

Universal Multiple-Octet Coded Character Set
International Organization for Standardization
Internationale Standardisierungs-Organisation
Organisation Internationale de Normalisation
Διεθνής Οργανισμός Τυποποίησης
Международная организация по стандартизации

Doc Type: Working Group Document

Title: Proposal to encode miscellaneous scientific symbols

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Version: 3rd revision

Status: forward to UTC / WG2

Action: for UTC review, for Unicode 18.0 pipeline

Date: January 19, 2026

Requester's reference: LUCP L-2604

1. Background – Miscellaneous symbols in historic sources




This proposal is part of the research program upon historical mathematical sources, conducted by the CNRS Philiumm project (headed by Prof. David Rabouin, University of Paris) and supported by researchers from the Landesbibliothek Hanover (Germany). The aim of this project work is to achieve a standardized encoding for special mathematical characters in historic texts, which is required for accurate facsimile editions of those sources.

For more background information about the Philiumm project and the related research work, please visit the [Philiumm website](#) or see doc. no. [N5277](#).

In this proposal we introduce a number of various symbols mainly from the field of historical mathematics.

2. Revision

2nd version: After discussion with SEWG and UTC experts and following the subsequent recommendations, we have dropped a few candidates (for various reasons) and propose now a selection of 13 new characters, of which 3 are combining characters.

In this 3rd revision the three combining characters ,  and  are omitted. They will be revised separately and will be subject to a new proposal, alongside further new characters of a similar kind. The remaining 10 characters have not caused any further critique during the January 15 meeting, so we understand that they are regarded mature for encoding.

3. Characters

If this proposal gets accepted, the following 10 characters will exist:

- ✕ CASTING-OUT-NINES
- ① LUNATE ENCIRCLED DIGIT ONE
- ⊢ PROPORTION WITH ONE STROKE
- ⊢⊢ PROPORTION WITH TWO STROKES
- ∞ INFINITY WITH TWO DOTS
- ⊙ INVOLVED
- ⊙ LEIBNIZIAN ENCIRCLED V
- ⊙ LEIBNIZIAN ENCIRCLED V IN BOX
- ~ SUPERScript WAVE SYMBOL
- ~ SUPERScript WAVE WITH OVERLINE SYMBOL

4. Figures and explanations

302
ARITHMETISCHE KREISQUADRATUR 1673-1676
N. 26

Als men de $\angle ACB$ wil 2 mahl in 2 gelijcke deel, deelen; om AF te vinden, soo kan men het dus oock doen[:]

Regel.

Gelijck als

5

$AC + BC$, sijn \square
 $-\square AB$, multipl. in BC

staet tot

$\square AB$, multipl. in AC

also het

$\square AC$

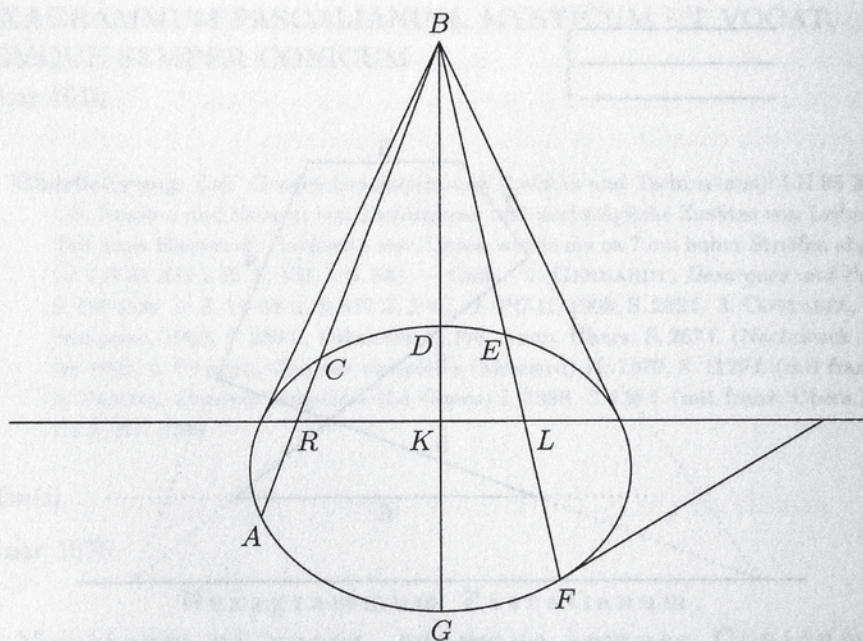
tot het

$\square AF$

⊢⊢ PROPORTION WITH TWO STROKES

This figure also shows the use of \square WHITE CUBE (proposed in doc. L-2514 for u1F7F7).
 LAA VII-6 P. 302; LH 35 VIII 30, fol. 68r (below)

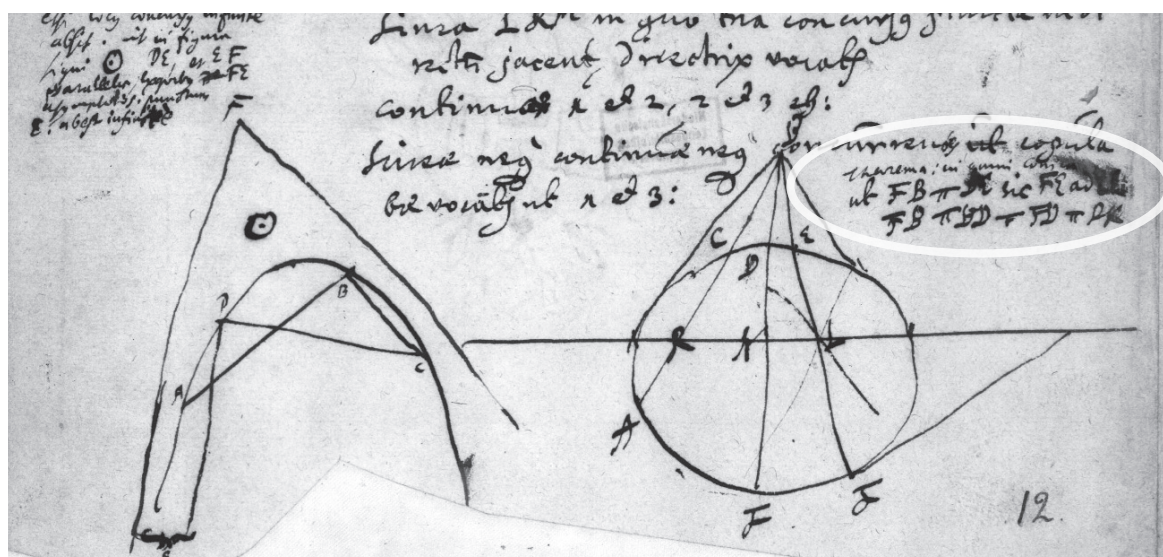
The image shows a handwritten manuscript page with several geometric diagrams of triangles and a handwritten text block. The text is in Dutch and describes a method for finding the length of a segment AF by bisecting an angle ACB. The diagrams show various configurations of triangles and points A, B, C, and F. The text includes a rule (Regel) and a proportion (Gelijck als) that relates the lengths of the sides and the segment AF. The rule is: $AC + BC$ sijn \square staet tot $\square AB$, multipl. in BC also het $\square AC$ tot het $\square AF$. The proportion is: $AC + BC$ sijn \square $\square AB$, multipl. in BC also het $\square AC$ tot het $\square AF$.



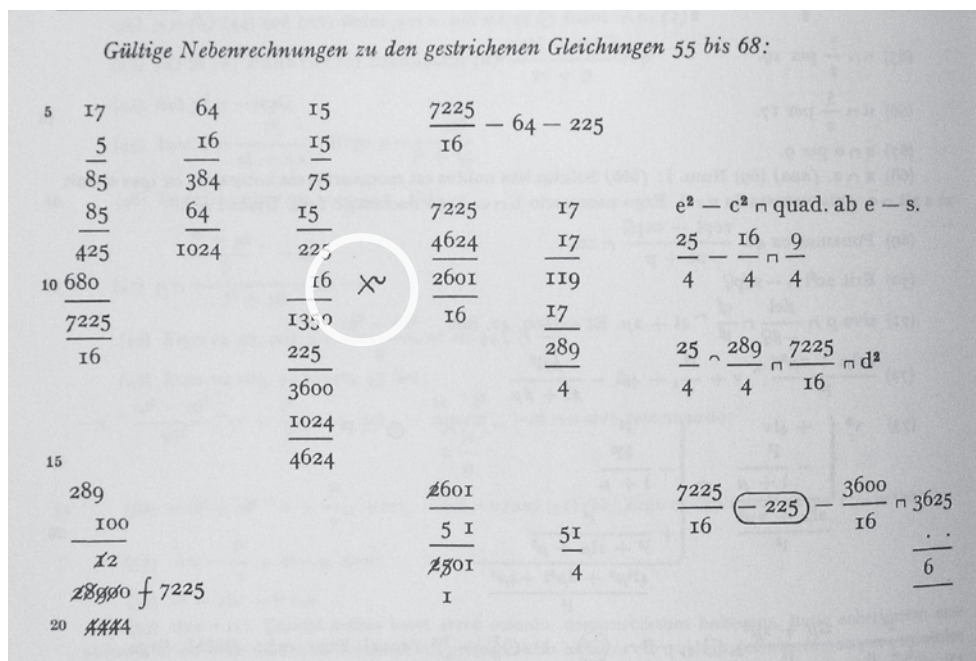
[Fig. 3]

Theorema: in omni Conica ut $FB \parallel BE$ sic FE ad $\langle EL \rangle$
 $GB \parallel BD \parallel GD \parallel DK$

⊢ PROPORTION WITH ONE STROKE, ∥ PROPORTION WITH TWO STROKES
 LAA VII-7 p. 578

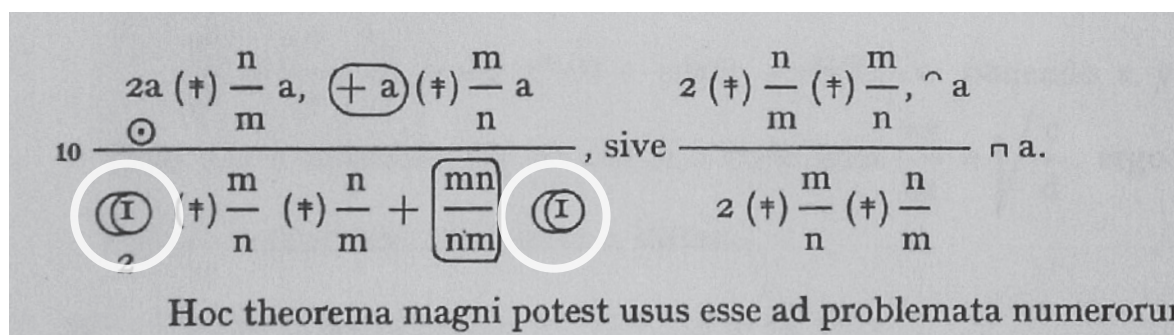
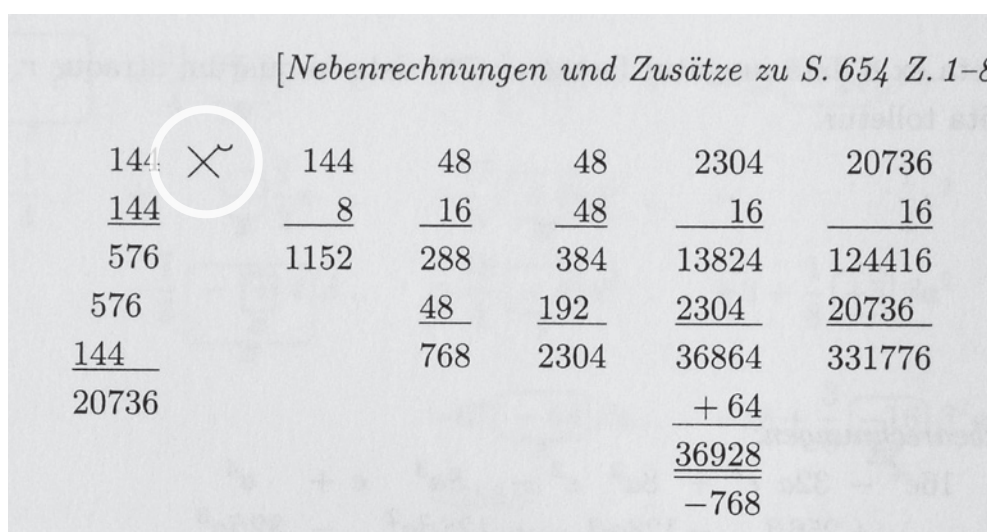


⊢ PROPORTION WITH ONE STROKE, ∥ PROPORTION WITH TWO STROKES
 LH 35 XV 1, fol. 12r



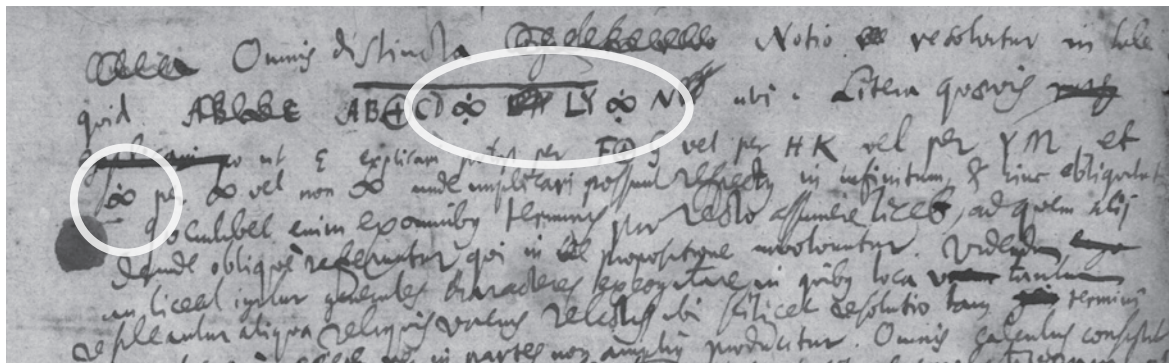
×~ CASTING-OUT-NINES

LAA VII-1 p. 408; VII-3 p.660 (below)



① LUNATE ENCIRCLED DIGIT ONE

LAA VII-1 p. 472



∞ INFINITY WITH DOTS

LH 4 VII B 2, fol. 73v (top), LAA VI-4 p. 873 (below)

Omnia ad haec videntur revocari posse. Aliquidditas. Essentia. Existentia. Realitas. 15
Perfectio. Uni[tas.] Convenientia. Veritas. Consequentia. Ordo. Causalitas. Mutatio.
Magnitudo. Sensus. Appetitus. Cogitatio. Qualitates Sensibiles.

⟨ − ⟩ in characteristica omnia distincte cogitabilia revocari possunt ad

$\overline{AB + CD} \overset{\infty}{\underset{\text{non } \infty}}{LM} \infty N$, hoc uno not(ato) ⟨ − − − ⟩ et contra explicari ⟨ − − − ⟩ quod
quaedam literae in ⟨ − ⟩ ut Y pro S pon(−) 20

Omnis distincta Notio resolvitur in tale quid $\overline{AB \oplus CD \infty LY \infty N}$ ubi Litera quaevis
ut E explicari potest per $F \oplus G$ vel per HK vel per YM et ∞ vel $\text{non } \infty$ unde
implicari possunt respectus in infinitum, et hinc obliquitat[es].

Quemlibet enim ex omnibus terminis pro recto assumere licet, ad quem alii deinde
oblique referuntur, qui in propositione involvuntur. Videndum an liceat igitur generales 25

		Rechenkunst.		73
		$D - E = 2B$		
		$D = E + 2B$		
		$\therefore D > E$		
		Aus D und F das übrig finden.		
$a = ?$	1	$a + b = D$		
$b = ?$	2	$ab = F$		
$i \odot 2$	3	$aa + 2ab + bb = DD$		
$i * 4$	4	$4ab = 4F$		
$3 - 4$	5	$aa - 2ab + bb = DD - 4F$		
$5 \sqrt{2}$	6	$a - b = \sqrt{DD - 4F}$		
		Weil $a + b$ Item $a - b$ bekant sind / so werr es ein überfluß weiter zuprocedieren / als da in den nächst hiervorstehenden auflösungen / die manier weiter zuschreiten / vor augen ligt.		
		Aus D und G.		
$a = ?$	1	$a + b = D$		
$b = ?$	2	$\frac{a}{b} = G$		

© INVOLVED – J. H. Rahn, Teutsche Algebra, 1659 (after Cajori).

In expressions of the form $a \odot b$, the sign \odot is used to denote the exponentiation of a by the power of b . In his “Teutsche Algebra” from 1659, the swiss mathematician Johann Heinrich Rahn refers to the operation as “involvierē” (= to involve).

		Involvierē in einfachen ungebrochenen quantiteten.				
		Das Hauptzeichen des Involvierens ist \odot heisset eingewickelt oder involviert.				
		Regel.				
		So vil mal eine quantitet erslich in sich selbs/darnach in ihr product / und drittens wider in das letztere product/2c. eingewickelt oder involviert wird ; so groß ist das vermögen besagter quantitet / so groß muß auch die zahl seyn die solchem zeichen in dem breiten reyen des nebenrands nachzusetzen ist.				
$i \odot 2$	1	$-a$	$++ab$	bcd	yx	zyy
$i \odot 3$	2	$++aa$	$++aabb$	$bbccdd$	$yyxx$	$4y^4$
$i \odot 4$	3	$-a^3$	$++a^3b^3$	$b^3c^3d^3$	y^3x^3	$8y^6$
	4	$++a^4$	$++a^4b^4$	$b^4c^4d^4$	y^4x^4	$16y^8$

© INVOLVED – J. H. Rahn, Teutsche Algebra, 1659.

In the time of Leibniz, the usual way of referring to curves or magnitudes is by giving equations that describe their specific relations. The concept of mapping as it is used in modern mathematics is not yet developed. Leibniz writes the signs \odot and $\boxed{\odot}$ to the right of an expression (such as $x \odot$ and $y+1, \boxed{\odot}$) in order to denote two different arbitrary rules by which the expressions given in the left position are treated. The result is an expression. By this, the meaning is similar to writing $f(x)$ or $g(y+1)$ in modern mathematical notation with f and g denoting arbitrary functions.

In a similar way, Johann Bernoulli uses the sign ϕ (see L-2535/N5335R) to denote a quantity depending on variables x and a (in modern terminology a function in x and a).

stantem numerum multiplicatam esse vel 1, vel multipulum facti ex denominatoribus duobus proximis, per numerum respondentem, ut 3. 35 etc. nempe:

Sunto duo termini: $\frac{b}{z \odot} \frac{b}{z+1, \odot}$ erit $b \frac{z+1, \odot, -z \odot}{z \odot, \wedge z+1, \odot} \sqcap \frac{1}{16z^2 - 16z + 3}$. Quod si nominator etiam sit inconstans, erunt termini $\frac{z \boxed{\odot}}{z \odot} \cdot \frac{z+\beta, \boxed{\odot}}{z+\beta, \odot}$ et fiet:

$$\frac{z+1, \odot, \wedge z \boxed{\odot}, -z \odot, \wedge z+\beta, \boxed{\odot}}{z \odot, \wedge z+\beta, \odot} \sqcap \frac{1}{16z^2 - 16z + 3}$$

Certum est semper destrui omnia quae non ducuntur in β . Sed hanc aequationem

\odot LEIBNIZIAN ENCIRCLED V, $\boxed{\odot}$ LEIBNIZIAN ENCIRCLED V IN BOX

LAA VII-1 p. 527 (above), LH 35 V 4, fol. 6 (below), LH 35 VIII 30, fol. 115r (bottom)

Investigamus paulo accuratius, post modum fieri possit, ut quicquid in differentibus dicitur aliter fiat.

Sic: $\frac{y \odot}{\boxed{\odot} y} - \frac{y+1, \odot}{y+1, \boxed{\odot}}$ unde $\frac{y \odot, y+1, \boxed{\odot} - y \boxed{\odot}, y+1, \odot}{\boxed{\odot} y, y+1, \boxed{\odot}}$

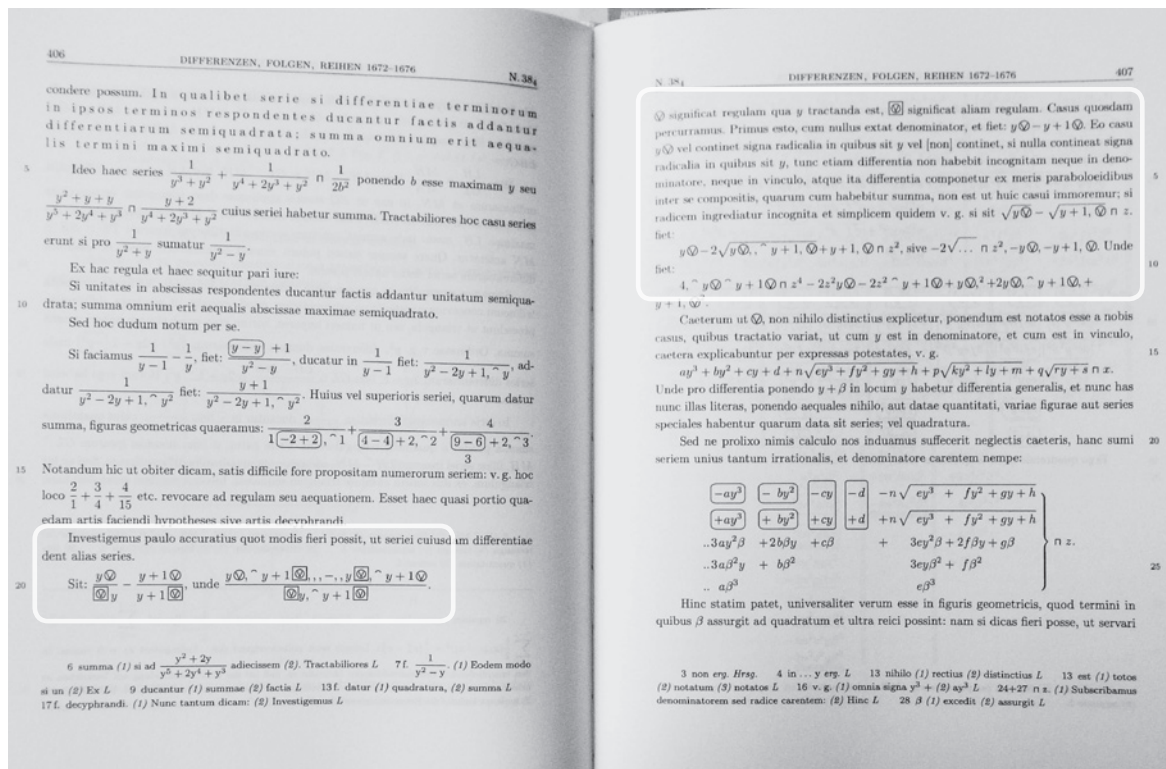
Observat regulas quae y tractantur. $\boxed{\odot}$ significat aliam regulam. Cuius quidem periculum. primum quod nullus est denominator et fieri: $y \odot - y+1, \odot$. Et casu $y \odot$ vel continet signa radicalia in quibus y vel continet si nulla sit talis signa radicalia tunc etiam differentia nulla est. unde habebit innumerabilem alq. in denominatoribus, nec differentia componetur ex mensurabilibus.

vel multipulum facti ex denominatoribus duobus proximis ipse numerus respondentem, ut 3. 35 etc. nempe:

Sunto duo termini: $\frac{b}{2 \odot} \frac{b}{2+1, \odot}$ erit $b \frac{2+1, \odot, -2 \odot}{2 \odot, \wedge 2+1, \odot} \sqcap \frac{1}{16z^2 - 16z + 3}$

Composui, quod erunt termini $\frac{z \boxed{\odot}}{z \odot} \cdot \frac{z+\beta, \boxed{\odot}}{z+\beta, \odot}$ et fiet: $\frac{z \odot, \wedge z+\beta, \boxed{\odot} - z \boxed{\odot}, \wedge z+\beta, \odot}{z \odot, \wedge z+\beta, \odot} \sqcap \frac{1}{16z^2 - 16z + 3}$

Equationem resolvere per hanc methodum. Sicut an sit in potestate humana, facilius quidem, nisi una methodus regeretur, per diversa successat. Cumque huius sit per methodum universalem ipse determinari non potest, quare ea est methodus in qua equationes omnes et omnia huiusmodi equationes incognita per se

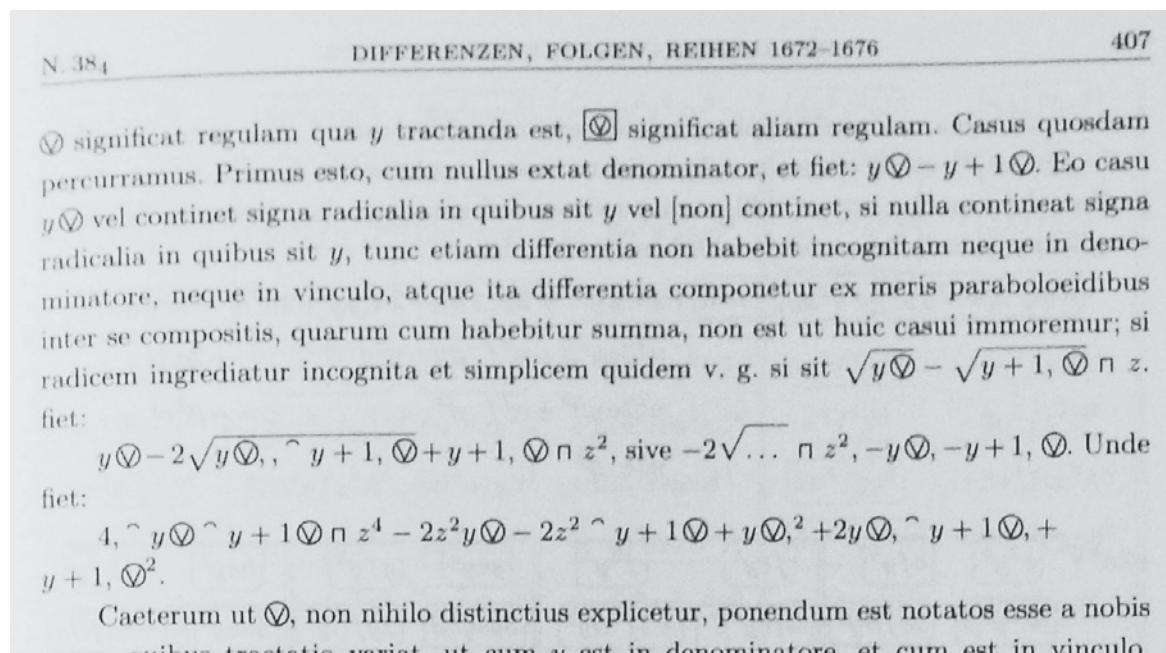


© LEIBNIZIAN ENCIRCLED V, ☉ LEIBNIZIAN ENCIRCLED V IN BOX

The typographical solution of these characters in the edition is bad. In fact the round shape has to be connected with the v, similar as in @ (0040). – LAA VII-3 p. 406-407

Investigemus paulo accuratius quot modis fieri possit, ut seriei cuiusdam differentiae dent alias series.

$$\text{Sit: } \frac{y \textcircled{v}}{\textcircled{v}y} - \frac{y+1 \textcircled{v}}{y+1 \textcircled{v}}, \text{ unde } \frac{y \textcircled{v}, \wedge y+1 \textcircled{v}, , , -, , y \textcircled{v}, \wedge y+1 \textcircled{v}}{\textcircled{v}y, \wedge y+1 \textcircled{v}}.$$



5. Unicode Character Properties

```
xf01;CASTING-OUT-NINES;Sm;0;ON;;;;;N;;;;;
xf02;LUNATE ENCIRCLED DIGIT ONE;Sm;0;ON;;;;;N;;;;;
xf03;PROPORTION WITH ONE STROKE;Sm;0;ON;;;;;N;;;;;
xf04;PROPORTION WITH TWO STROKES;Sm;0;ON;;;;;N;;;;;
xf05;INFINITY WITH TWO DOTS;Sm;0;ON;;;;;N;;;;;
xf06;INVOLVED;Sm;0;ON;;;;;N;;;;;
xf07;LEIBNIZIAN ENCIRCLED V;Sm;0;ON;;;;;N;;;;;
xf08;LEIBNIZIAN ENCIRCLED V IN BOX;Sm;0;ON;;;;;N;;;;;
xf09;SUPERSCRIT WAVE SYMBOL;Sm;0;ON;;;;;N;;;;;
xf10;SUPERSCRIT WAVE WITH OVERLINE SYMBOL;Sm;0;ON;;;;;N;;;;;
```

6. Bibliography

LAA – refers to: Leibniz, Gottfried Wilhelm: Sämtliche Schriften und Briefe. (‘Leibniz-Akademie-Ausgabe’, many volumes)

LH – refers to: Leibniz’s original manuscripts, GWLB Hanover

Bombelli, Rafael: L’Algebra. Bologna 1579

— : L’Algebra. Milan 1966

Cajori, Florian: A history of mathematical notations. Chicago 1928

Probst, Siegmund: Édition des symboles de Leibniz. PDF. Hanover 2023 (presentation Paris 2023)

Rahn, Johann Heinrich: Teutsche Algebra. Zurich 1659

Rinner, Elisabeth: List of glyphs in Leib.mf. PDF. Hanover 2022

**ISO/IEC JTC 1/SC 2/WG 2
PROPOSAL SUMMARY FORM TO ACCOMPANY SUBMISSIONS
FOR ADDITIONS TO THE REPERTOIRE OF ISO/IEC 10646¹**

Please fill all the sections A, B and C below.

Please read Principles and Procedures Document (P & P) from
<http://std.dkuug.dk/JTC1/SC2/WG2/docs/principles.html> for guidelines and details before filling this form.

Please ensure you are using the latest Form from
<http://std.dkuug.dk/JTC1/SC2/WG2/docs/summaryform.html>.

See also <http://std.dkuug.dk/JTC1/SC2/WG2/docs/roadmaps.html> for latest *Roadmaps*.

A. Administrative

1. Title:	Proposal to encode 21 miscellaneous scientific symbols		
2. Requester's name:	Uwe Mayer, Siegmund Probst, David Rabouin, Elisabeth Rinner, Andreas Stötzner, Achim Trunk, Charlotte Wahl		
3. Requester type (Member body/Liaison/Individual contribution):	Individual (work group)		
4. Submission date:	2026-01-19.		
5. Requester's reference (if applicable):	LUCP L-2604		
6. Choose one of the following:			
This is a complete proposal:			Yes
(or) More information will be provided later:			

B. Technical – General

1. Choose one of the following:			
a. This proposal is for a new script (set of characters):			No
Proposed name of script:			
b. The proposal is for addition of character(s) to an existing block:			No
Name of the existing block:	[Miscellaneous Symbols Supplement]		
2. Number of characters in proposal:			10
3. Proposed category (select one from below - see section 2.2 of P&P document):			
A-Contemporary	B.1-Specialized (small collection)	Yes	B.2-Specialized (large collection)
C-Major extinct	D-Attested extinct		E-Minor extinct
F-Archaic Hieroglyphic or Ideographic			G-Obscure or questionable usage symbols
4. Is a repertoire including character names provided?			Yes
a. If YES, are the names in accordance with the “character naming guidelines” in Annex L of P&P document?			Yes
b. Are the character shapes attached in a legible form suitable for review?			Yes
5. Fonts related:			
a. Who will provide the appropriate computerized font to the Project Editor of 10646 for publishing the standard?	Andreas Stötzner		
b. Identify the party granting a license for use of the font by the editors (include address, e-mail, ftp-site, etc.):	Andreas Stötzner Gestaltung, Klaufügelweg 21, 88400 Biberach/R., Germany, as@signographie.de		
6. References:			
a. Are references (to other character sets, dictionaries, descriptive texts etc.) provided?			Yes
b. Are published examples of use (such as samples from newspapers, magazines, or other sources) of proposed characters attached?			Yes
7. Special encoding issues:			
Does the proposal address other aspects of character data processing (if applicable) such as input, presentation, sorting, searching, indexing, transliteration etc. (if yes please enclose information)?			No

¹ Form number: N4502-F (Original 1994-10-14; Revised 1995-01, 1995-04, 1996-04, 1996-08, 1999-03, 2001-05, 2001-09, 2003-11, 2005-01, 2005-09, 2005-10, 2007-03, 2008-05, 2009-11, 2011-03, 2012-01)

C. Technical - Justification

1. Has this proposal for addition of character(s) been submitted before?	Yes
If YES explain	see N5336 (L-2536, L-2521), N5277 (L-2402n)
2. Has contact been made to members of the user community (for example: National Body, user groups of the script or characters, other experts, etc.)?	Yes
If YES, with whom?	Leibniz-Archiv, Forschungsstelle der Leibniz-Edition, Niedersächsische Landesbibliothek (GWLb), Hanover, Göttingen Academy of Science and Humanities in Lower Saxony (DE), Philium research group of CNRS (UMR 7219, laboratoire SPHERE) / Université de Paris VII; general: scholars, researchers, authors and editors working in the field of science history and upon editions of historic text corpora (e.g. of G. W. Leibniz, but also many others)
If YES, available relevant documents:	L-2409, L-2410
3. Information on the user community for the proposed characters (for example: size, demographics, information technology use, or publishing use) is included?	Yes
Reference:	
4. The context of use for the proposed characters (type of use; common or rare)	Common
Reference:	mainly specialist usage, scholarly, worldwide
5. Are the proposed characters in current use by the user community?	Yes
If YES, where? Reference:	mainly Europe, Americas; other countries
6. After giving due considerations to the principles in the P&P document must the proposed characters be entirely in the BMP?	No
If YES, is a rationale provided?	
If YES, reference:	
7. Should the proposed characters be kept together in a contiguous range (rather than being scattered)?	No
8. Can any of the proposed characters be considered a presentation form of an existing character or character sequence?	No
If YES, is a rationale for its inclusion provided?	
If YES, reference:	
9. Can any of the proposed characters be encoded using a composed character sequence of either existing characters or other proposed characters?	No
If YES, is a rationale for its inclusion provided?	
If YES, reference:	
10. Can any of the proposed character(s) be considered to be similar (in appearance or function) to, or could be confused with, an existing character?	No
If YES, is a rationale for its inclusion provided?	
If YES, reference:	
11. Does the proposal include use of combining characters and/or use of composite sequences?	No
If YES, is a rationale for such use provided?	
If YES, reference:	
Is a list of composite sequences and their corresponding glyph images (graphic symbols) provided?	No
If YES, reference:	
12. Does the proposal contain characters with any special properties such as control function or similar semantics?	No
If YES, describe in detail (include attachment if necessary)	
13. Does the proposal contain any Ideographic compatibility characters?	No
If YES, are the equivalent corresponding unified ideographic characters identified?	
If YES, reference:	