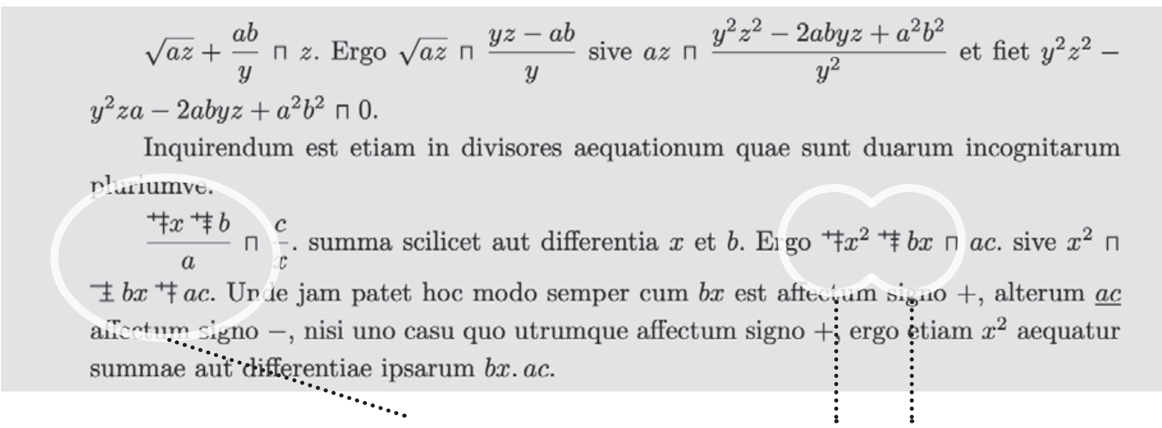




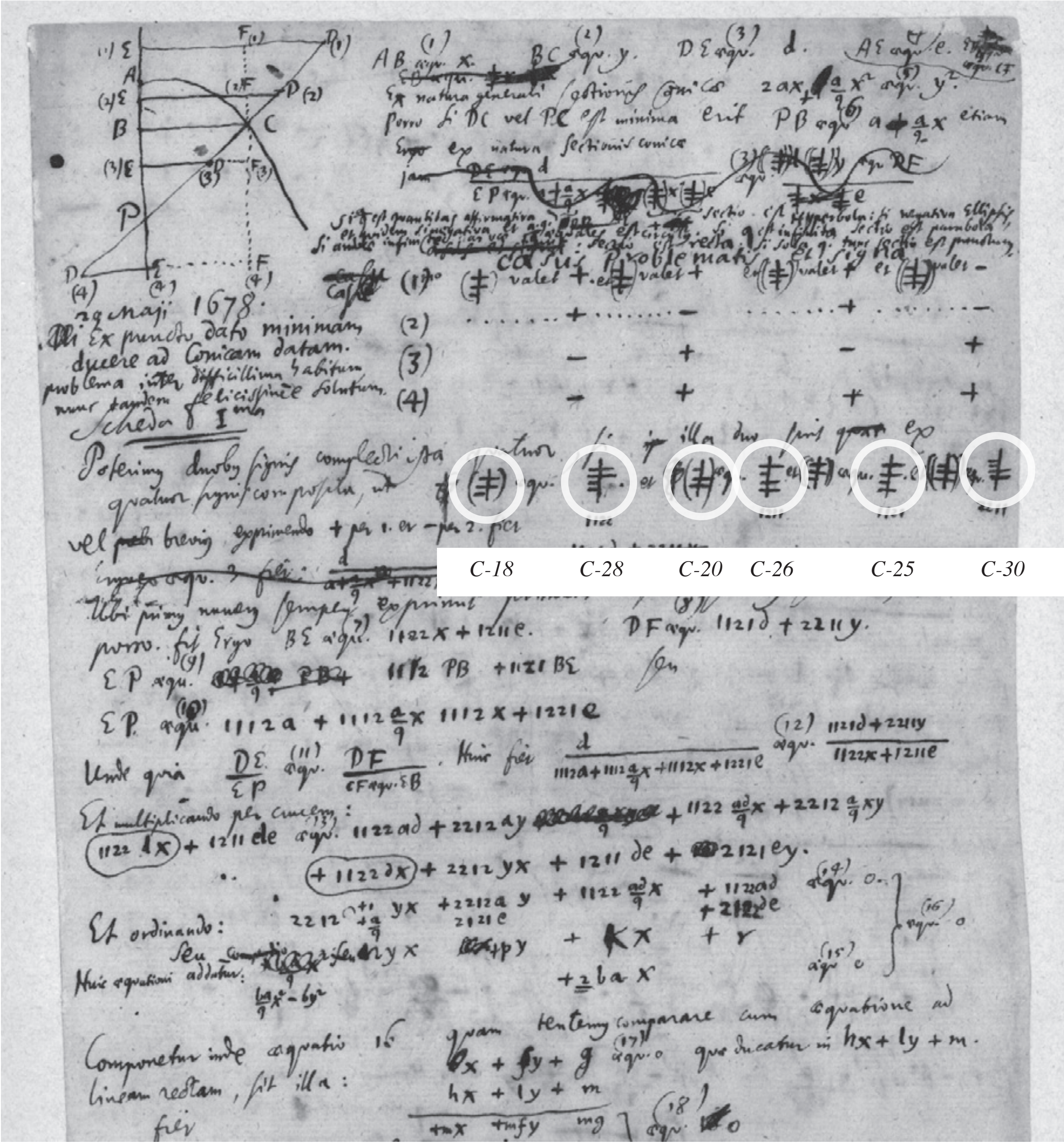
Ambiguous signs, 5th system. LAA VII-2, p. 54



Example of ambiguous signs, 2nd and 5th system. LAA VII-7, introduction p. XLV

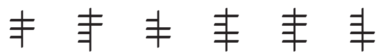


Ambiguous signs, 2nd system.
LAA VII-5, p. 233

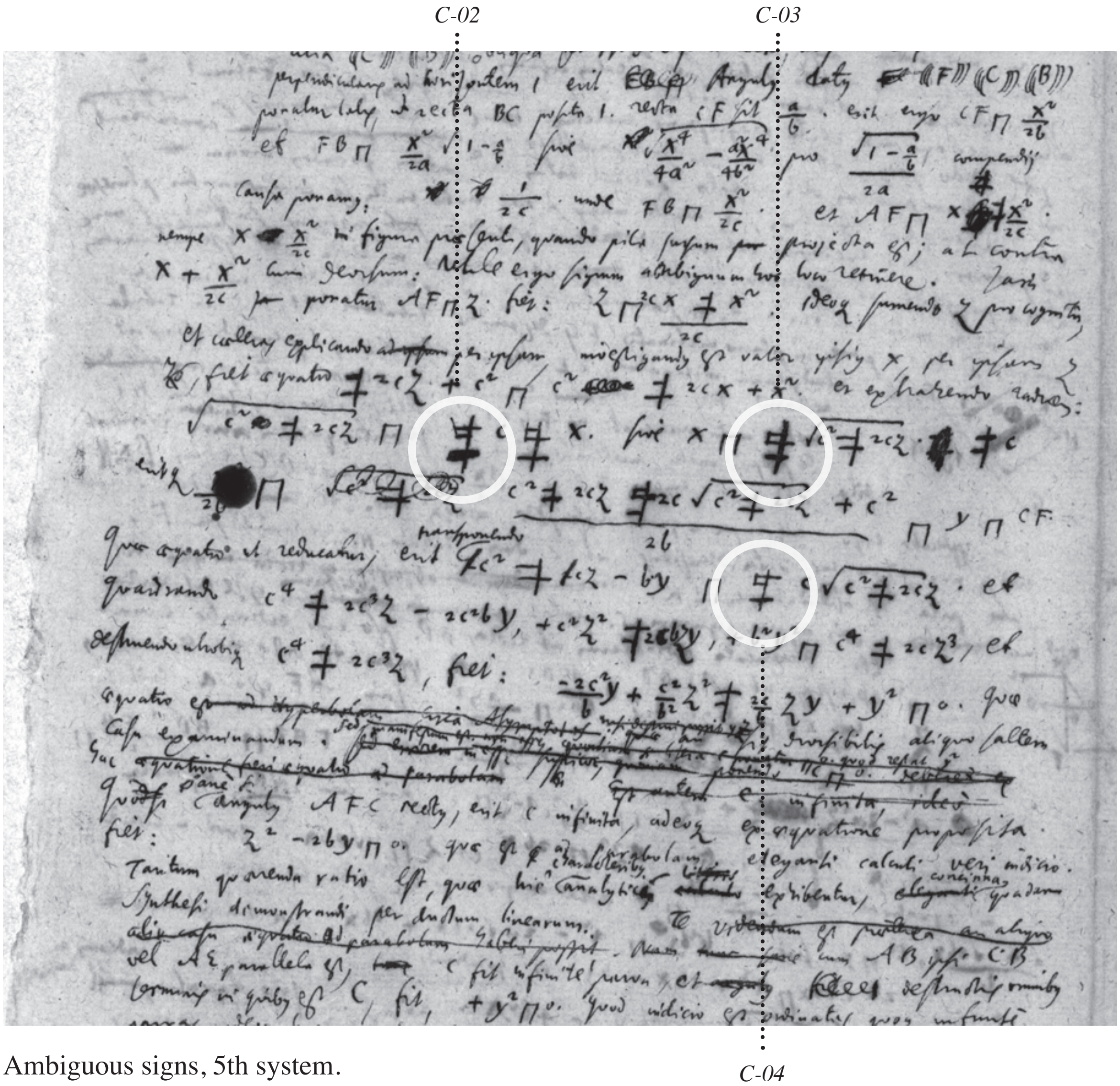


Ambiguous signs of the 5th system, LH 35 XII 1, 217v.

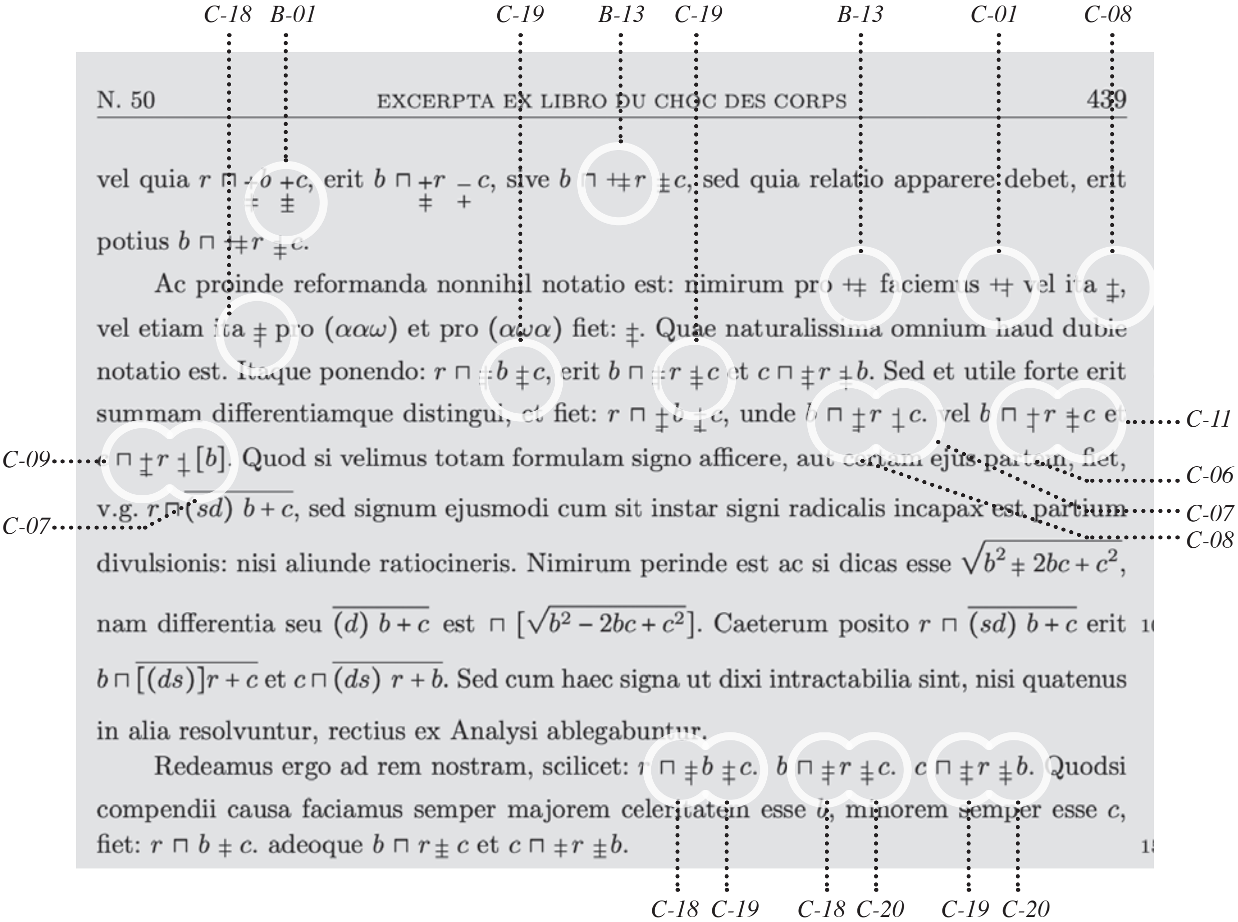
The edition of this manuscript is in preparation.



C-18 *C-28* *C-20* *C-26* *C-25* *C-30*



Ambiguous signs, 5th system.
LH 35 XII 1, fol. 227v



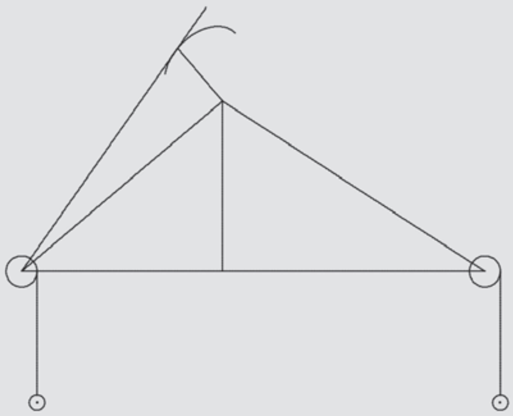
N. 50 EXCERPTA EX LIBRO DU CHOC DES CORPS 439

vel quia r ⊓ ∓b ±c, erit b ⊓ ±r ∓c, sive b ⊓ ⧺r ‡c, sed quia relatio apparere debet, erit potius b ⊓ ⧺r ‡c.

Ac proinde reformanda nonnihil notatio est: nimirum pro ⧺ faciemus ⧺ vel ita ‡, vel etiam ita ‡ pro (ααω) et pro (αωα) fiet: ‡. Quae naturalissima omnium haud dubie notatio est. Itaque ponendo: r ⊓ ‡b ‡c, erit b ⊓ ‡r ‡c et c ⊓ ‡r ‡b. Sed et utile forte erit summam differentiamque distingui, et fiet: r ⊓ ‡b ‡c, unde b ⊓ ‡r ‡c. vel b ⊓ ‡r ‡c et c ⊓ ‡r ‡[b]. Quod si velimus totam formulam signo afficere, aut certam ejus partem, fiet, v.g. $r \sqcap \overline{(sd)\ b+c}$, sed signum ejusmodi cum sit instar signi radicalis incapax est partium divulsionis: nisi aliunde ratiocineris. Nimirum perinde est ac si dicas esse $\sqrt{b^2 ‡ 2bc + c^2}$, nam differentia seu $\overline{(d)\ b+c}$ est $\sqcap [\sqrt{b^2 - 2bc + c^2}]$. Caeterum posito $r \sqcap \overline{(sd)\ b+c}$ erit $b \sqcap \overline{[(ds)]r+c}$ et $c \sqcap \overline{(ds)\ r+b}$. Sed cum haec signa ut dixi intractabilia sint, nisi quatenus in alia resolvuntur, rectius ex Analysi ablegabuntur.

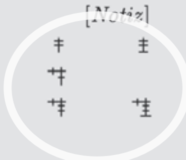
Redeamus ergo ad rem nostram, scilicet: r ⊓ ‡b ‡c. b ⊓ ‡r ‡c. c ⊓ ‡r ‡b. Quodsi compendii causa faciamus semper majorem celeritatem esse b, minorem semper esse c, fiet: r ⊓ b ‡ c. adeoque b ⊓ r ‡ c et c ⊓ ‡r ‡b.

Example of ambiguous signs, 2nd and 5th system. LAA VIII-2, p. 439



[*Fig. 7, ohne Bezug zum Haupttext, quer zur Schreibrichtung*]

[*Notiz*]



B-02, B-06, B-03; LAA VII-3, p. 360

6. Unicode Character Properties

ID	
A-01	X001;LEIBNIZIAN SYSTEM 1 AND 2 PLUS-MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
A-02	X002;LEIBNIZIAN SYSTEM 1 MINUS-PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
A-03	X003;LEIBNIZIAN SYSTEM 1 PLUS OR PLUS-MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
A-04	X004;LEIBNIZIAN SYSTEM 1 PLUS OR MINUS-PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
A-05	X005;LEIBNIZIAN SYSTEM 1 MINUS OR PLUS-MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
A-06	X006;LEIBNIZIAN SYSTEM 1 MINUS OR MINUS-PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
A-07	X007;LEIBNIZIAN SYSTEM 1 PLUS-MINUS TIMES PLUS-MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
A-08	X008;LEIBNIZIAN SYSTEM 1 MINUS-PLUS TIMES PLUS-MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
B-01	X009;LEIBNIZIAN SYSTEM 2 MINUS-PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
B-02	X010;LEIBNIZIAN SYSTEM 2 PLUS OR PLUS-MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
B-03	X011;LEIBNIZIAN SYSTEM 2 PLUS OR MINUS-PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
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B-08	X016;LEIBNIZIAN SYSTEM 2 PLUS-MINUS OR PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
B-09	X017;LEIBNIZIAN SYSTEM 2 PLUS-MINUS OR MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
B-10	X018;LEIBNIZIAN SYSTEM 2 NEGATED PLUS-MINUS OR PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
B-11	X019;LEIBNIZIAN SYSTEM 2 NEGATED PLUS OR MINUS-PLUS PARTIAL AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
B-12	X020;LEIBNIZIAN SYSTEM 2 NEGATED MINUS OR MINUS-PLUS PARTIAL AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
B-13	[X021] X010 FE00; variant form; # LEIBNIZIAN SYSTEM 2 PLUS OR PLUS-MINUS AMBIGUOUS SIGN
B-14	[X022] X011 FE00; variant form; # LEIBNIZIAN SYSTEM 2 PLUS OR MINUS-PLUS AMBIGUOUS SIGN
B-15	[X023] X014 FE00; variant form; # LEIBNIZIAN SYSTEM 2 NEGATED PLUS OR MINUS-PLUS AMBIGUOUS SIGN
B-16	X024 LEIBNIZIAN SYSTEM 2 NEGATED PLUS-MINUS OR PLUS PARTIAL AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
B-17	[X025] X019 FE00; variant form 1; # LEIBNIZIAN SYSTEM 2 NEGATED PLUS OR MINUS-PLUS PARTIAL AMBIGUOUS SIGN
B-18	[X026] X019 FE01; variant form 2; # LEIBNIZIAN SYSTEM 2 NEGATED PLUS OR MINUS-PLUS PARTIAL AMBIGUOUS SIGN
C-01	X027;LEIBNIZIAN SYSTEM 5-1 PLUS OR PLUS-MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-02	X028;LEIBNIZIAN SYSTEM 5-2 PLUS-MINUS OR PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-03	X029;LEIBNIZIAN SYSTEM 5-2 MINUS-PLUS OR PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-04	X030;LEIBNIZIAN SYSTEM 5-3 PLUS-MINUS OR PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-05	X031;LEIBNIZIAN SYSTEM 5-3 MINUS-PLUS OR PLUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
C-06	X032;LEIBNIZIAN SYSTEM 5-4 PLUS OR MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;
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C-31	X057;LEIBNIZIAN SYSTEM 5 MINUS-PLUS-MINUS-MINUS AMBIGUOUS SIGN;Sm;0;ON;;;;;N;;;;;

7. Bibliography

LAA – refers to: Leibniz, Gottfried Wilhelm: Sämtliche Schriften und Briefe. ('Leibniz-Akademie-Ausgabe', many volumes)
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ISO/IEC JTC 1/SC 2/WG 2
PROPOSAL SUMMARY FORM TO ACCOMPANY SUBMISSIONS
FOR ADDITIONS TO THE REPERTOIRE OF ISO/IEC 10646[1]
Please fill all the sections A, B and C below.
Please read Principles and Procedures Document (P & P) from http://std.dkuug.dk/JTC1/SC2/WG2/docs/principles.html **for guidelines and details before filling this form.**
Please ensure you are using the latest Form from http://std.dkuug.dk/JTC1/SC2/WG2/docs/summaryform.html**.**
See also http://std.dkuug.dk/JTC1/SC2/WG2/docs/roadmaps.html **for latest *Roadmaps*.**

A. Administrative

1. **Title:** **Proposal to encode Leibnizian ambiguous signs**
2. Requester's name: Uwe Mayer, Siegmund Probst, David Rabouin, Elisabeth Rinner, Andreas Stötzner, Achim Trunk, Charlotte Wahl
3. Requester type (Member body/Liaison/Individual contribution): Individual (work group)
4. Submission date: 2025-11.24.
5. Requester's reference (if applicable): LUCP L-2529
6. Choose one of the following:
   This is a complete proposal: Yes
   (or) More information will be provided later:

B. Technical – General

1. Choose one of the following:
   a. This proposal is for a new script (set of characters): No
      Proposed name of script:
   b. The proposal is for addition of character(s) to an existing block: No
      Name of the existing block:
2. Number of characters in proposal: 57
3. Proposed category (select one from below - see section 2.2 of P&P document):

A-Contemporary		B.1-Specialized (small collection)	Yes	B.2-Specialized (large collection)	
C-Major extinct		D-Attested extinct		E-Minor extinct	
F-Archaic Hieroglyphic or Ideographic		G-Obscure or questionable usage symbols			

4. Is a repertoire including character names provided? Yes
   a. If YES, are the names in accordance with the "character naming guidelines" in Annex L of P&P document? Yes
   b. Are the character shapes attached in a legible form suitable for review? Yes
5. Fonts related:
   a. Who will provide the appropriate computerized font to the Project Editor of 10646 for publishing the standard?
      Andreas Stötzner
   b. Identify the party granting a license for use of the font by the editors (include address, e-mail, ftp-site, etc.):
      Andreas Stötzner Gestaltung, Klauflügelweg 21, 88400 Biberach/R., Germany, as@signographie.de
6. References:
   a. Are references (to other character sets, dictionaries, descriptive texts etc.) provided? Yes
   b. Are published examples of use (such as samples from newspapers, magazines, or other sources) of proposed characters attached? Yes
7. Special encoding issues:
   Does the proposal address other aspects of character data processing (if applicable) such as input, presentation, sorting, searching, indexing, transliteration etc. (if yes please enclose information)? No

8. Additional Information:
Submitters are invited to provide any additional information about Properties of the proposed Character(s) or Script that will assist in correct understanding of and correct linguistic processing of the proposed character(s) or script. Examples of such properties are: Casing information, Numeric information, Currency information, Display behaviour information such as line breaks, widths etc., Combining behaviour, Spacing behaviour, Directional behaviour, Default Collation behaviour, relevance in Mark Up contexts, Compatibility equivalence and other Unicode normalization related information. See the Unicode standard at http://www.unicode.org for such information on other scripts. Also see Unicode Character Database (http://www.unicode.org/reports/tr44/) and associated Unicode Technical Reports for information needed for consideration by the Unicode Technical Committee for inclusion in the Unicode Standard.

[1] Form number: N4502-F (Original 1994-10-14; Revised 1995-01, 1995-04, 1996-04, 1996-08, 1999-03, 2001-05, 2001-09, 2003-11, 2005-01, 2005-09, 2005-10, 2007-03, 2008-05, 2009-11, 2011-03, 2012-01)

C. Technical - Justification

1. Has this proposal for addition of character(s) been submitted before? Yes

If YES explain *N5329 (L-2526); N5329; previous as a part of* N5277 / L-2402n

2. Has contact been made to members of the user community (for example: National Body, user groups of the script or characters, other experts, etc.)? Yes

If YES, with whom? Leibniz-Archiv, Forschungsstelle der Leibniz-Edition, Niedersächsische Landesbibliothek (GWLB), Hanover, Göttingen Academy of Science and Humanities in Lower Saxony (DE), Philiumm research group of CNRS (UMR 7219, laboratoire SPHERE) / Université de Paris VII; general: scholars, researchers, authors and editors working in the field of science history and upon editions of historic text corpora (e.g. of G. W. Leibniz, but also many others)

If YES, available relevant documents: L-2409, L-2410

3. Information on the user community for the proposed characters (for example: size, demographics, information technology use, or publishing use) is included? Yes

Reference:

4. The context of use for the proposed characters (type of use; common or rare) Common

Reference: mainly specialist usage, scholarly, worldwide

5. Are the proposed characters in current use by the user community? Yes

If YES, where? Reference: mainly Europe, Americas; other countries

6. After giving due considerations to the principles in the P&P document must the proposed characters be entirely in the BMP? No

If YES, is a rationale provided?

If YES, reference:

7. Should the proposed characters be kept together in a contiguous range (rather than being scattered)? Yes

8. Can any of the proposed characters be considered a presentation form of an existing character or character sequence? No

If YES, is a rationale for its inclusion provided?

If YES, reference:

9. Can any of the proposed characters be encoded using a composed character sequence of either existing characters or other proposed characters? Yes

If YES, is a rationale for its inclusion provided? *Yes*

If YES, reference: *5 variation sequences; see introduction and p. 25*

10. Can any of the proposed character(s) be considered to be similar (in appearance or function) to, or could be confused with, an existing character? No

If YES, is a rationale for its inclusion provided?

If YES, reference:

11. Does the proposal include use of combining characters and/or use of composite sequences? No

If YES, is a rationale for such use provided?

If YES, reference:

Is a list of composite sequences and their corresponding glyph images (graphic symbols) provided? No

If YES, reference:

12. Does the proposal contain characters with any special properties such as control function or similar semantics? No

If YES, describe in detail (include attachment if necessary)

13. Does the proposal contain any Ideographic compatibility characters? No

If YES, are the equivalent corresponding unified ideographic characters identified?

If YES, reference: